

**Electricity and Magnetism** pervades our modern world.

We generate **electric power** (energy/second: **Watts**) in power stations and transmit it down power lines to our homes where we use it for

heating & cooling,  
lighting,  
cooking  
hot showers,  
computing,  
television,  
... and much more.



While its hard to imagine living without these wonderful modern conveniences electricity and magnetism are much more fundamental to our lives than even this suggests.

Our understanding of the matter making up the world (including ourselves and all life) is that it is comprised of **atoms** of the elements:

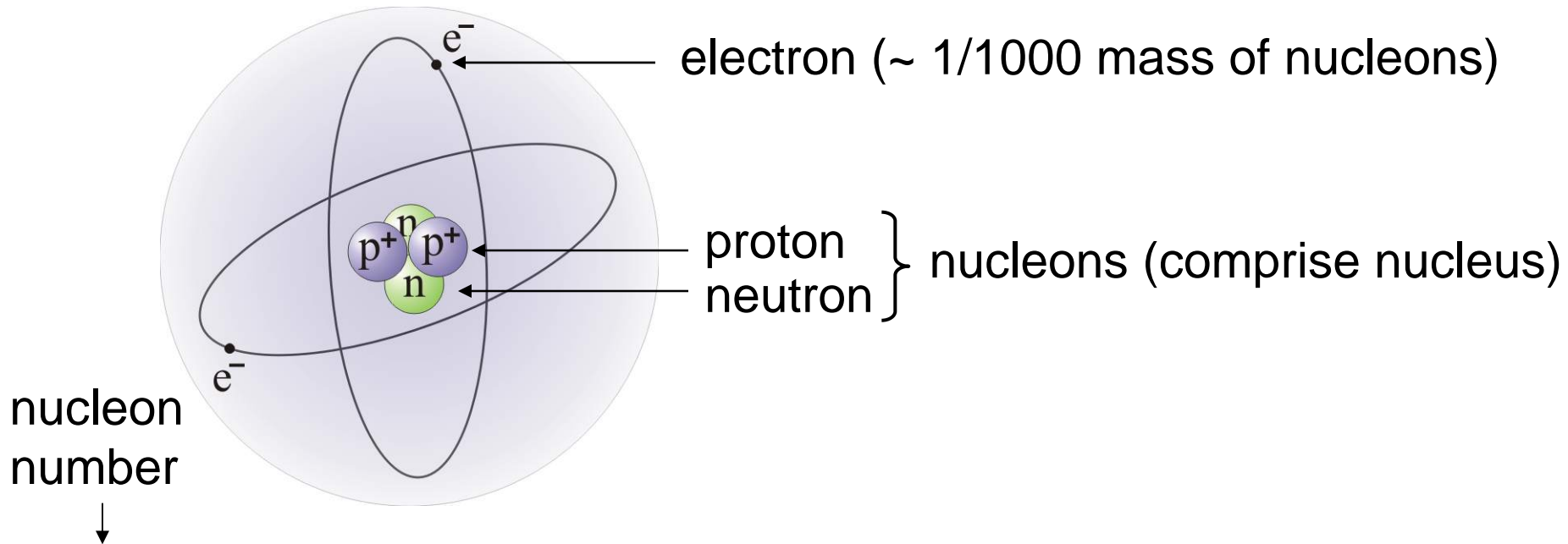
Group IA												VIII																							
1	H											2	He																						
3	Li	IIA												4	Be	5	B	6	C	7	N	8	O	9	F	10	Ne								
11	Na	12	Mg	13	Al	14	Si	15	P	16	S	17	Cl	18	Ar																				
19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
55	Cs	56	Ba	71	Lu	72	Hf	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt	79	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn
87	Fr	88	Ra	103	Lr	104	Unq	105	Unp	106	Unh	107	Uns	108	Uno	109	Une	110	Uun	111	Uuu	112	Uub												
Lanthanide Series				57	La	58	Ce	59	Pr	60	Nd	61	Pm	62	Sm	63	Eu	64	Gd	65	Tb	66	Dy	67	Ho	68	Er	69	Tm	70	Yb				
Actinide Series				89	Ac	90	Th	91	Pa	92	U	93	Np	94	Pu	95	Am	96	Cm	97	Bk	98	Cf	99	Es	100	Fm	101	Md	102	No				

These atoms themselves are comprised of three, still smaller, constituents:

**Electrons, Protons and Neutrons**

## The atom

Planetary model (Neils Bohr): electrons orbit around nucleus held in place by the **electrostatic force**.



${}^4\text{He}$  – 2 protons + 2 neutrons in nucleus (2 electrons “orbiting”)

( ${}^3\text{He}$  also exists 2 p<sup>+</sup>, but 1 n (still 2 e<sup>-</sup>) called an isotope)

(This picture has been greatly refined by quantum mechanics but that’s another course you may get later. This model remains conceptually useful for our purposes.)

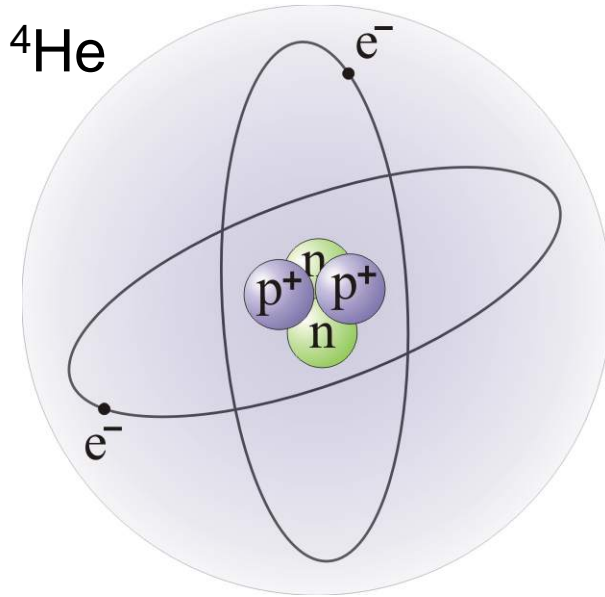
Electrons and protons have a property we call **charge** that is measured in units of **Coulombs** (C).

The charge of the electron is  $-1.602 \times 10^{-19} \text{ C}$

The charge of the proton is  $+1.602 \times 10^{-19} \text{ C}$

Electrons and protons have charge of exactly the **same magnitude** but **different sign**.

This **smallest magnitude** of charge is often designated  $e = 1.602 \times 10^{-19} \text{ C}$



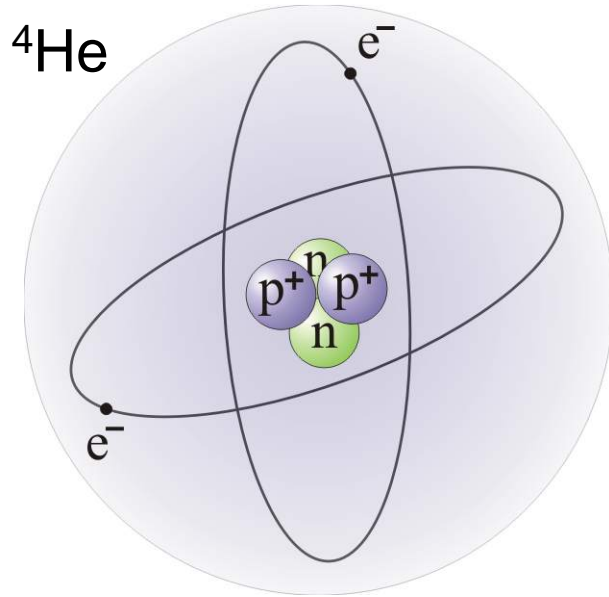
The significance of the sign is the following:

**Like charges** (same sign) **repel** each other with the **electrostatic force**

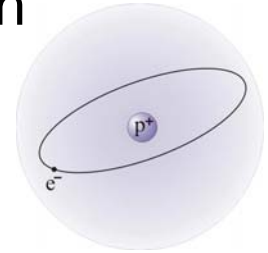
**Opposite charges** (opposite sign) **attract** with the **electrostatic force**

Neutrons are neutral and are not affected by electrostatic forces.

Electrons and protons can be liberated from the atom to **exist as free charged particles**.



A free electron and free proton (opposite sign charges) will attract each other with the electrostatic force to form an electrically neutral hydrogen atom.

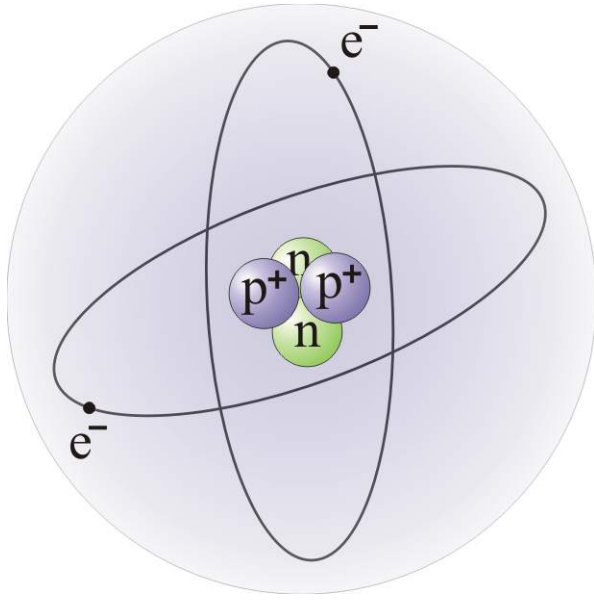


Stable **atoms are electrically neutral** because they have an **equal number** of electrons and protons.

Aluminum (Al) has 13 protons in its nucleus. To be electrically neutral it must have 13 electrons. If an atom of Al were missing 2 electrons it would have a charge of  $+2e$ .

Placed in the vicinity of free electrons it would attract them with the electrostatic force, capturing 2 of them, after which it would become electrically neutral and no longer attract further electrons.

You may wonder: if like charges repel **how come the protons can be so close together in the nucleus?**



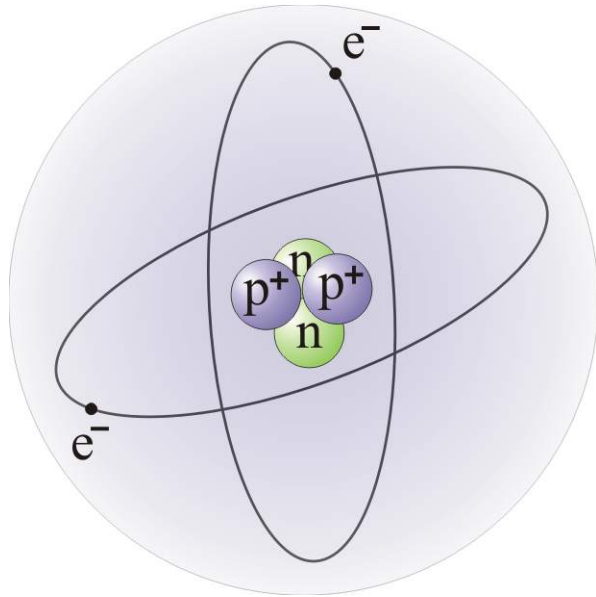
This is because **another force** acts in the nucleus called the **strong force** (not a very imaginative name... but there it is).

Both the electrostatic force and the strong force fall off (decrease) with increasing distance between the particles.

The strong force is much stronger at very close particle distances but it falls off much more quickly with increasing distance.

The strong force dominates inside the nucleus while the electrostatic force dominates outside the nucleus.

You may also wonder **why electrons don't just spiral in to merge with the protons there?**



No one has a real answer to that question.

In the early 1900's Neils Bohr, inventor of this planetary model and one of the founders of quantum mechanics, stamped his foot and declared it so.



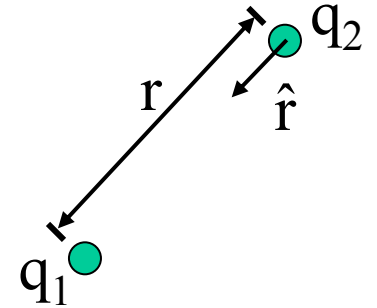
Actually, using the assumption that electrons can only exist in particular, stable (quantized) orbits in an atom he was able to explain a number of otherwise very puzzling phenomena (garnering him the Nobel Prize for Physics in 1922).

But we digress.



The **electrostatic force** between elementary (point) charges is given by Coulomb's law,

$$\vec{F}_{1,2} = k \frac{q_1 q_2}{r^2} \hat{r}$$



Where:

$q_1$  is the signed amount of charge on particle 1 (in Coulombs)

$q_2$  is the signed amount of charge on particle 2 (in Coulombs)

The subscripts mean that this is the force experienced **by** particle 1 **due to** particle 2.

$r$  is the distance between the charges

$\hat{r}$  is a unit vector pointing **from** particle 2 **to** particle 1

$k$  is a constant, often written as  $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$

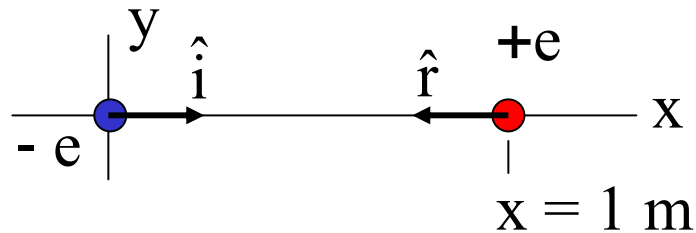
where the permittivity constant,  $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$



Example: Consider  $q_1 = -e$  (an electron) fixed at the origin and  $q_2 = +e$  (a proton) fixed along the x axis at  $x = 1$  m.

What is the force **on** the electron ( $q_1$ ) **from** the proton?

$$\vec{F}_{1,2} = k \frac{q_1 q_2}{r^2} \hat{r}$$



In this case,  $\hat{r} = -\hat{i}$  and  $r = 1$  m, so,

$$\vec{F}_{1,2} = k \frac{(-e)(+e)}{r^2} (-\hat{i}) = k \frac{e^2}{r^2} \hat{i} = (8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(1.60 \times 10^{-19} \text{C})^2}{(1 \text{m})^2} \hat{i}$$

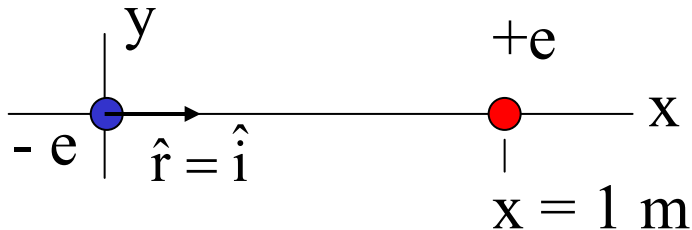
$$\vec{F}_{1,2} = 2.3 \times 10^{-28} \text{ N } \hat{i}$$

Note that the **direction** of the **vector force** ends up being toward positive  $\hat{i}$ , as it should, because the charges have opposite sign and thus the proton is attracting the electron.

What is the **force acting on the proton** at  $x = 1$  m due to the electron at the origin?

$$\vec{F}_{2,1} = k \frac{q_1 q_2}{r^2} \hat{r}$$

(switched)



The diagram shows a Cartesian coordinate system with a horizontal x-axis and a vertical y-axis. A blue circle representing an electron, labeled  $-e$ , is located at the origin  $(0,0)$ . A red circle representing a proton, labeled  $+e$ , is located on the x-axis at  $x = 1$  m. A horizontal arrow labeled  $\hat{r} = \hat{i}$  points from the electron to the proton. The x-axis is labeled  $x$  and the y-axis is labeled  $y$ .

Now  $\hat{r} = \hat{i}$  , so,

$$\vec{F}_{2,1} = k \frac{(-e)(+e)}{r^2} \hat{i} = -k \frac{e^2}{r^2} \hat{i} = -2.30 \times 10^{-28} \text{ N} \hat{i}$$

The **magnitude** is the **same** but the vector force on the proton is in the opposite direction, now toward the origin, the location of the electron (this must be true by Newton's third law ).

We used an explicit **labeling scheme** and **vector convention** above to show how it goes, but it is often simpler to just **calculate the magnitude** of the electrostatic force and **insert** the appropriate vector direction based on whether the charges repel or attract.

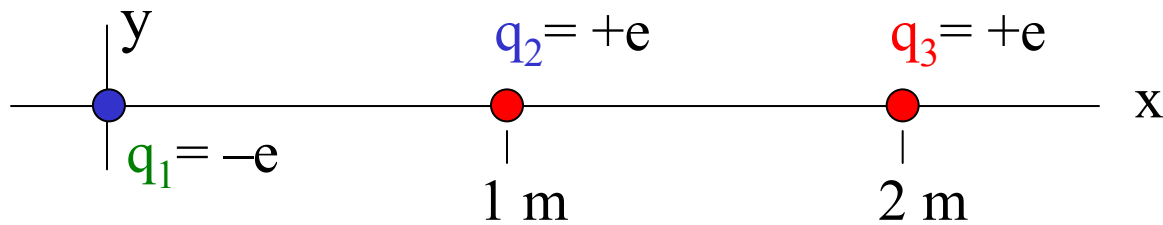
In that case we drop the subscripts on  $F$  (the vector electrostatic force on  $q_1$  due to  $q_2$ , will always be equal and opposite of that on  $q_2$  due to  $q_1$ ) and use the absolute values of  $q_1$  and  $q_2$  so that the magnitude of the force is,

$$F = k \frac{|q_1| |q_2|}{r^2} \quad \text{or} \quad F = k \frac{q_1 q_2}{r^2} \quad \text{where the absolute values of } q_1 \text{ and } q_2 \text{ are understood}$$

The **sign** for the vector force is then put in depending on the signs and locations (in the coordinate system) of the charges  $q_1$  and  $q_2$ .

Electrostatic forces obey the **principle of superposition** so if there are more than two charges the net force on any one of them is given by the **vector sum** of the electrostatic forces **from each** of the others.

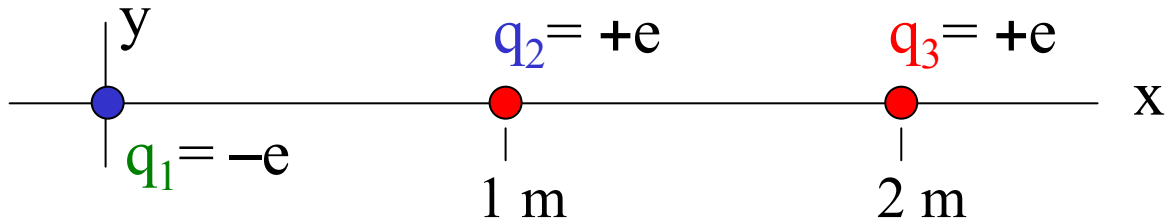
Consider charges  $-e$  at the origin and  $+e$  at  $x = 1$  m. We ask what is the net force experienced by a third charge  $+e$  at  $x = 2$  m?



The force on  $q_3$  due to  $q_2$  (only) has magnitude,

$$F_{3,2} = \frac{1}{4\pi\epsilon_0} \frac{|q_3| |q_2|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2\text{m} - 1\text{m})^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(1\text{m})^2} = \frac{e^2}{4\pi\epsilon_0} (1\text{m}^{-2})$$

Since  $q_2$  and  $q_3$  have the same sign  $q_3$  is repelled, so the force on  $q_3$  from  $q_2$  (only) has this magnitude in direction  $+\hat{i}$ .



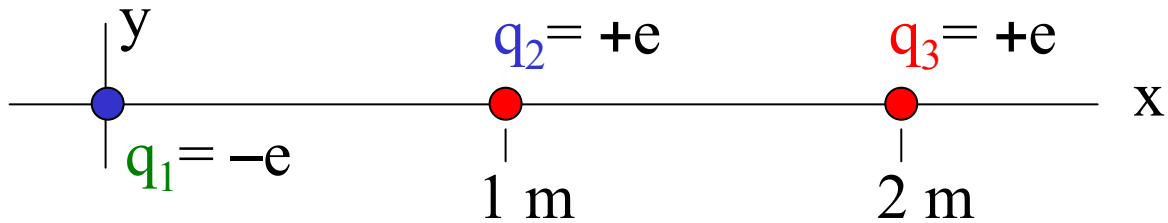
The force on  $q_3$  due to  $q_1$  (only) meanwhile has magnitude,

$$F_{3,1} = \frac{1}{4\pi\epsilon_0} \frac{|q_3| |q_1|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2\text{m} - 0\text{m})^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{(2\text{m})^2} = \frac{e^2}{4\pi\epsilon_0} \left[ \frac{1}{4} \text{m}^{-2} \right]$$

Since  $q_3$  and  $q_1$  have **opposite sign** the force between them is **attractive** so this force on  $q_3$  has direction  $-\hat{i}$ .

The net force on  $q_3$  is the vector sum of these,

$$\vec{F}_3 = \vec{F}_{3,2} + \vec{F}_{3,1} = \frac{e^2}{4\pi\epsilon_0} \left[ 1\text{m}^{-2} \right] \hat{i} + \frac{e^2}{4\pi\epsilon_0} \left[ \frac{1}{4} \text{m}^{-2} \right] (-\hat{i})$$



So,

$$\vec{F}_3 = \frac{e^2}{4\pi\epsilon_0} \left[ \left( 1 - \frac{1}{4} \right) \text{m}^{-2} \right] \hat{i} = \frac{e^2}{4\pi\epsilon_0} \left( \frac{3}{4} \text{m}^{-2} \right) \hat{i}$$

Pointing away from  $q_1$  and  $q_2$  along  $+\hat{i}$ . Does this make sense?

The force from  $q_2$  (only) was in this direction but equal to,

$$\vec{F}_{3,2} = \frac{e^2}{4\pi\epsilon_0} (1 \text{m}^{-2}) \hat{i} \text{ which is } > \text{ the net force } \vec{F}_3 = \frac{e^2}{4\pi\epsilon_0} \left( \frac{3}{4} \text{m}^{-2} \right) \hat{i}$$

That the net force is smaller makes sense since the attractive force from  $q_1$  charge at the origin partly compensates the repulsive force from  $q_2$  causing the decrease. The compensation is far from complete since the electrostatic force falls off as  $1/r^2$  and  $q_1$  was further away.

Materials bodies (made up of **atoms**), can also be electrically **charged**.

They can have an **excess of electrons** (beyond the number that neutralize all the positive atomic cores) charging the overall body **negative**, or they can have a **deficit of electrons** resulting in the body having an overall **positive** charge.

Note that **what matters** is **not** the total number of electrons and protons but **only the excess number** of each.

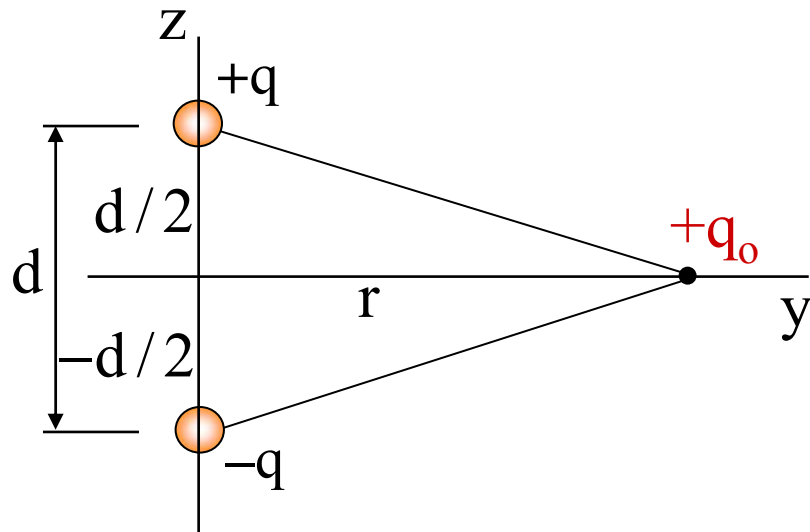
While charge is **quantized** in the smallest fundamental charge ( $e$ ) this is so small ( $0.0000000000000000000000001602$  C) that we often treat it as a continuous variable.

Thus some configuration of charged bodies can be specified by giving the locations of signed quantities of charge e.g.  $+q_1$  ,  $-q_2$  , ...

In many cases we can assume that the separation of the charges on the bodies is so small (compared to the distance between bodies) that the charges  $+q_1$  ,  $-q_2$  , ... themselves are point particles.



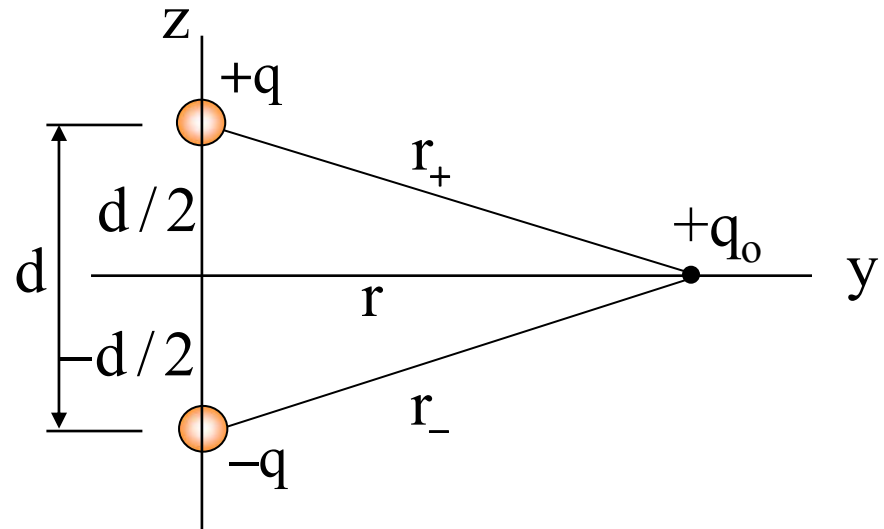
## Example



A charge  $+q_0$  lies on the y axis a distance  $r$  from the origin. What is the electrostatic force on it from the charges  $+q$  and  $-q$  (equal but opposite sign) lying on the z axis at  $\pm d/2$  ( $+q$  and  $-q$  separated by  $d$ ).

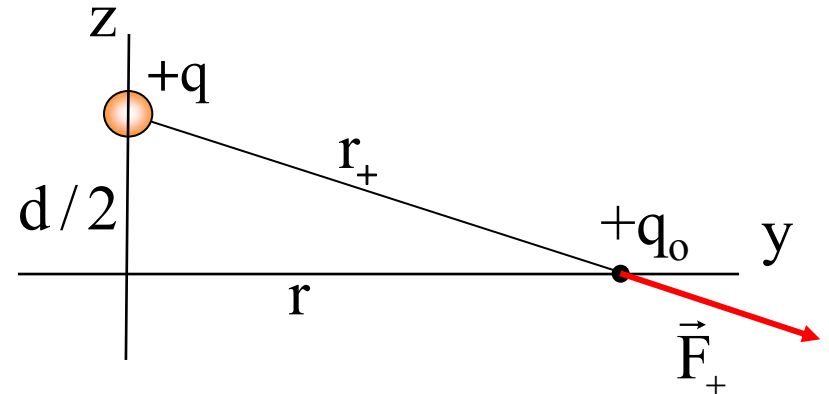
We apply Coulomb's force law between  $+q_0$  and each of the other the other charges, in turn.

The relevant distance in the denominator of the force law is the distance between the particles so we label these distances  $r_+$  and  $r_-$ , as shown.



The **magnitude** of the force between  $+q$  and  $+q_0$  is then,

$$F_+ = k \frac{q_0 q}{r_+^2}$$

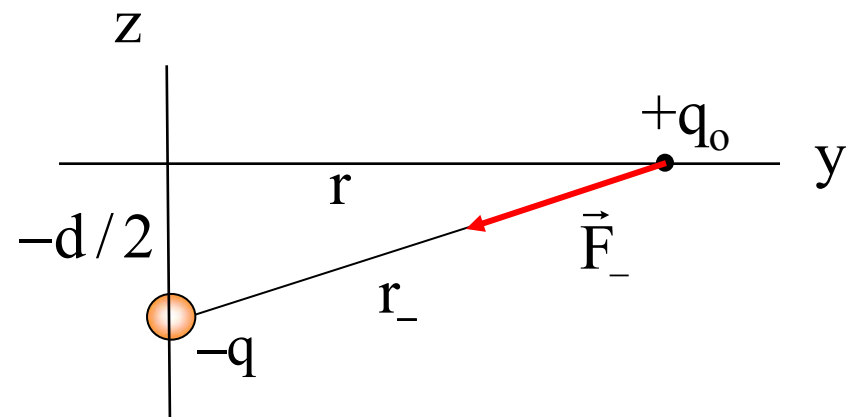


This force acts along the line between  $+q$  and  $+q_0$ , and because they have the same sign, the force is repulsive, resulting in the force vector (of magnitude  $F_+$ ) shown.

Similarly, the magnitude of the force between  $-q$  and  $q_0$  is,

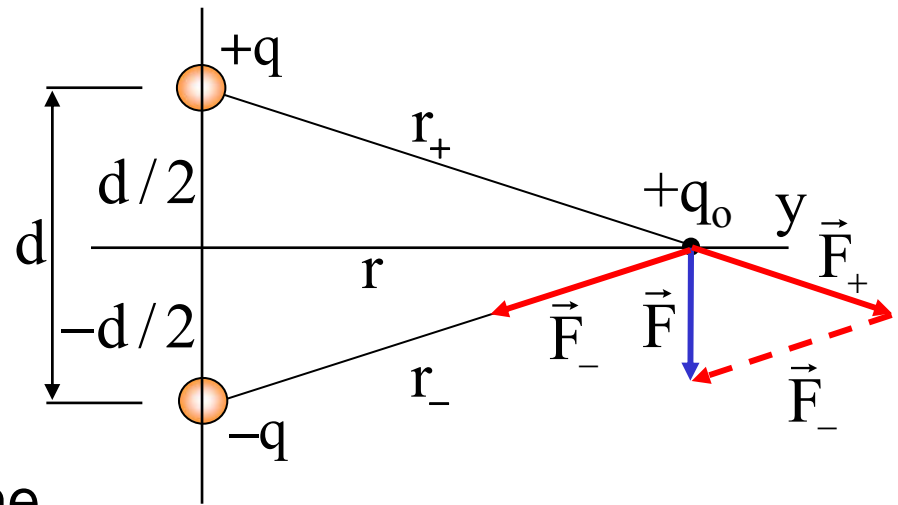
$$F_- = k \frac{q_0 q}{r_-^2}$$

But the opposite signs on the charges result in an attractive force giving the vector (of magnitude  $F_-$ ) shown.



The net force  $\vec{F}$  on  $+q_0$  is the vector sum of  $\vec{F}_+$  and  $\vec{F}_-$ .

By symmetry we see that the components of  $\vec{F}_+$  and  $\vec{F}_-$  along the y axis are equal and opposite, such that when they are summed they cancel each other, while their components along the z axis are equal and in the same



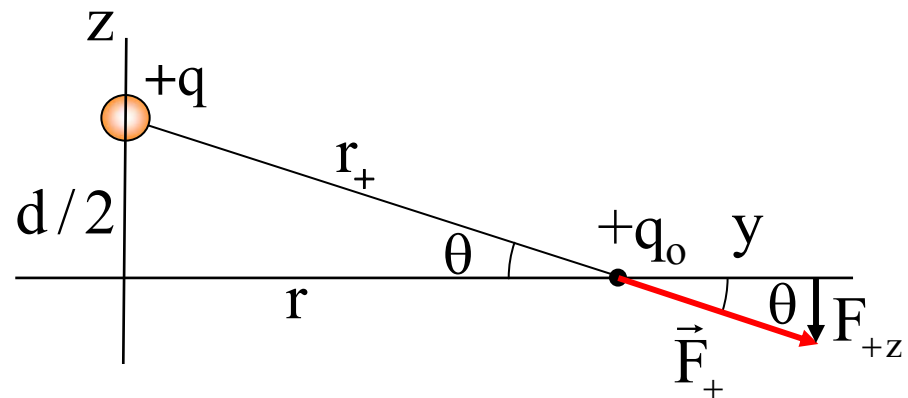
direction. These mean that the resultant  $\vec{F}$  lies in the  $-\hat{k}$  direction with magnitude given by twice *the z component* of either  $\vec{F}_+$  or  $\vec{F}_-$ .

The z component of  $\vec{F}_+$  is

$$F_{+z} = F_+ \sin \theta$$

So the net force on  $+q$  is,

$$\vec{F} = 2F_+ \sin \theta (-\hat{k})$$

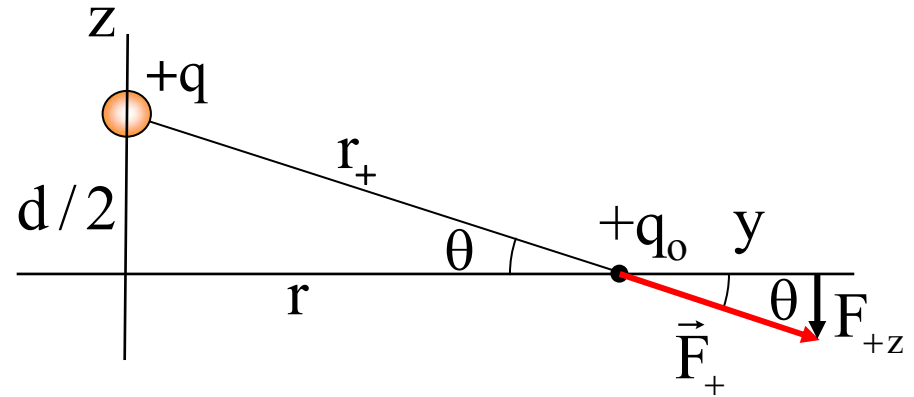


From the sketch we see that

$$\sin \theta = \frac{d/2}{r_+}$$

while,

$$F_+ = k \frac{q_0 q}{r_+^2}$$



Then,

$$\vec{F} = 2F_+ \sin \theta (-\hat{k}) = 2k \frac{q_0 q}{r_+^2} \frac{d/2}{r_+} (-\hat{k})$$

$$\vec{F} = k \frac{q_0 q d}{r_+^3} (-\hat{k})$$

Also from the sketch,

$$r_+ = \sqrt{r^2 + (d/2)^2} = r \sqrt{1 + \left(\frac{d}{2r}\right)^2}$$

so finally,

$$\vec{F} = k \frac{q_0 q d}{r^3 \left(1 + \left(\frac{d}{2r}\right)^2\right)^{\frac{3}{2}}} (-\hat{k})$$