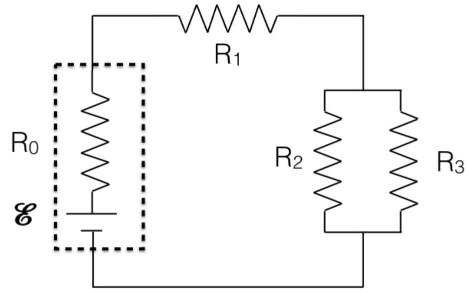
Exam 2, Phy 2049, Spring 2017. Solutions:

1. A battery, which has an emf of EMF = 10V and an internal resistance of $R_0 = 50\Omega$, is connected to three resistors, as shown in the figure. The resistors have the resistances of $R_1 = 25\Omega$, $R_2 = 50\Omega = R_3$. What is the current through R_3, in mA?



(1) 50 (2) 80 (3) 125 (4) 10 (5) 180

Solution:

First, we need to calculate the equivalent resistance seen by the voltage source. Then we can calculate the current going through the circuit. From the values of the parallel resistors R_2 and R_3 we realize that each of them will let half of that current through. Equivalent resistance:

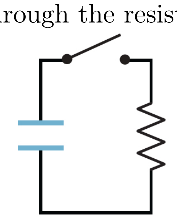
$$R_{eq} = R_0 + R_1 + R_2||R_3$$

$$R_2||R_3 = \frac{R_2R_3}{R_2 + R_3} = 25\Omega \quad \Rightarrow \quad R_{eq} = 100\Omega$$

So the total current and the current through R_3 are

$$I_0 = \frac{V_0}{R_{eq}} = 0.1\text{A} \quad \Rightarrow \quad I_{R3} = I_0/2 = 50\,\text{mA}$$

2. Consider an RC circuit with $R = 100\,\Omega$ and $C = 10\mu$F, shown in the figure. The initial charge on the capacitor is $Q = 1\mu$C. At time t = 0, the switch is closed. What is the magnitude of the current (in mA) through the resistor at $t = 500\mu$s?



(1) 0.61 (2) 1.35 (3) 1.0 (4) 0.25 (5) 2.2

Solution: The charge on the capacitor is exponentially decaying

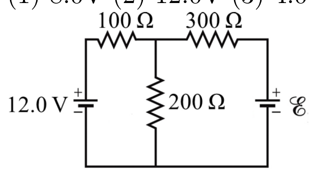
$$q(t) = Q_0 e^{-t/\tau} \qquad \tau = RC = 100\Omega 10^{-5}F = 1\text{ms}$$

The current is

$$i(t = 500\mu\text{s}) = \frac{dq(t = 500\mu s)}{dt} = -\frac{Q_0}{\tau}e^{-t/\tau} = -\frac{10^{-6}\text{C}}{10^{-3}\text{s}}e^{-0.5} = -0.61\,\text{mA}$$

3. In the circuit shown, find the EMF E that makes the current through the 300 Ω resistor zero.

(1) 8.0V (2) 12.0V (3) 4.0V (4) 10.0V (5) 6.0V



Solution: You can use Kirchhoff's laws (loop rule and current law) to set up a system of equations and then solve it. But the easier way is to realize that when the current through the 300 Ω resistor is zero, then the current through the 100 Ω and 200 Ω resistors has to be equal. If that is the case, the voltage differences across these two resistors is:

$$I = \frac{V_{100\Omega}}{100\Omega} = \frac{V_{200\Omega}}{200\Omega} \quad \Rightarrow \quad V_{200\Omega} = 2 \cdot V_{100\Omega}$$

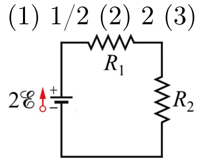
We need twice as much voltage across the two times larger resistor. Still, from Kirchhoff's loop rule:

$$V_{200\Omega} + V_{100\Omega} = 12V$$

That means, that the voltage drop across $V_{200\Omega} = 8V$. And our initial assumption that no current goes through the 300Ω resistor means that the EMF E on the other side has to be 8V as well.

4. When resistors R_1 and R_2 are connected separately, one at a time, to a battery whose emf is E, R_1 dissipates twice as much power as R_2. When the resistors are connected in series to another battery whose emf is $2E$, as shown, R_1 dissipates a power P_1, and R_2 dissipates a power P_2. What is the ratio P_1/P_2? Assume that the batteries have no internal resistance.

(1) 1/2 (2) 2 (3) 1 (4) 4 (5) $\sqrt{2}$



Solution: Power dissipation in general is equal to

$$P = V \cdot I = \frac{V^2}{R} \qquad \text{used:} \qquad V = RI$$

So if R_1 dissipates twice as much power as R_2 when they are connected individually to the same emf, then $2R_1 = R_2$ or R_1 has to be half as large as R_2 (the smaller resistor lets more current through for the same voltage, so it dissipates more power).

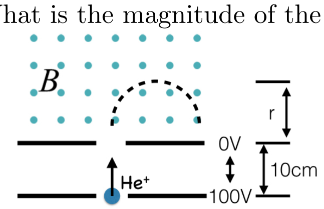
If we connect them in series, the current through both of them is equal but the voltage across the two times smaller resistor is two times smaller compared to the voltage across the larger resistor:

$$2V_1 = V_2$$

If we now look at the power dissipated:

$$P_1 = \frac{V_1{}^2}{R_1} \qquad P_2 = \frac{V_2^2}{R_2} = \frac{4V_1^2}{2R_1} = 2P_1$$

5. A helium ion He^+ is first accelerated by a potential difference of 100V over a distance of 10 cm before it enters a region of uniform magnetic field pointing perpendicular to the path of the ion. There is no electric field in that region. The magnetic field causes the ion to undergo a circular motion with radius $r = 10\,\text{cm}$. What is the magnitude of the magnetic field in mT? The mass of a helium ion is $m = 6.64 \times 10^{-27}$kg.



(1) 28.8 (2) 127 (3) 54.3 (4) 92.7 (5) 4.7

Solution: The electric field accelerates the helium ion to an energy of 100eV. From that, we can calculate the velocity. From the relation between the force of the magnetic field on a moving charge and the centrifugal force, we can get a relation between the B-field and the radius.

$$U_K = \frac{1}{2} m_{He} v^2 = 100eV \qquad \Rightarrow \qquad v = \sqrt{\frac{2 \cdot 100eV}{m_{He}}} = \sqrt{\frac{200 \times 1.6 \times 10^{-19}\text{J}}{6.64 \times 10^{-27} kg}} = 69421\text{m/s}$$

The motion in the B-field region is then described by

$$m\frac{v^2}{r} = qvB \qquad \Rightarrow \qquad B = \frac{m}{q}\frac{v}{r} = 28.8\,\text{mT}$$

6. A 10-turn circular loop of wire of area $200\,\text{cm}^2$ carries a current of 10 A. The magnetic dipole moment of the loop is oriented at an angle 50° to a uniform magnetic field of 4T. How much work was required to rotate the loop from its lowest energy position to this orientation?
(1) 2.9 J (2) 35.1 J (3) 4.3 J (4) 1.28 J (5) 17.4 J

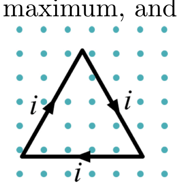
Solution: The magnetic moment of the loop is μ = NiA. The potential energy of a loop in a magnetic field U = −μB cos(ϑ). The work required to move from the lowest energy configuration, ϑ = 0 and U = −μB, to the configuration with ϑ = 50◦ and U = −μB cos(50) is W = μB(1 − cos(50)). In numbers:

$$\mu = NiA = 10 \cdot 0.02\,\text{m}^2 \cdot 10\text{A} = 2\text{Am}^2 \qquad \mu B = 8\,\text{J} \qquad \cos(50^\circ) = 0.64$$

Which gives:

$$W = \mu B\,(1 - 0.64) = 2.9\,\text{J}$$

7. A wire loop, which has the shape of an equilateral triangle, is placed in a uniform magnetic field, with the plane of the triangle perpendicular to the field such that the field points out of the plane, as shown in the figure. The loop carries a current i in the direction indicated by arrows. There is no gravity, and the loop is free to move. Which of the following statements is correct about the magnitude of the torque, τ, on the loop and the potential energy U of the loop?
(1) is zero, and U is maximum. (2) is zero, and U is minimum. (3) is maximum, and U is minimum. (4) is maximum, and U is maximum. (5) is maximum, and U is zero.

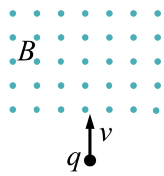


Solution: See for example Fig 28-20 in the book. The magnetic moment $\vec{\mu}$ points into the plane of the triangle (right hand rule). So it is anti-parallel with the B-field. Equations:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = 0 \qquad U = -\vec{\mu} \cdot \vec{B} = \mu B$$

The cross product of anti-parallel vectors is zero, the dot product is negative with max magnitude. The additional - sign makes this the maximum energy. You can also look at the directions of the forces on each wire and how they would turn the triangle around by 180deg.

8. Travelling upward in the plane of the page at speed v, a particle enters a region of uniform magnetic field B, which points out of the plane of the page, as shown in the figure. The particle has mass m and positive charge q. What is the uniform electric field E that is required to make the particle travel on a straight trajectory in this region?



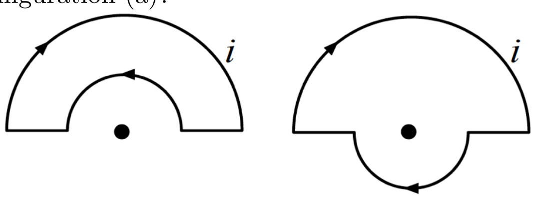
(1) E = vB pointing to the left. (2) E = vB pointing to the right. (3) E = qvB/m pointing to the left. (4) E = qvB/m pointing to the right. (5) There is no such E.

Solution: Look at

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

The E-field has to have a magnitude equal to vB. The magnetic force $\vec{v} \times \vec{B}$ points to the right, so E has to point to the left.

9. The figure shows two configurations of loops with identical currents. Each loop consists of a larger semicircle of radius 20 cm, a smaller concentric semicircle of radius 13.3 cm, and two straight segments, all in the same plane. The magnitude of the magnetic field produced at the center marked by the dot is 25 µT in configuration (b). What is the magnitude of the magnetic field at the center marked by the dot in configuration (a)?



(a) (b)

(1) 5 μT (2) 10 μT (3) 2.0 μT (4) 25 μT (5) 50 μT

Solution: We know the equations for currents of an arc and superposition tells us that we can simply add the two B-fields produced by the two arcs. The straight sections do not create any B-field at the marked spot. In case a, the B-fields from the two contributions point in opposite directions, in case b, they point in the same direction. General equation to use:

$$B = \frac{\mu_0 i}{4\pi R}\pi$$

which describes the B-field of a semi-circle of wire with current i and radius R. In case (a) and (b), the net fields will be:

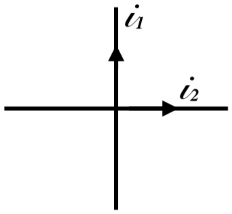
$$\frac{\mu_0 i}{4}\left(\frac{1}{r_{in}} - \frac{1}{r_{out}}\right) = B_a \qquad \frac{\mu_0 i}{4}\left(\frac{1}{r_{in}} + \frac{1}{r_{out}}\right) = B_b$$

Dividing both equations gives:

$$\frac{B_b}{B_a} = \frac{\left(\frac{1}{r_{in}} + \frac{1}{r_{out}}\right)}{\left(\frac{1}{r_{in}} - \frac{1}{r_{out}}\right)} = \frac{r_{in} + r_{out}}{r_{in} - r_{out}} = \frac{33.3}{6.7} = 5$$

So $B_a = 5\mu$T

10. Two long straight current-carrying wires cross each other at an angle of 90°, as shown in the figure. The arrows indicate the directions of the currents. What are the directions of the forces on the upper and lower sections of wire 1, which carries a current i_1? The answers are given for the upper section first, the lower section second after a semicolon.



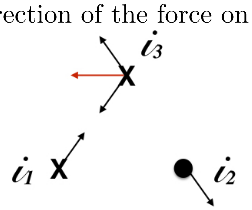
Upper section first/lower section second

(1) to the right/to the left (2) to the right/to the right (3) 0 (4) to the left/to the right (5) to the left/to the left

Solution: Step one is to figure out the direction of the B-field created by current i_2. Above wire 2 (upper section of wire 1), the B-field comes out of the plane. Below wire 2 (lower section of wire 1), the B-field goes into the plane. Now we need the right hand rule to calculate the direction of $\vec{v} \times \vec{B}$ where $\vec{v}$ is in the direction of the current. The force on the upper part of wire 1 points to the right, the one on the lower part to the left.

Another way to get the same result faster: Remember that parallel currents attract each other and anti-parallel currents repel. This also means that two currents want to be parallel to each other and the forces will have to point in ways that would rotate wire 1 such that it would end up being parallel to wire 2.

11. Three long straight current-carrying wires pierce the plane of the page at vertices of an equilateral triangle, as shown in the figure, with a 5 cm separation between the wires. Currents i_1 and i_3 in wires 1 and 3 go into the plane, whereas current i_2 in wire 2 comes out of the plane. Each current is 1A. What is the direction of the force on wire 3, which lies above wires 1 and 2?

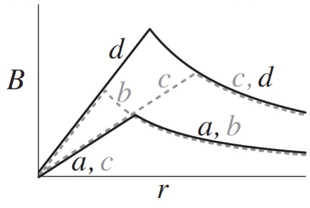


(1) to the left (2) up (3) 0 (4) down (5) to the right

Solution: Current carrying wires attract each other when the current goes in the same direction and repel each other when they go into the opposite direction. As the distances and currents are all identical, the magnitude of the force from wire 1 on wire 3 will be equal to the magnitude of the force from wire 2 on wire 1. These two forces are shown as black arrows attached to wire 3 in the drawing. Their vector addition generates the red arrow pointing to the left.

12. The figure gives, as a function of radial distance r, the magnitude B of the magnetic field inside and outside four long straight wires (a, b, c, and d), each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots are indicated by double labels. Rank the wires according to the value of the current, greatest first.

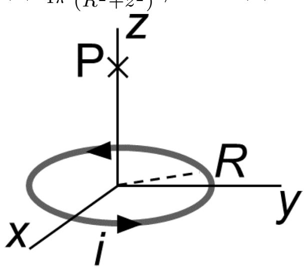
(1) c = d > a = b (2) d > c > b > a (3) b = d > a = c (4) c > d > a > b (5) c > d = b > a



Solution: The B-field outside a current carrying wire only depends on the current itself and not on the radius of the wire. So we look at the region larger than the wire radii where the B-field falls with 1/r. The magnitude at large r will then only depend on the currents. That means that the currents in cases c and d are equal and larger than the currents in cases a and b which are also equal to each other.

13. A circular loop of radius R, carrying a current i, lies flat on the xy plane, as shown. The z axis runs through the center of the loop. Consider a point P on the z axis, at distance z from the center. At this point, what is the magnitude of the z component of the infinitesimal magnetic field due to the current in a very short segment of the loop of length ds?

(1) $\frac{\mu_0}{4\pi}\frac{iR}{(R^2+z^2)^{3/2}}ds$ (2) 0 (3) $\frac{\mu_0}{4\pi}\frac{iz}{(R^2+z^2)^{3/2}}ds$ (4) $\frac{\mu_0}{4\pi}\frac{iR}{R^2+z^2}ds$ (5) $\frac{\mu_0}{4\pi}\frac{iRz}{(R^2+z^2)^2}ds$

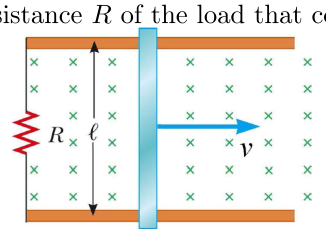


Solution: The hard way is to look at the Biot-Savart law and rewrite the integrand. The better way is to think about what you know about the B-field to exlude the other solutions. 1. The B field of a loop current points up at P = (0,0,z). But it also points up at (0.0,-z), the position on the z-axis below the loop. That means that the solution can not change sign if we replace z with (-z). This excludes solutions 3 and 5. Answer 2 is also expluded because the field is not zero. We also know that. That leaves solutions 1 and 4. From the B-field inside the loop at the center:

$$B = \frac{\mu_0 i}{2R}$$

we know that the units of solution (4) will not work out. That leaves solution 1 as the only possible solution.

14. As shown in the figure, a sliding bar is pulled at constant speed v on two conducting rails, which are separated by distance $l = 20\,\text{cm}$. A uniform magnetic field of 2.0 T is directed into the page, as indicated by crosses. What is the work (in mJ) required to pull the bar over a distance of 40 cm at $v = 25\text{cm/s}$, if the resistance R of the load that connects the two rails is 50Ω ?



(1) 0.32 (2) 0.11 (3) 2.7 (4) 12 (5) 74

Solution: The work is $W = F \cdot d$. The force, we have to work against is the force on the wire caused by the current through the resistor R. This current is generated by the induced EMF. This allows us to derive a plan to solve this problem. First, we can calculate the EMF. From that, we get the curent through the bar. And then we have the force on the bar.

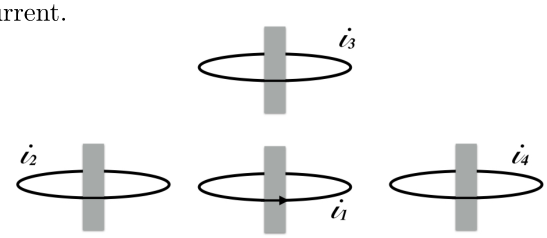
$$EMF = \frac{d\Phi_B}{dt} = Blv = 2.0\text{T} \cdot 0.2\text{m} \cdot 0.25\text{m/s} = 0.1\text{V} \qquad \Rightarrow \qquad i = EMF/R = 2\,\text{mA}$$

The force on the bar due to the current and the magnetic field is:

$$F_b = ilB = 2\,\text{mA} \cdot 0.2\,\text{m} \cdot 2.0\,\text{T} = 0.8\,\text{mN} \qquad \Rightarrow \qquad W = F_b d = 0.8\text{mN} \cdot 0.4\,\text{m} = 0.32\,\text{mJ}$$

15. Current i_1 flows through the circular loop of wire in the center, as shown in the figure. The direction of the current is counterclockwise, when viewed from above. (The grey bars, which indicate the central axes of the loops, have been added to help you visualize the geometry of the problem.) If this current increases with

time, what are the directions of the currents — i_2, i_3, i_4 — induced in the three adjacent circular loops of wire? Loop 3, whose current is i_3, is located above loop 1, whose current is i_1. In the answers, “cw” stands for clockwise, and “ccw” for counterclockwise, both when viewed from above, and “0” means no induced current.



(1) ccw, cw, ccw (2) 0, cw, 0 (3) cw, cw, cw (4) cw, 0, ccw (5) cw, ccw, cw

Solution: The key here is to understand how the B-field ’flows’ in a dipol. The B-field points up inside the loop (see problem 13 for this as well) and also in the loop above it (current i_3). It then curves around and points downward in the loops with currents i_2 and i_4. If we increase the counterclockwise current i_1, the induced current i_3 in the upper loop has to go the other way (cw) to reduce the change in flux again. The other two currents have to ccw to create a B-field which points upwards.

16. A 2.5 mH inductor is connected in series with a resistor and a 10 V battery. After the current reaches its maximum value, the energy stored in the magnetic field of the inductor is 0.8 mJ. What is the resistance of the circuit (in ohms)?

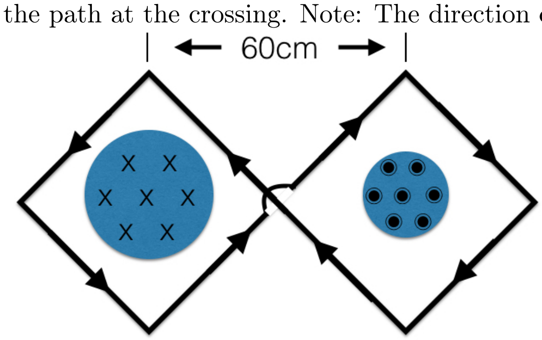
(1) 12.5 (2) 10.0 (3) 2.83 (4) 2.25 (5) 7.3

Solution:

$$U_B = \frac{1}{2}Li^2 = 80\,\text{mJ} \qquad i = \sqrt{\frac{2 \cdot 0.8\text{mJ}}{2.5\text{mH}}} = \sqrt{\frac{1.6\text{mJ}}{2.5\text{mH}}} = \frac{4}{5}A = 0.8\text{A}$$

$$R = \frac{V}{I} = \frac{10V}{0.8A} = 12.5\Omega$$

17. The figure shows two circular regions — region 1 of radius $r_1 = 15\,$cm and region 2 of radius $r_2 = 10\,$cm — separated by 60 cm. The magnetic field in region 1 is 50mT going into the plane of the page, and that in region 2 is 30mT coming out of the plane of the page. The magnitudes of both fields are decreasing at a rate of 5mT/s. Calculate the path integral $\oint \vec{E} \cdot d\vec{s}$ along the path drawn in the figure; ignore the bending of the path at the crossing. Note: The direction of the path integral matters.



(1) $-0.51\,$mV (2) $0.32\,$mV (3) $2.3\,$mV (4) $-76\,$mV (5) 0

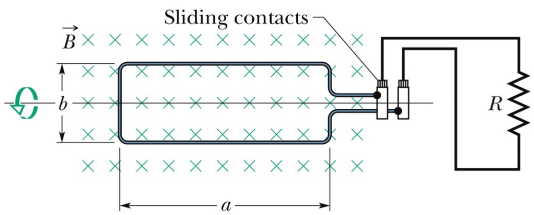
Solution: The first step to solving this is to recall that the path integral over a closed path over the electric field is equal to the change in magnetic flux through the closed path. The second step is to remember that this depends on the direction of the change in flux but also on the direction through the loop:

$$\oint \vec{E} d\vec{s} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}$$

In this particular case, the left part will induce an E-field which 'rotates' clockwise around the decreasing magnetic flux. This is against the direction of integration and the result will be negative. In the right half, the E-field rotates counterclockwise to the decreasing magntic flux which is again against the direction of integration. Both contributions will be negative.

$$\oint \vec{E}d\vec{s} = -\pi\frac{dB}{dt}\left(r_1^2 + r_2^2\right) = -0.51\text{mV}$$

18. A rectangular coil of 10 turns and of length $a = 10\,\text{cm}$ and width $b = 5\,\text{cm}$ is rotated 60 times per second in a uniform magnetic field $\vec{B}$ as indicated in the figure. The coil is connected to co-rotating cylinders, against which metal brushes slide to make electrical contact. If the amplitude of the resulting AC current through the resistor $R = 1\,\text{k}\Omega$ is $1\,\text{mA}$, what is the magnitude of the magnetic field $\vec{B}$? The resistance of the coil is negligible.



(1) 530 mT (2) 120 mT (3) 200 mT (4) 640 mT (5) 63 mT

Solution:

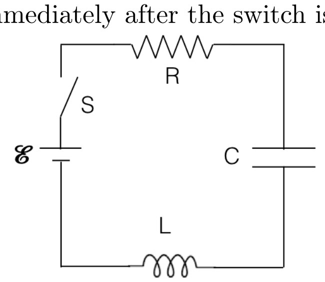
The EMF is:

$$EMF = RI = 1V\cos\omega t \qquad EMF = 2\pi fBA\cos 2\pi ft$$

So we solve for B:

$$B = \frac{EMF_{max}}{2\pi fA} = \frac{1V}{2\pi 60\text{Hz}\cdot 0.005\text{m}^2} = 530\text{mT}$$

19. A constant emf of 10V is connected through switch S to a series RLC circuit with $R = 10\,\Omega$, $L = 20\,\text{mH}$, and $C = 500\,\mu\text{F}$. Across which of the three elements will the potential difference be the largest (a) immediately after the switch is closed, and (b) a long time later?

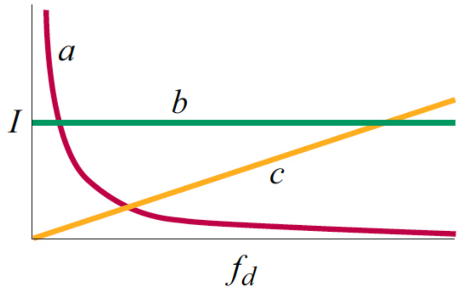


(1) L, C (2) C, R (3) R, L (4) L, R (5) R, C

Solution: This is similar to an HITT question we had. Let's start with part (b). A long time after the switch is closed, the capacitor will be charged up to the 10V coming from the emf and resist any further current flow. So C is the correct answer for part (b). When the switch is turned on, the emf tries to increase the current through the inductor rapidly. The inductor pushes back with an induced voltage which is initially equal to the emf. So L will see the potential difference immediately after the switch is closed.

20. An alternating emf source with a certain emf amplitude is connected, in turn, to a resistor R, a capacitor C, and then an inductor L. Once connected to one of these three elements, the driving frequency f_d is varied and the amplitude I of the resulting current through the element is measured and plotted. Which of the three plots — labeled a, b, c — in the figure corresponds to which of the three elements? (In the answers,

the label for a plot is followed by an equal sign, then by the symbol — R, C, or L— for the corresponding element.)



Solution: This part looks at the three reactances.

$$X_R = R \qquad X_C = \frac{1}{\omega C} \qquad X_L = \omega L$$

or how the amount of current that we can get through a resistor, a capacitor, and an inductor changes with frequency for a given oscillating emf:

$$I_R = \frac{EMF}{R} \qquad I_C = EMF \cdot \omega C \qquad I_L = \frac{EMF}{\omega L}$$

Which shows the frequency dependence of the current.
In words: A resistor is frequency independent. So (b) has to be the resistor R. A capacitor does not let any current at DC (directed current) through. So in a current - frequency plot, the curve has to start at 0 (no current) when the frequency is 0. So curve c has to be the capacitor. We can also argue that the inductor does not allow fast current changes. So it will reduce the amplitude of any oscillating current at high frequencies.