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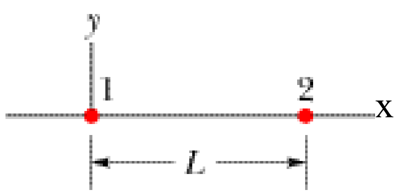
Exam 1 Solutions

Note: Most problems have more than one version with different answers. Be careful that you check your exam against your version of the problem.

1. Two charges, $q_1 = -1$ C and $q_2 = -4$ C *(or vice versa)*, are placed along the x-axis a distance L apart with charge q_1 at the origin and q_2 at x=L (see figure). A third charge, $q_3 = +4/9$ C, is also placed along the x-axis such that there is no net Coulomb force on any of the charges. What is position of this charge along the x axis in units of L, i.e. what is x/L?

(1) 1/3 (2) 2/3 (3) 1/2 (4) 4/3 (5) -2/3



Solution:

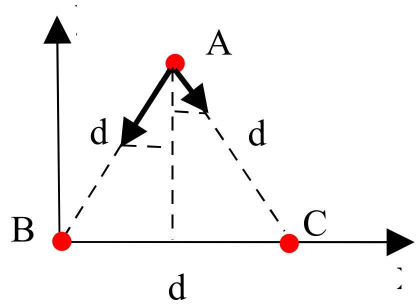
The only place to place a third charge that is positive is between the two negative charges. If there is no net Coulomb force, we must satisfy for charge 1 the following:

$$F_x = k\frac{|q_1 q_3|}{x^2} - k\frac{|q_1 q_2|}{L^2} = 0$$

$$\frac{x^2}{L^2} = \frac{|q_3|}{|q_2|} \qquad \frac{x}{L} = \sqrt{\frac{4/9}{4}} = \frac{1}{3}$$

If q_1 and q_2 are swapped, then the answer would be 2/3.

2. Three charges form an equilateral triangle of side length d = 20cm as shown in the figure. If q_A = -1 nC, q_B=+2nC, and q_C=+1nC what is the horizontal x component *(or y component)* of the net electrostatic force on particle A?
(1) -1.13 × 10^{-7} N
(2) -5.85 × 10^{-7} N
(3) -1.95 × 10^{-7} N
(4) -2.25 × 10^{-7} N
(5) 0 N



Solution:

Work out the x and y components of the force on particle A:

$$F_x = -K\frac{|q_A q_B|}{d^2}\sin 30° + K\frac{|q_A q_C|}{d^2}\sin 30°$$
$$= 1.1\ \ 10^{-7}\ \text{N}$$
$$F_y = -K\frac{|q_A q_B|}{d^2}\cos 30° - K\frac{|q_A q_C|}{d^2}\cos 30°$$
$$= 5.8\ \ 10^{-7}\ \text{N}$$

3. Two electrons each with mass m_e = 9.11× 10^{-31} kg are spaced 1mm apart. What is the magnitude of the acceleration for one of the electrons?

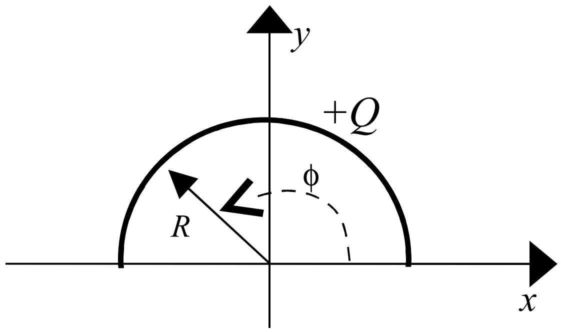
(1) 2.5 x 10^8 m/s^2
(2) 2.3 x 10^{-22} m/s^2
(3) 1.25 x 10^8 m/s^2
(4) 2.5 x 10^{46} m/s^2
(5) 250 m/s^2

Solution:

Set the coulomb force equal to m*a and solve for acceleration:

$$a = \frac{|\mathbf{F}|}{m} = k\frac{e^2}{(0.001m)^2\left(9.11\ \ 10^{-31}kg\right)} = 2.5\ \ 10^8\ \text{m/s}^2$$

4. Consider electric charge Q = +3nC distributed uniformly along a one-dimensional path in the shape of a semicircle of radius R=0.1m. Find the component of the electric field along the y-axis at the origin (0,0).
(1) -1700 N/C (2) -2700 N/C (3) 0 (4) +1700 N/C (5) +2700 N/C



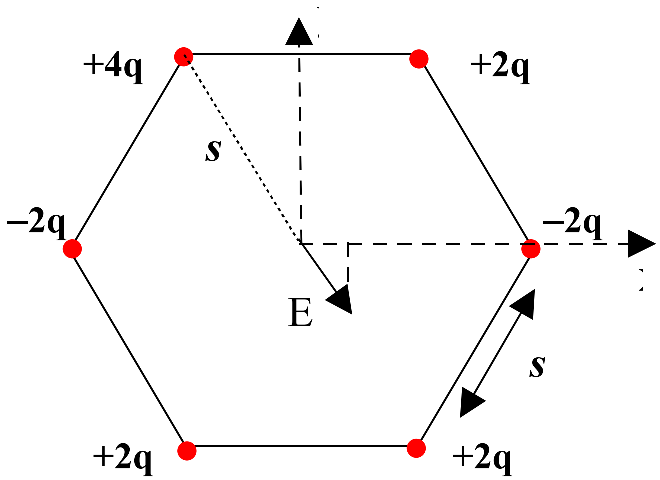
Solution:

By symmetry, the x component of the field cancels at the origin and we only have the y component, which points in the negative y direction since the field points away from positive charge. The linear charge density is $\lambda = +Q/\pi R$ for the semicircle. So computing the y component of the field:

$$E_y = \quad dE_y = \quad -k\frac{dq}{R^2}\sin\phi = -{}_{0}^{\pi}\, k\frac{\lambda R d\phi}{R^2}\sin\phi$$

$$E_y = \frac{k\lambda}{R}\cos\phi\Big|_0^{\pi} = -\frac{2k\lambda}{R} = -\frac{2kQ}{\pi R^2} = -1700 \text{ N/C}$$

5. What is the x component (or the y component) of the electric field at the origin (center) of the hexagonal array of charged particles. The side length s=20cm and q=5 x 10^{-9}C.
(1) 1130 N/C (2) 2250 N/C (3) 3380 N/C (4) -1950 N/C (5) 0

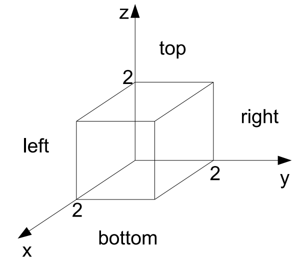


Solution:

Note that the field cancels for all opposing pairs of particles **except** for +2q in the top left corner. The net field points down and to the right. The distance from that charge to the origin is s. So the components of the field are:

$$E_x = k\frac{2q}{s^2}\cos 60° = \left(9\ \ 10^{9}\right)\frac{2\left(5\ \ 10^{-9}\right)}{0.2^2}\frac{1}{2} = 1125\text{ N/C}$$

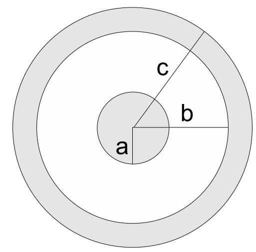
$$E_y = -k\frac{2q}{s^2}\sin 60° = -\left(9\ \ 10^{9}\right)\frac{2\left(5\ \ 10^{-9}\right)}{0.2^2}\frac{\sqrt{3}}{2} = -1950\text{ N/C}$$



6. A cube of side 2 m has one corner at the origin as shown in the figure. If $\vec{E} = (1+x^2)\hat{\imath} + (2+2y^2)\hat{\jmath} + (3+3z^2)\hat{k}$ V/m when x, y, and z are measured in meters, what is the flux through the top face?

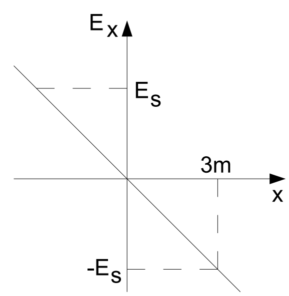
Solution: Because the electric field is constant on each face of the cube, the flux is $\Phi = \hat{n}\cdot\vec{E}\,A$:

$$\begin{aligned}
\Phi_{top} &= \hat{k}\cdot\vec{E}(z=2)\,A = 60\,V\cdot m \\
\Phi_{bottom} &= -\hat{k}\cdot\vec{E}(z=0)\,A = -12\,V\cdot m \\
\Phi_{left} &= -\hat{\jmath}\cdot\vec{E}(y=0)\,A = -8\,V\cdot m \\
\Phi_{right} &= \hat{\jmath}\cdot\vec{E}(y=2)\,A = 40\,V\cdot m
\end{aligned}$$



7. A conducting sphere is inside a conducting shell as shown in the figure. The net charge on the sphere is -3μC, and the net charge on the shell is 5μC. If $a = 1$ m, $b = 2$ m, and $c = 2.5$ m, what is the magnitude and direction of the electric field at $r = 1.5$ m?

Solution: Apply Gauss' Law, $\Phi = \hat{n}\cdot EA = Q_{enc}/\epsilon_o$, to a sphere of radius $r = 1.5m$, which is in between the sphere and the outside shell ($A = 4\pi r^2$). The charge enclosed is thus the charge on the sphere. For $-3\mu C$ on the sphere, $\hat{n}\cdot\vec{E} = -12000\,V/m$, and for $-2\mu C$ on the sphere, $\hat{n}\cdot\vec{E} = -8000\,V/m$. The fact that these are negative implies that the field is directed inwards.



8. The electric field in the x direction is plotted in the figure. If $V(0) = 2$ V and $E_s = 6$ V/m, what is the voltage at $x = 2$ m?

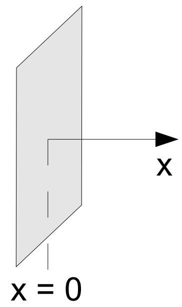
Solution: The voltage is related to the area under the electric field graph.

$$V(x) = V(0) - \int_0^x E_x dx'.$$

The slope of the graph is $-E_s/3$, and consequently the electric field at $x = 2m$ is $-2E_s/3$. The integral is the area under the graph, which is negative.

$$V(2) = V(0) + \frac{1}{2}\,2\,\frac{2E_s}{3}$$

For $E_s = 6V/m$, $V(2) = 6V$, and for $E_s = 3V/m$, $V(2) = 4V$.



9. An infinite plane with charge density $\sigma = 4 \times 10^{-6} C/m^2$ lies in the $x = 0$ plane (see figure). A particle with charge $3 \times 10^{-6} C$ is moved from $(x, y, z) = (1, 1, 0)$ to $(x, y, z) = (2, 2, 0)$, where distances are measured in meters. What is the change in potential energy of the particle, $U_f - U_i$?

Solution: The electric field on the right is $\sigma/2\epsilon_o$ to the right. If you do not remember this, it can easily be derived from Gauss' Law. The force on the particle is $|F| = q\sigma/2\epsilon_o$ to the right. The change in the potential energy is

$$U_f - U_i = -\int \vec{F} \cdot \vec{s} = -|F|(x_f - x_i) = -0.68J.$$

10. Two protons are initially at rest and separated by a distance of 1 cm. They are both released and move away from each other. What is the speed of one of the protons when they are infinitely far away?

Solution: This is a conservation of energy problem, $K_i + U_i = K_f + U_f$,

$$0 + \frac{e^2}{4\pi\epsilon_o d} = \frac{1}{2} m_p v_f^2 + \frac{1}{2} m_p v_f^2,$$

which implies that

$$v_f = \sqrt{\frac{e^2}{4\pi\epsilon_o d m_p}}.$$

For $d = 1\,cm$, $v_f = 3.7\,m/s$, and for $d = 1\,mm$, $v_f = 11.7\,m/s$. Because these answers are not exactly on the answer sheet, for $d = 1$ cm we are counting both 5.2 m/s and 3.3 m/s as correct, and for $d = 1$ mm we are counting both 16.6 m/s and 12.4m/s as correct. This also accounts for students who thought one of the protons was fixed.