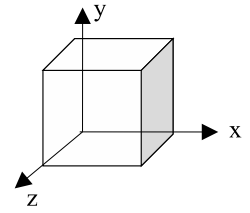# Exam 1 Solutions

1. What is the flux through the right side face of the shown cube if the electric field is given by $\vec{E} = -2x\hat{i} + 3y\hat{j}$ and the cube has a side length of 2?
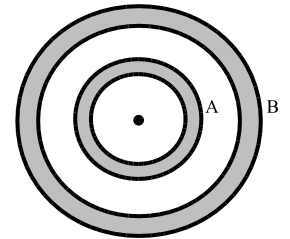
Answer: -16

Solution: The flux is $\oint \hat{n} \cdot \vec{E} dA$, where $\hat{n}$ is directed out of the surface. For the right face this means that the normal is $\hat{n} = \hat{i}$ and $x = 2$ so $\hat{i} \cdot \vec{E} A = -2 \cdot 2 \cdot A = -16$.

2. Two large nonconducting plates each with surface area 5 m^2 are parallel to each other and separated by 1 cm. If the total charge distributed uniformly on one plate is $+5 \, \mu$C, and the total charge distributed uniformly on the other plate is $-7.5 \, \mu$C, what is the magnitude of the electric field midway between the two plates?

Answer: 1.4×10^5 N/C

Solution: The electric field for a single charged plate has magnitude $|\sigma|/(2\epsilon_o)$. The field goes away from the positively charged plate and towards the negatively charged plate. Thus, for this problem the electric fields of the two plates add constructively between the plates: $|\vec{E}| = (5\mu C + 7.5\mu C)/(2\epsilon_o \, 5 \, m^2)$.

3. A charged particle is held at the center of two concentric conducting spherical shells. A cross section is shown in the figure. If the charged particle at the center has charge $+2 \, \mu$C, and the two conducting shells A and B have charges $-3 \, \mu$C and $+4 \, \mu$C deposited on them, respectively, what is the charge on the inner surface of shell B?

Answer: $+1 \, \mu$C

Solution: The electric field inside the conducting shells is zero. Thus, the charge enclosed in a Gaussian surface inside shell B is zero by Gauss's law. The charge enclosed is $2 \, \mu C + -3 \, \mu C + q_{in} = 0$ so that $q_{in} = +1 \, \mu C$.

4. A long charged cylinder with a radius of $R_1 = 3$ cm has a charge density per unit length of $4 \, \mu$C/m. It is surrounded by a concentric charged cylindrical shell of radius $R_2 = 5$ cm with a charge density per unit length of $-3 \, \mu$C/m. What is the magnitude of the electric field at a radial distance of 6 cm from the centerline?
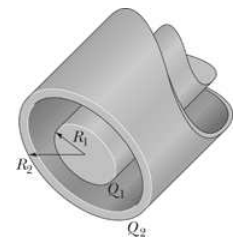
Answer: 3.0×10^5 N/C

Solution: A cylindrical Gaussian surface with radius 6 cm encloses both the inner and outer cylinder. The flux through the surface is $2\pi r L|E|$. The charge enclosed is $(4 \, \mu C - 3\mu C)L$. Using Gauss's law, the magnitude of the electric field is $|E| = (1\mu C)/(2\pi r \epsilon_o)$.

5. In the figure shown, what is the net electric potential at the origin of the coordinate system due to the four other charged particles if $V = 0$ at infinity? Express your answer in terms of the constants q, d, and $k = 1/4\pi\epsilon_0$.
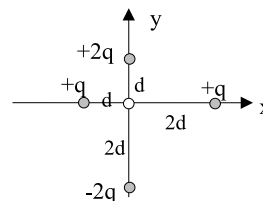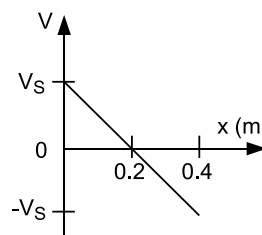
Answer: $5kq/(2d)$

Solution: The potential due to a single point charge is $V = kq/r$. Add the potentials due to all four point charges.

$$\frac{k2q}{d} + \frac{kq}{2d} + \frac{k(-2q)}{2d} + \frac{kq}{d} = \frac{5kq}{2d}$$

6. The figure shows a graph of the electric potential as a function of x. The scale of the vertical axis is set by $V_S = 200$ V. What is x component of the force acting on a proton located at $x = 0.2$ m?

Answer: 1.6×10^{-16} N

Solution: For a proton the force in the x direction is $eE_x = -e\partial V/\partial x$, where $\partial V/\partial x = -200V/0.2\,m = -1000V/m$ and $E_x = 1000\,V/m$.

7. An electron traverses between two parallel conducting plates. It leaves one plate with an electric potential of $V = 0$ at a velocity of 3×10^6 m/s toward the second plate. What is the electric potential of the second plate if the electron slows to a stop just as it reaches it? (The mass of the electron is 9.11×10^{-31} kg)

Answer: -2.6×10^1 V

Solution: This is a conservation of energy problem.

$$-eV_i + \frac{1}{2}m_e v_i^2 = -eV_f + \frac{1}{2}m_e v_f^2$$

Since $V_i = 0$ and $v_f = 0$, this implies that $V_f = -m_e v_i^2/(2e)$.

8. A charged particle of charge $+q$ is fixed in its position near the surface of the earth. A second charged particle of the same charge, $+q$, and of mass m levitates, in equilibrium, directly above the first. Ignoring the very small gravitational attraction between the two particles (but not ignoring Earth's gravity), the distance between the charged particles must be:

Answer: $\sqrt{\dfrac{q^2}{4\pi\epsilon_o mg}}$

Solution: For the charge to levitate the gravitational force must balance the electrostatic force: $kq^2/r^2 = mg$. Solve for r with $k = 1/(4\pi\epsilon_o)$.

9. As shown in the figure the particle of charge $q_1 = +3.0\mu C$ sits at the origin and is surrounded by a spherical shell of uniformly distributed total charge $Q = -3.0\mu C$, radius r = 50.0 cm centered at the origin. Another charge $q_2 = -1.0\mu C$ is located on the x axis at $x = -2.0m$. What are the net electrostatic forces on q_1 and q_2 (respectively) in unit vector notation. Answers are in the order of the force experienced by q_1; followed by the force experienced by q_2:
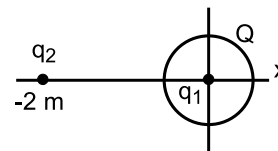
Answer: -6.7×10^{-3} N ; 0

Solution: Q does not exert a force on q_1 because the electric field due to Q at q_1 is zero. q_1 is attracted to q_2 with a force of magnitude $k|q_1||q_2|/(2\,m)^2$. The fact that the force is attractive means that it is in the negative x direction. The electric field due to Q at q_2 is the same as for a point charge of Q at the origin. Thus, the electric fields of Q and q_1 cancel at q_2. There is no force on q_2.

10. A line of charge extending along the x axis has a continuous charge density given by $\lambda = 3x^2\,nC/m$. How much charge is contained on the line from x = 8 to x = 10 meters?

Answer: 488 nC

Solution: Integrate the linear charge density from 8 to 10 meters.

$$\int_8^{10} \lambda dx = x^3\,nC|_8^{10}$$

11. The electric field lines representing the electric field in a particular region are as shown in the figure. An electron is placed successively at points A, B, and C. Rank order the electric field strength at the three points, greatest first, and give the direction of the force experienced by the electron (toward the left or toward the right) in this field.
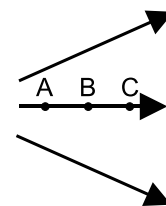
Answer: A, B, C, left

Solution: Where the field lines are closer together the magnitude of the field is larger. The force on an electron is in the opposite direction as the field because electrons are negatively charged.

12. Three equal charges q = +2.00 nC lie along the x axis with one at the origin and the other two at x = ± 1.00 m, respectively (see figure). The magnitude of the electric field due to these charges at point P along the z axis at z = 1.00 m is,
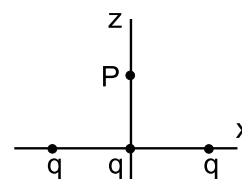

Answer: 30.7 N/C

Solution: By symmetry the net electric field is in the positive z direction. Add the z components of the electric field at P due to each charge starting from the left most charge:

$$E_z = \frac{kq}{(\sqrt{2})^2}\sin(45°) + \frac{kq}{(1)^2} + \frac{kq}{(\sqrt{2})^2}\sin(45°)$$

13. A semicircular thin rod of uniformly distributed total charge Q = -22.0 nC lies in the x-y plane as shown in the figure. The distance from the origin to the rod is r = 0.50 m. The magnitude of the electrostatic field it creates at the origin is,
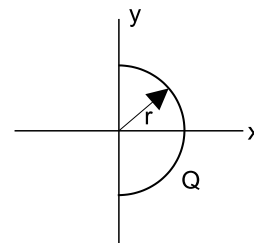
Answer:(1) 504 N/C

Solution: By symmetry the net electric field at the origin is in the x direction. Compute the x component of the electric field:

$$E_x = \int \frac{kdq}{r^2} \cos(\theta) = \int_{-\pi/2}^{\pi/2} \frac{k(|Q|/\pi)d\theta}{r^2} \cos(\theta) = \frac{2k|Q|}{\pi r^2}$$

14. In a uniform electric field of 1.00×10^3 N/C a proton starting from rest travels 1.00 meter in how much time?

Answer: 4.57 μs

Solution: The acceleration of the proton is $a = F/m = eE/m$. Starting from rest the proton travels $x = \frac{1}{2}at^2$ in time t. This implies that the time it takes to travel a distance x is $t = \sqrt{2xm/eE}$.

15. Four charges are arranged on the corners of a square as shown in the figure. Assuming that $q > 0$, what is the direction of the net force, F, on a negative charge, $Q < 0$, placed at the center of the square? Measure the angle θ from the x-axis going counterclockwise as shown in the figure.
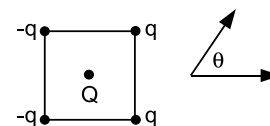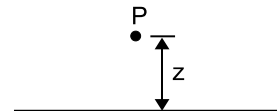
Answer: 0°

Solution: Because Q is negatively charged and q is positively charged, Q is attracted to q and repelled from $-q$. Consequently the forces due to the q charges are at $\pm45°$, and the forces due to the $-q$ charges are at $\pm45°$. These add to give a net force at 0°.

16. A 4 C charge is placed at the $x = 0$, and a 1 C charge is placed $x = 2$ m. Where can a third charge be placed on the x-axis so that the net force on that charge is zero?

Answer:(1) 1.33 m

Solution: In the region $0 < x < 2\,m$ the forces on a third charge due to the 4 C charge and the 1 C charge are in opposite directions irrespective of the sign of the third charge. Set the magnitude of the two forces equal to each other: $k(4\,C)Q/x^2 = k(1\,C)Q/(2-x)^2$. Cross multiply and solve: $4(2-x)^2 = x^2 \rightarrow 2(2-x) = x \rightarrow 4 = 3x$.

17. An infinite line of charge has charge per unit length of λ. Which of the following expressions gives the magnitude of the electric field at point P, which is a distance z from the line of charge?


Answer: $\displaystyle\int_{-\infty}^{\infty} \frac{k\lambda z dx}{(x^2 + z^2)^{3/2}}$

Solution: The distance from a point on the line of charge to P is $r = \sqrt{x^2 + z^2}$. An element of charge is equal to $dq = \lambda dx$. By symmetry the net electric field is away from the line of charge (z direction). Thus, we only need to compute the z component of the field. To do that we must multiply by z/r.

$$E_z = \int \frac{kdq}{r^2}\frac{z}{r} = \int_{-\infty}^{\infty} \frac{k\lambda z dx}{(x^2 + z^2)^{3/2}}$$

18. The electric field in the x-direction is shown in the figure. If the electrical potential at the origin is $2V$, what the potential at $x = 4m$?
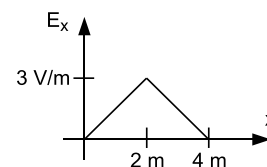
Answer: -4 V

Solution:

$$V(4) = V(0) - \int_0^4 E_x dx$$

The integral of E_x is the area under the curve, which is 6 V.

19. An insulating sphere of radius 0.5 m has a uniform charge density of $\rho = 6\mu C/m^3$. What is the magnitude of the electric field a distance of 0.25 m from the center of the sphere?

Answer: 0.56×10^5 V/m

Solution: Use a Gaussian surface of radius $r = 0.25$ m. The flux through the surface is $4\pi r^2|E|$, and the charge enclosed in the surface is $(4\pi/3)r^3\rho/\epsilon_o$. Applying Gauss's law, $|E| = \rho r/(3\epsilon_o)$.

20. Two positive $2\mu C$ charges are placed at $x = 0$ m and $x = 1$ m. A $-3\mu C$ charge is initially at $x = 0.25$ m. The -3μ charge is then moved to $x = 0.75$ m. What is the change in potential energy, $U_f - U_i$, of the system of three charges?

Answer: 0 J

Solution: The potential energy of the system of charges is the sum of the potential energies of each pair of charges, kq_iq_j/r^2. Initially the $-3\mu C$ charge is 0.25 m from one charge and 0.75 m from the other. Finally the $-3\mu C$ charge is 0.75 m from one charge and 0.25 from the other. The net change on the potential energy is zero.