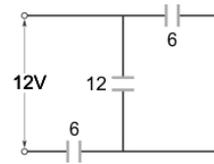


Exam 2 Solution

1. In the Figure the capacitances are in micro-Farads (μF). The equivalent capacitance of the network of capacitors is:



Answer: $4.5 \mu\text{F}$

Solution: The $6 \mu\text{F}$ and $12 \mu\text{F}$ capacitors in parallel have capacitance $18 \mu\text{F}$. These capacitors are in series with the other $6 \mu\text{F}$ capacitor to the effective capacitance is $4.5 \mu\text{F}$.

2. In the previous problem, the charge on the positive plate of the **top** $6 \mu\text{F}$ capacitor is:

Answer: $18 \mu\text{C}$

Solution: 12 volts across an effective $4.5 \mu\text{F}$ capacitor gives a charge of $CV = 54 \mu\text{C}$. The $6 \mu\text{F}$ and $12 \mu\text{F}$ capacitors in parallel have effective capacitance of $18 \mu\text{F}$ so the voltage across them is $Q/C = 54 \mu\text{C}/18 \mu\text{F} = 3\text{V}$. A voltage of 3V across a $6 \mu\text{F}$ capacitor corresponds to a charge of $(6 \mu\text{F})(3\text{V}) = 18 \mu\text{C}$.

3. A parallel plate capacitor has a plate area of 45 cm^2 a plate separation of 30 micrometers and is filled with a dielectric material having a dielectric constant of 280. When charged to 6.0 V the potential energy stored in the device is:

Answer: $6.7 \mu\text{J}$

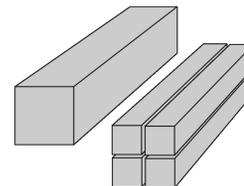
Solution: The capacitance of the parallel plate capacitor is $C = \kappa\epsilon_0 A/d$. The energy stored in the capacitor is $U = (1/2)CV^2 = 6.7 \mu\text{J}$.

4. A particular wire has a circular cross-section. For the first 1/2 meter of its one meter length its radius is 4 mm while for the second half it is 2 mm. 14 Coulombs of charge pass a point in the 4 mm section in 30 seconds. The current in the 2 mm section is:

Answer: 467 mA

Solution: The current through both sections of the wire is the same because current-in is equal to current-out. The current in the wider section is $14\text{C}/(30\text{s}) = 467\text{mA}$.

5. A copper wire has a length L , a square cross-section of side length W and an end to end resistance R . Without loss of material the wire is sliced along its long direction to make 4 equal length wires of equal square cross-sections (see Figure). These are then welded together end-to-end to make one wire that is 4 times the original length. The new resistance is:



Answer: $16R$

Solution: The resistance of a wire is $R = \rho L/A$. If $L \rightarrow 4L$ and $A \rightarrow A/4$, then the resistance increases by a factor of 16.

6. A 24.0V DC motor raises a 1.00 kg mass with a constant speed through 10.0 meters in 20.0 seconds. Assuming no other energy losses the current drawn by the motor must be:

Answer: 204 mA

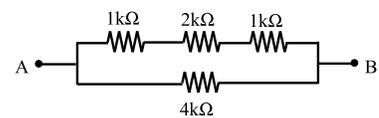
Solution: If there are no other energy losses, the power output of the motor is equal to the power input into the motor. The power output is $F \cdot v$, where F is the force, mg , and v is the velocity that the mass is raised. The power input is $P = IV$. Since the voltage is 24 volts, the current must be 0.204A.

7. Three resistors A , B , and C are connected in parallel with a 12 V power supply. You feel each resistor in turn and B is hottest while C is coolest. The resistance values of the three resistors must be such that:

Answer: $C > A > B$

Solution: The power dissipated in heat is $P = V^2/R$. The lower resistance resistor generates more heat. Thus, B has the smallest resistance, and C has the largest resistance.

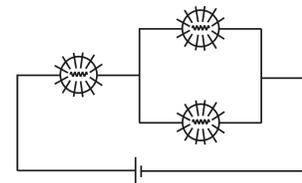
8. The resistor network in the diagram has 12V across its end terminals labeled A and B. The current through, and voltage drop across, the $2k\Omega$ resistor are, respectively:



Answer: 3mA, 6V

Solution: The resistance of the top segment is $4k\Omega$. Thus, the current through the top is $12V/4k\Omega = 3mA$. The voltage across the $2k\Omega$ resistor is $IR = 6V$.

9. The Figure shows 3 identical light bulbs wired as shown. With all bulbs lit the power supply (potential difference V) sources current I . One of the parallel pair of light bulbs is unscrewed. Ignoring temperature dependent changes of the filament resistivity, the new current sourced by the power supply is:

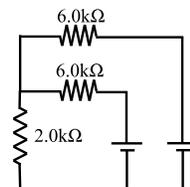


Answer: $0.75I$

Solution: Let each of the light bulbs have resistance R . The effective resistance of the circuit show is $(3/2)R$ so the current is initial $I = (2/3)(V/R)$. When one of the parallel light bulbs is removed the resistance becomes $2R$ because there are just two light bulbs in series. The current is $V/(2R) = (3/4)I$.

10. In the circuit shown the ideal power supplies are each 12 V. The current through the $2.0 k\Omega$ resistor is

Answer: 2.4 mA

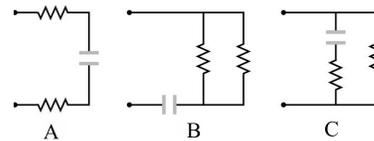


Solution: Because of the symmetry of this problem, the current through each of the $6k\Omega$ resistors is the same. Let the current through one of the $6k\Omega$ resistors be $i/2$. Then the current through the $2k\Omega$ resistor is i . Kirchoff's loop rule implies that

$$0 = -(2k\Omega)i - (6k\Omega)\frac{i}{2} + 12V,$$

which has solution $i = 2.4mA$.

11. In the circuits shown all the capacitors have the same capacitance and all the resistors have the same resistance. The capacitors are all charged to the same initial charge Q_0 . At time $t = 0$ the terminals of each circuit are electrically connected together. Rank order the time taken for the capacitor in each circuit to reach 10% of Q_0 , *shortest* time first.



Answer: B, C, A

Solution: The product RC sets the time scale for discharging (and charging). Let each of the resistors have resistance r . The resistance of the first circuit is $2r$, the resistance of the second circuit is $r/2$, and the resistance of the third circuit is r because the resistor on the far right is short circuited when the two ends are connected.

12. An ideal 6.00 V power supply is connected to a series RC circuit in which the resistor is 120Ω and the capacitor (initially uncharged) is 47.0 pF. The charge accumulated on the positive plate of the capacitor at a time $t = 10RC$ is approximately,

Answer: 0.282 nC

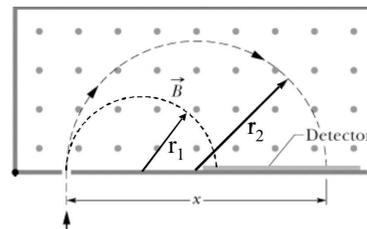
Solution: The charge as a function of time is $Q(t) = CV(1 - e^{-t/(RC)})$. Substituting in for C , V , and t , $Q(t)$ is equal to $0.282nC$.

13. A proton with velocity $\vec{v} = 56 \text{ m/s } \hat{i}$ enters a region of space in which there is a magnetic field $\vec{B} = 0.5T\hat{j}$. The electric field that must permeate this region of space to keep the proton moving in the direction of its original velocity without deflection is (in N/C):

Answer: $-28\hat{k}$

Solution: The electric force, $q\vec{E}$, must be opposite to the magnetic force, $q\vec{v} \times \vec{B}$, for there to be no net force on the proton. This implies that $\vec{E} = -\vec{v} \times \vec{B}$.

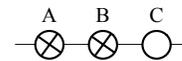
14. The Figure shows a rudimentary mass spectrometer based on charged particles entering a region with a velocity that is perpendicular to a uniform magnetic field that permeates the region (the gray dots represent a uniform magnetic field pointing out of the page). Assume that two particles having the same charge enter the B-field region with the same velocity. If $r_2 = 2r_1$ then m_2 (the mass of particle 2), is:



Answer: $2m_1$

Solution: For circular motion $F = mv^2/r = qvB$ implies that the mass is $m = qBr/v$. Since q , B , and v are the same for the two particles, the mass, m , is proportional to the radius, r , and hence $m_2 = 2m_1$.

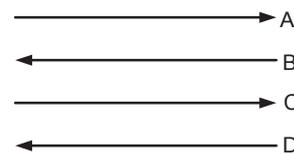
15. Three wires are equally spaced along a line (perpendicular to the page) as shown in the Figure. Wires A and B each carry current i into the page. When wire C carries no current, wire B experiences a force F . With no other currents changed, in order for wire B to experience a force $3F$ in the same direction as the original F , the current in wire C must be set equal to



Answer: $2i$, out

Solution: Wires A and B attract with force per unit length $F/l = \mu_0 i^2 / (2\pi r)$, where r is the separation between the wires. In order for wire B to experience a force of $3F$, wire C must repel wire B with a force of $2F$. This means that wire C must have a current opposite to that of wire B and have twice the current.

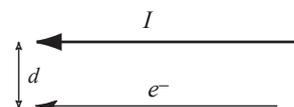
16. Four long parallel wires are arranged on a plane, as shown in the attached figure, with 2.0 cm gaps between them. Each wire carries a 6.0 A current in the direction indicated by the arrow. On the wire labeled C, what is the magnetic force per meter in N/m?



Answer: 1.8×10^{-4}

Solution: The forces on wire C due to wire B and wire D cancel. The force on wire C due to wire A is $F/l = \mu_o(6A)^2/2\pi(4cm)$.

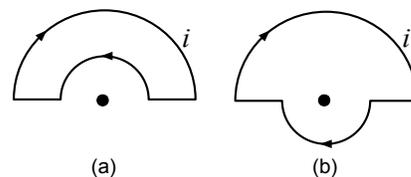
17. An electron moves at a speed of $v = 3 \times 10^7$ m/s parallel to and a distance $d = 10$ cm from a conducting wire carrying a current of $I = 40$ A. If the directions of the electron and current are as shown in the figure, what is the magnitude (in N) and direction of the force experienced by the electron?



Answer: 3.8×10^{-16} away from the wire

Solution: The magnetic field at the electron is $B = \mu_o i / (2\pi d)$ and coming out of the page. The force on the electron is $q\vec{v} \times \vec{B}$ has magnitude $|e|vB$ and is away from the wire.

18. In the Figure two loop configurations are shown with identical currents. Each wire loop consists of a semicircle of radius 10 cm, a smaller concentric semicircle of radius 6.67 cm, and two radial straight lengths, all in the same plane. The magnitude of the magnetic field produced at the center marked by the dot is $10 \mu\text{T}$ in (a). What is the magnitude of the magnetic field at the dot in (b)?



Answer: $50 \mu\text{T}$

Solution: Let the radii of the inner and outer semicircles be r_{in} and r_{out} , respectively. The magnitudes of the magnetic fields in cases (a) and (b) are

$$|B_a| = \frac{1}{2} \left(\frac{\mu_o i}{2r_{in}} - \frac{\mu_o i}{2r_{out}} \right)$$

$$|B_b| = \frac{1}{2} \left(\frac{\mu_o i}{2r_{in}} + \frac{\mu_o i}{2r_{out}} \right)$$

The ratio of the fields for these two cases is $|B_b|/|B_a| = (r_{out} + r_{in})/(r_{out} - r_{in}) = 5$.

19. A proton executes helical motion in a uniform B field at a frequency of 820 kHz. It moves at an angle of 30° relative to the direction of the field. What is the magnitude of the B field?

Answer: 0.054 T

Solution: The magnetic force on the proton has magnitude $F = qv_{\perp}B = ma = mv_{\perp}^2/r$. The angular frequency of rotation is $\omega = 2\pi f = v_{\perp}/r = qB/m$. One can solve for the magnetic field because we know f , q , and m .

20. A 5-turn circular loop of wire of radius 23 cm carries a current of 15 A. Its magnetic moment is oriented at an angle 50° to a uniform magnetic field of 3.7 T. What energy is required to rotate the loop from its lowest energy position to this orientation?

Answer: 16 J

Solution: The magnetic moment of the loop is $\mu = NiA$, where $A = \pi r^2$. The potential energy of a loop in a magnetic field $U = -\mu B \cos(\theta)$. The work required to move from the lowest energy configuration, $\theta = 0$ and $U = -\mu B$, to the configuration with $\theta = 50^\circ$ and $U = -\mu B \cos(50)$ is $W = \mu B(1 - \cos(50))$.