Prof. Darin Acosta
Prof. Selman Hershfield
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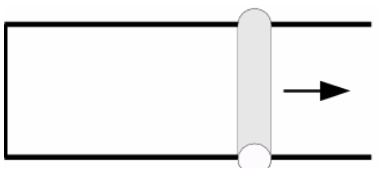
Exam 3 Solutions

1. A metal rod is forced to move with constant velocity of 60 cm/s [or 90 cm/s] along two parallel metal rails, which are connected by metal at one end (see figure). The rails are separated by 20 cm, and there is a magnetic field of 2 T coming out of the page. If the rod has resistance 50 Ohms and the rails and connector have negligible resistance, what force is required to keep the rod moving at constant velocity?

(1) 2×10^{-3} N (2) 1×10^{-3} N (3) 3×10^{-3} N (4) 4×10^{-3} N (5) 5×10^{-3} N



The induced EMF (magnitude) is given by Faraday's Law

$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d}{dt} BL\left(x + vt\right) = BLv$$

The current is given by

$$i = \frac{\varepsilon}{R}$$

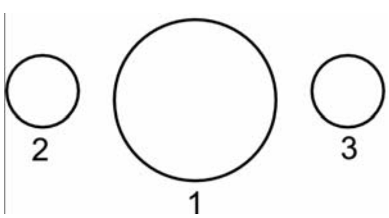
This gives rise to a force on the rod carrying this current (recall Ch.28):

$$F = iLB$$

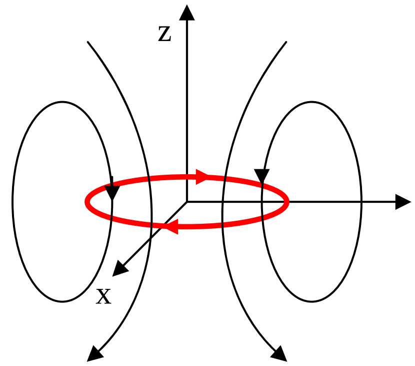
Plugging in gives $F = 2 \times 10^{-3}$ N for v=60 cm/s, and $F = 3 \times 10^{-3}$ N for v=90 cm/s

2. The current in the large loop is in the clockwise [or counterclockwise] direction and increasing. What is the direction of the induced current in loop 2 and in loop 3, respectively?

(1) clockwise, clockwise
(2) clockwise, counterclockwise
(3) counterclockwise, clockwise
(4) counterclockwise, counterclockwise
(5) no induced current



For a clockwise current, the field points into the page within ring 1 and out of the page outside of ring 1. For example, the field lines look like below (use the right-hand rule):



Thus, since the current is increasing, the field strength is increasing, which means in the vicinity of rind 2 and 3 the field is growing out of the page. This induces and EMF and current in rings 2 and 3 which must oppose this increase in magnetic flux (Lenz's Law), thus those rings also have clockwise currents induced which create fields pointing into the page inside the loops (thus subtracting from the increasing magnetic flux from ring 1).

3. An alternating source drives a series RLC circuit with an EMF amplitude of 5V. The current lags the EMF by 45º. When the potential difference across the inductor, v_L, reaches its maximum positive value of 6V [or 4V], what is the potential difference across the capacitor, v_C, including sign? The potential difference across the resistor is 0.
(1) -2.5V (2) -0.5V (3) 1V (4) -1V (5) 3.5V

This is alternating current and EMF, not DC! Recall that for AC circuits:

$$\varepsilon(t) = \varepsilon_m \sin(\omega t)$$

$$i(t) = i_m \sin(\omega t - \phi)$$

The potential difference across the inductor is $v_C = L\dfrac{di}{dt} = \omega L \cos(\omega t - \phi)$

This is a maximum when $\omega t - \phi = 0, 2\pi, \ldots$ *Let's take 0, then the EMF is:*

$$\varepsilon(t) = \varepsilon_m \sin(\omega t) = \varepsilon_m \sin(\omega t - \phi + \phi) = \varepsilon_m \sin(\phi) = 5\sin 45 = \frac{5}{\sqrt{2}} = 3.5\text{V}$$

Then by Kirchoff's loop rule:

$$\varepsilon - v_R - v_L - v_C = 0$$

$$\Rightarrow 3.5 - 0 - 6 = v_C = -2.5\text{V for } v_L = 6$$

$$3.5 - 0 - 4 = v_C = -0.5\text{V for } v_L = 4$$

4. A solenoid that is 13m long and 3m in radius has 2070 total windings and carries a current of 20,000 A. Calculate the energy density of the magnetic field inside the solenoid.

(1) 6.4 x 10^6 J/m^3 (2) 4.0 J/m^3 (3) 1.1 x 10^9 J/m^3 (4) 12.0 J/m^3 (5) 2 x 10^4 J/m^3

$$u_B = \frac{|B|^2}{2\mu_0} = \frac{(\mu_0 n i)^2}{2\mu_0} = \frac{\left(\mu_0 \frac{N}{L} i\right)^2}{2\mu_0} = \frac{1}{2}\mu_0\left(\frac{2070}{13\text{ m}}\right)^2(20000\text{ A})^2$$
$$u_B = 6.4\times10^6\text{ J/m}^3$$

5. An oscillating LC circuit consists of a 2 mH inductive coil and a 4μF capacitor. The capacitor has a potential difference of 0.75 V when the current through the coil is 30 mA. What is the maximum possible charge on the capacitor [or current through the inductor]?

(1) 4 x 10^{-6}C (2) 3 x 10^{-6} C (3) 2 x 10^{-6} C (4) 8 x 10^{-9} C (5) 0.75 C

Two ways to approach this problem. First, we can try conservation of energy, where the total energy at one instant of time must equal that when all the energy is in the capacitor:

$$U = \frac{1}{2}Li_1^2 + \frac{1}{2}\frac{q_1^2}{C} = \text{constant} = \frac{1}{2}\left[0.002(0.03)^2 + \left(4\times10^{-6}\right)(0.75)^2\right] = 2\times10^{-6}\text{ J}$$
$$U = \frac{1}{2}Li_{\max}^2 \Rightarrow i_{\max} = \sqrt{\frac{2U}{L}} = 0.045\text{ A}$$
$$U = \frac{1}{2}\frac{q_{\max}^2}{C} \Rightarrow q_{\max} = \sqrt{2CU} = 4\times10^{-6}\text{ C}$$

The other way to solve this problem is to use the sinusoidal solution to the charge on the capacitor:

$$q = q_{\max}\sin\omega t$$
$$i = \frac{dq}{dt} = q_{\max}\omega\cos\omega t$$
$$\frac{q_1}{i_1} = \frac{1}{\omega}\tan\omega t_1$$
$$\Rightarrow t_1 = \frac{1}{\omega}\tan^{-1}\left(\omega\frac{q_1}{i_1}\right)$$
$$q_{\max} = \frac{q_1}{\sin\omega t_1} = \frac{CV_1}{\sin\left[\tan^{-1}\left(\frac{1}{\sqrt{LC}}\frac{CV_1}{i_1}\right)\right]} = 4\times10^{-6}\text{ C}$$

6. A parallel-plate capacitor with circular plates of radius 0.05 m is being charged by a constant current of 0.5 A. What is the magnitude of the magnetic field in μT at a radius of 0.025 m from the central axis connecting the centers of the plates?

(1) 1 (2) 2 (3) 3 (4) 4 (5) 0

The displacement current between the capacitor plates equals the real current going into the capacitor. It is distributed uniformly between the plates since the electric field there is uniform. Thus the fraction of the current enclosed by a circular path of radius r is

$$i_{disp,encl} = i\frac{\pi r^2}{\pi R^2}$$

Thus by the Ampere-Maxwell Law

$$\oint B \cdot ds = 2\pi r B = \mu_0 i \frac{r^2}{R^2}$$

$$\Rightarrow B = \frac{\mu_0 i r}{2\pi R^2} = 1\times 10^{-6} \text{ T}$$

7. A paramagnetic [or diamagnetic] material is placed in an external magnetic field B_{ext}. The magnitude of the magnetic field inside the material is:

(1) > $|B_{ext}|$ (paramagnetic)
(2) < $|B_{ext}|$ (diamagnetic)
(3) = $|B_{ext}|$
(4) exactly zero
(5) cannot be determined from information given

Recall that the material magnetization goes as: $B_M = \mu_0 M = \chi B_{ext}$. *For paramagnetic substances* $\chi>0$ *and for diamagnetic ones,* $\chi<0$. *So the total field* $B_{tot} = B_{ext} + B_M$ *is larger for paramagnetic substances and smaller for diamagnetic ones.*

8. The average intensity of light from an incandescent light bulb is 300 mW/m^2 [or 600] on a particular surface. Assuming that the light is in the form of an electromagnetic plane wave, what is the maximum magnetic field amplitude, B_m?

(1) 5 x 10^{-8} T (2) 7 x 10^{-8} T (3) 3.5 x 10^{-8} T (4) 21 T (5) 15 T

$$I = S_{av} = \frac{1}{2\mu_0}E_m B_m = \frac{1}{\mu_0}E_{rms}B_{rms}$$

since $\frac{E_m}{B_m} = c$

$$I = S_{av} = \frac{c}{2\mu_0}|B_m|^2 = 0.3 \quad \text{[or 0.6]}$$

$$\Rightarrow |B_m| = 5\times 10^{-8} \text{ T} \quad (\text{or } 7\times 10^{-8} \text{ T})$$

9. A beam of initially unpolarized light is sent into a stack of 4 polarizing sheets, where the polarizing direction of each sheet is rotated +60º with respect to the previous sheet. What fraction of the incident intensity is transmitted by the stack of 4 sheets?

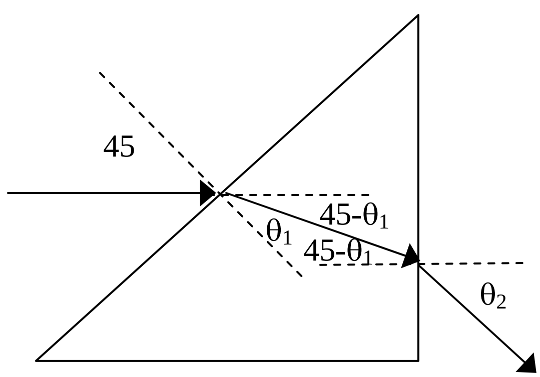
(1) 1/128 (2) 1/256 (3) 1/64 (4) 1/16 (5) 27/128

Unpolarized light passing through one Polaroid reduces the intensity by 1/2. After that, each Polaroid reduces the intensity by a factor $\cos^2\theta$*. So the over reduction is:*

$$\frac{I}{I_0} = \left(\frac{1}{2}\right)\left(\cos^2 60^\circ\right)^3 = \frac{1}{2}\left(\frac{1}{2}\right)^{2\cdot 3} = \frac{1}{128}$$

10. Light traveling horizontally enters a right prism as shown in the figure. The index of refraction of the prism is n=1.6. At what angle is the light deflected from horizontal?

(1) 31º (2) 26º (3) 19º (4) 45º (5) 0º



There are 2 refractions from the 2 surfaces. And since the second surface is not parallel to the first, the light does not return to the incident direction.

$$\sin 45 = 1.6\sin\theta_1$$
$$1.6\sin\left(45-\theta_1\right) = \sin\theta_2$$
$$\Rightarrow \theta_2 = 31^\circ$$