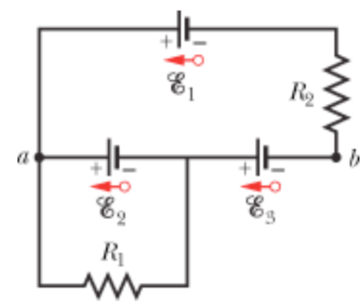
Exam 2 Solutions

Problem 1

In the circuit shown, R_1=100 Ω, R_2=25 Ω, and the ideal batteries have EMFs of ε_1 = 6.0 V, ε_2 = 3.0 V, and ε_3 = 1.5 V. What is the magnitude of the current flowing through resistor R_2?



(1) 60 mA (2) 30 mA (3) 40 mA (4) 20 mA (5) 240 mA

Let's use Kirchoff's loop rule for the upper loop, where we go around clockwise from point b:

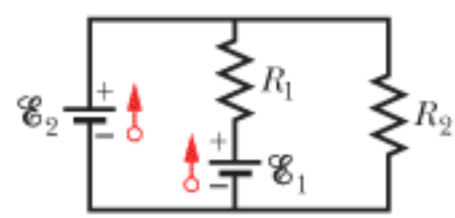
$$\varepsilon_3 + \varepsilon_2 - \varepsilon_1 - i_2 R_2 = 0$$

$$i_2 = \frac{\varepsilon_3 + \varepsilon_2 - \varepsilon_1}{R_2} = \frac{-1.5}{25} = -0.06V$$

$$\Rightarrow |i_2| = 60 \text{ mV}$$

Problem 2

In the circuit shown, R_1=15 Ω, R_2=60 Ω, and the ideal batteries have EMFs of ε_1 = 4.0 V and ε_2 = 9.0 V. What is the magnitude of the current flowing through resistor R_1?



(1) 0.3 A (2) 0.15 A (3) 0.5 A (4) 0.9 A (5) 0.08 A

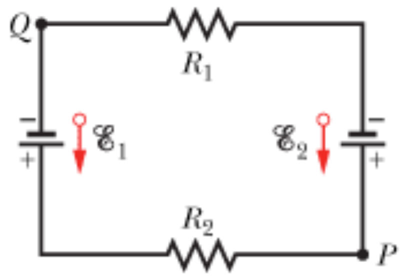
Let's use Kirchoff's loop rule for the left loop, where we go around clockwise:

$$\varepsilon_2 - i_1 R_1 - \varepsilon_1 = 0$$

$$i_1 = \frac{\varepsilon_2 - \varepsilon_1}{R_1} = \frac{5}{15} = 0.33$$

Problem 3

In the circuit shown, the ideal batteries have EMFs of $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6$ V and the resistances are R_1=30 Ω and R_2=10 Ω. If the potential at Q is defined to be 4.5 V, what is the potential at P?



(1) 15.0 V (2) 10.5 V (3) 6.0 V (4) 24.0 V (5) -10 V

First solve for the current in the circuit:

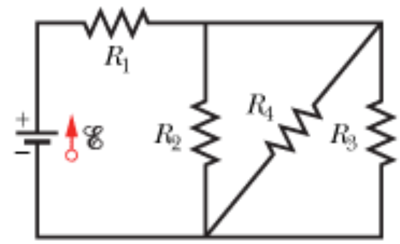
$$\varepsilon_1 - iR_2 - \varepsilon_2 - iR_1 = 0$$
$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{6}{40} = 0.15$$

Now let's add the potential difference to go from point Q to P:

$$V_P = V_Q + \varepsilon_1 - iR_2$$
$$V_P = 4.5 + 12 - 0.15(10) = 15$$

Problem 4

In the shown figure, $R_1 = R_2 = R_3 = 50\ \Omega$, $R_4 = 100\ \Omega$, and the ideal battery has EMF = 6 V. What is the equivalent resistance of the circuit?



(1) 70 Ω (2) 120 Ω (3) 20 Ω (4) 90 Ω (5) 250 Ω

The 3 resistors R_2, R_3, and R_4 are in parallel with equivalent resistance:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = \left(\frac{5}{100}\right)^{-1} = 20$$

Then adding resistor R_1 in series we have $R_{eq}=50+20=70\Omega$

Problem 5

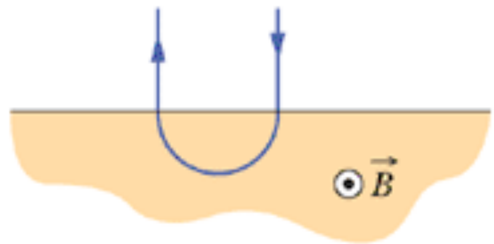
A capacitor with an initial potential difference of 50 V is discharged through a $10^6\ \Omega$ resistor when a switch between them is closed at t=0. At t=2 s, the potential difference across the capacitor is 25 V. What is the capacitance of the capacitor?

(1) $2.9\text{x}10^{-6}$ F (2) $2.0\text{x}10^{-6}$ F (3) $3.5\text{x}10^{5}$ F (4) $3.5\text{x}10^{-7}$ F (5) 0.5 F

$$V = \frac{q(t)}{C} = \frac{q_0}{C} e^{-t/RC}$$
$$\Rightarrow V = V_0 e^{-t/RC}$$
$$V_0 = 50$$
$$V(2) = 25 = 50 e^{-2/RC}$$
$$\Rightarrow \ln 2 = \frac{2}{RC}$$
$$\Rightarrow C = \frac{2}{R \ln 2} = 2.9 \times 10^{-6} \text{ F}$$

Problem 6

An ion of charge q= +2e and unknown mass is sent into a region with a uniform magnetic field of magnitude B=0.5 T as shown in the figure. The charged ion makes a U-turn in the region of the magnetic field as a semicircle of radius 1 m and exits after a time t=7.8 x10-6 s. What is the mass of the ion in kg?



(1) 4.0x10^{-25} (2) 1.6x10^{-19} (3) 2.0x10^{-25} (4) 1.2x10^{-24} (5) 1.0x10^{-25}

We can use the relation between momentum and radius of curvature for a particle moving in a magnetic field:

$$mv = qBr$$

$$\Rightarrow m = \frac{qBr}{v}$$

The velocity can be found from the time it takes to complete the semicircle:

$$d = \pi r = vt$$

$$\Rightarrow v = \frac{\pi r}{t} = \frac{\pi}{7.8\times10^{-6}s} = 4\times10^{5}\ \text{m/s}$$

So the mass is given by

$$m = \frac{qBr}{v} = \frac{2eBr}{v} = \frac{2\left(1.6\times10^{-19}\right)(0.5)1}{4\times10^{5}} = 4\times10^{-25}\ \text{kg}$$

This is the mass of Uranium.

Problem 7

A beam of electrons (``cathode rays'') with a velocity of v = $3.0\text{x}10^7$ i m/s is sent into a region where there is a uniform magnetic field of B = $5.0\text{x}10^{-4}$ j T. What electric field E is necessary (direction and magnitude) so that the electrons continue traveling in a straight line without deflection by the magnetic field?

(1) $-1.5\text{x}10^4$ k T (2) $1.5\text{x}10^4$ k T (3) $5.0\text{x}10^{-4}$ j T (4) $-5.0\text{x}10^{-4}$ j T (5) $2.4\text{x}10^{-15}$ i T

The Lorentz Force equation is
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

So the condition of no net force (so no deflection) is:
$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$
$$= -\left(3 \times 10^7 \hat{\mathbf{i}}\right) \times \left(5 \times 10^{-4} \hat{\mathbf{j}}\right)$$
$$= -1.5 \times 10^4 \hat{\mathbf{k}}$$

Problem 8

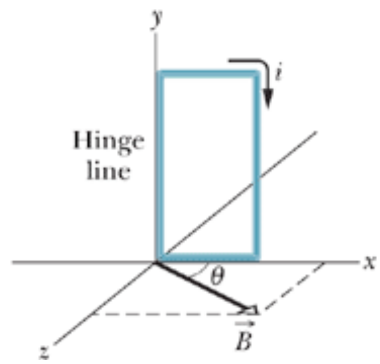
An electron moves in the -i direction, through a uniform magnetic field in the -j direction. The magnetic force on the electron is in the direction:

(1) -k (2) k (3) -j (4) j (5) -i

The magnetic force is $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$
So working with directions only:

$$F = (-e)\left(-\hat{\mathbf{i}}\right) \times \left(-\hat{\mathbf{j}}\right) = -\hat{\mathbf{k}}$$

Problem 9



The figure shows a rectangular loop of wire of dimensions 10 cm by 5.0 cm.
It carries a current of 0.2 A and it is hinged along one long side. It is mounted in the xy plane, and it makes an angle of Θ=30° to the direction of a uniform magnetic field of 0.25 T. What is the magnitude of the torque acting on the loop about the hinge line?

(1) 2.2×10^{-4} N m (2) 1.3×10^{-4} N m (3) 5.0×10^{-3} N m (4) 1.0×10^{-3} N m (5) 0 Nm

The torque on the loop is calculated below. Note that the angle between the magnetic dipole moment and the magnetic field is 90° + θ, or 120°.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$\Rightarrow \tau = \mu B \sin\theta$$
$$\mu = iA = (0.2A)(0.05m)(0.01m)$$
$$\tau = (0.2A)(0.05m)(0.01m)(0.25T)\sin 120^\circ$$
$$\tau = (0.2A)(0.05m)(0.01m)(0.25T)\frac{\sqrt{3}}{2}$$
$$\tau = 2.2 \times 10^{-4} \text{ Nm}$$

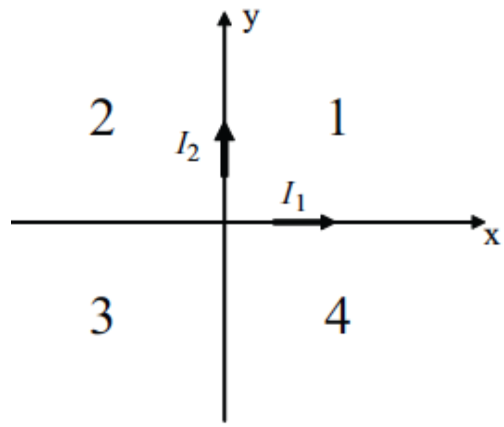
Problem 10

A magnetic field CANNOT:

(1) change the kinetic energy of a charge
(2) exert a force on a charge
(3) accelerate a charge
(4) change the momentum of a charge
(5) exist

The only thing a magnetic field cannot do is perform work on an object, hence it cannot change the kinetic energy of a charge.

Problem 11



Two wires are aligned with x- and y-axes and carry currents I_1 along x-axis and I_2 along y-axis as shown. Which of the four quadrants have points in (x, y)-plane where the magnetic field is zero.

(a) 1 & 3
(b) 2 & 4
(c) all
(d) none
(e) the answer depends on the relative magnitudes of the two currents

Answer:

Using right hand rule (align the right hand thumb along the current and curl the fingers around the thumb), the direction of magnetic fields induced by the currents are as follows:

Current	Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
I_1	out	out	in	in
I_2	in	out	out	in

Hence, the two fields can potentially cancel only in quadrants 1 and 3, where they point in different directions. This eliminates answers (b) and (c).

The magnitudes of fields in Quadrant 1 are:

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{y} \quad \text{and} \quad B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{x}$$

The two fields are equal in magnitude along the line:

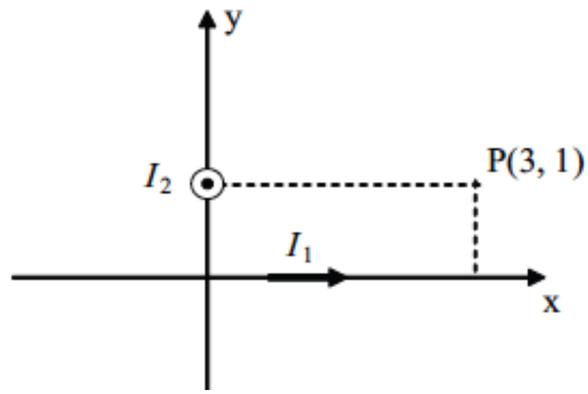
$$y = \frac{I_1}{I_2}x$$

Hence, the net field along this line is zero.
The exact same argumentation holds true for Quadrant 3.

Therefore, the right answer is (a)

Problem 12

One wire is aligned with x-axis and carries currents I_1=1A. Another wire carries current I_2=2 A out of page through the point (x, y)=(0 m, 1 m) as shown. What is the magnitude of the magnetic field in Tesla at point P(3 m, 1 m)?



(a) 2.4E-7
(b) (b) 3.3E-7
(c) (c) 6.7E-8
(d) (d) 1.5E-6
(e) (e) 4.2E-7

Answer:

Using right hand rule (align the right hand thumb along the current and curl the fingers around the thumb), the direction of magnetic fields induced by the two currents at point P are as follows:

Current I_1

out of the page; its magnitude is $B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{y} = 2\cdot 10^{-7}\frac{1}{1} = 2\cdot 10^{-7}$ T

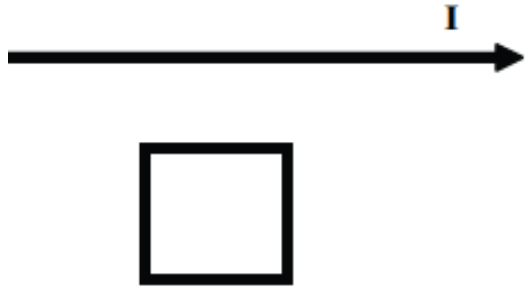
Current I_2

up; its magnitude is $B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{x} = 2\cdot 10^{-7}\frac{2}{3} = 1.333\cdot 10^{-7}$ T

Since the two fields are perpendicular to each other, we use the Pythagorean theorem for calculating the total field magnitude: $B_{TOT} = 2.4\cdot 10^{-7}$ T

Problem 13

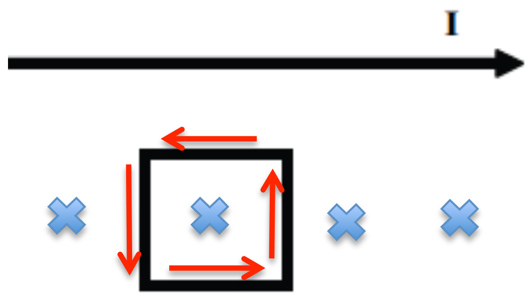
A current I through an infinitely long wire increases. A square loop made of a conductor is placed next to the wire carrying the current as shown (the loop and the wire are in the plane of the sheet). What is the direction of a net force exerted on the loop?



(a) down
(b) up
(c) out of the page
(d) into the page
(e) zero

Answer:

As the current increases, so does the flux of magnetic field through the loop. The field is into the page, so is its flux through the loop. Since the current I is increasing, the flux is also increasing. The change in the flux is into the page.



Induced EMF, $E = -\frac{d\Phi}{dt}$, is counterclockwise (right hand thumb is along the change in the flux, curl your fingers, and change the direction). The induced currents (shown in red), being in the magnetic field of the wire, will experience forces as follows:

Up side of the loop: force F_1, down

Left side of the loop: F_2, right

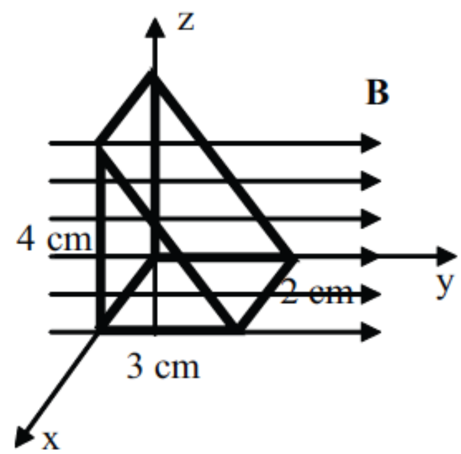
Bottom side of the loop: force F_3, up, but it is smaller than F_1 as the filed is weaker away from the wire (B ~ I/r)

Right side of the loop: F_4, left, same in magnitude as F_2, by symmetry

Therefore, F_2 and F_4 cancel each other and the net force of F_1 and F_3 points down.

Problem 14

A prism with sides 2, 3, 4 cm is placed in uniform magnetic field of 1 T pointing along y-direction (see drawing). Find the magnetic field flux (in Webbers) through the entire surface area of the prism.



(a) 0
(b) 10×10^{-4}
(c) 8×10^{-4}
(d) 18×10^{-4}
(e) 16×10^{-4}

Answer:

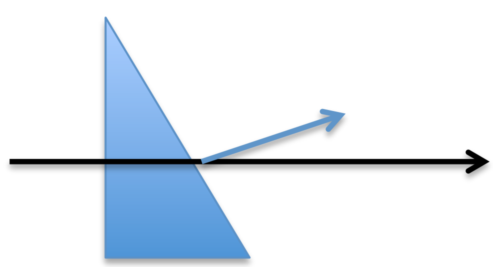
$\Phi = B \cdot A \cdot \cos\theta$, where θ is an angle between a vector normal to a surface of area A and a vector of a magnetic field of strength B.

a) The flux through the front, back, and bottom sides of the prism are zero since the field is parallel to these three surfaces (θ=90°, cosθ=0).

b) The flux through the left side is

$(1\text{ T}) \times (0.04 \times 0.02\text{ m}^2)$, inward

c) The right side of the prism has sides 2 cm (as marked) and 5 cm (from Pythagorean theorem). The angle between a vector normal to a surface of the right side and the magnetic field vector is the same as top angle of the front side of the prism. Cosine of that angle is 4/5. Therefore, the flux through the right side is

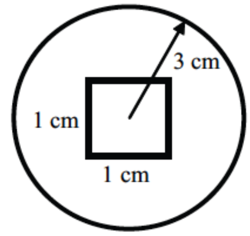


$(1\text{ T}) \times (0.05 \times 0.02\text{ m}^2) \times (4/5)$, outward.

d) The net flux through all five surfaces is: 0 + 0 + 0 + (-0.008) + (+0.008) = 0

Problem 15

A square, single-turn wire loop 1 cm on a side is placed inside a solenoid that has a circular cross section of radius 3 cm, as shown. The solenoid is 20 cm long and wound with 1000 turns of wire carrying a current of 3 A. If the current in the solenoid is reduced to zero in 0.3 s, find the magnitude of the average induced emf in the loop.



(1) 6.3μV (2) 0.63μV (3) 35μV (4) 18μV (5) 1.3μV

Answer:

Magnetic field inside the solenoid is $B = \mu_0 In$, where n=1000/(0.2 m).

Induced EMF is

$$|E| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{Ba^2}{\Delta t} = \frac{\mu_0 In \cdot a^2}{\Delta t} = \frac{\mu_0 I \cdot (N/L) \cdot a^2}{\Delta t} = \frac{4\pi \cdot 10^{-7} \cdot 3 \cdot (1000/0.2) \cdot 0.01^2}{0.3} = 6.28 \cdot 10^{-7}\ \text{V}$$

Problem 16

A 25-turn circular coil of wire has a diameter of 1 m. It is placed with its axis along the direction of Earth's magnetic field of 50μT. Then, the coil is flipped 180° in 0.2 s. An average emf of what magnitude is generated in the coil?

(1) 9.8 mV (2) 4.9 mV (3) 0.39 mV (4) 0.20 mV (5) 1.6 mV

Answer:

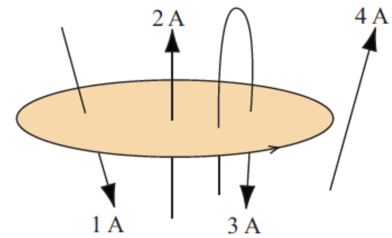
Induced EMF is

$$|E| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{B\pi r^2 N - (-B\pi r^2 N)}{\Delta t} = \frac{2B\pi r^2 N}{\Delta t} = \frac{2 \cdot 50 \cdot 10^{-6} \cdot 3.14 \cdot 0.5^2 \cdot 25}{0.2} = 9.8 \cdot 10^{-3}\ \text{V}$$

Problem 17

There are a number of wires in space carrying different currents (see figure). What is the result of integrating the **B**-field along a circular path of radius $r = 1$ m in the direction as shown in figure?

(1) 1.3×10^{-6} Tm
(2) -1.3×10^{-6} Tm
(3) 1.2×10^{-7} Tm
(4) -1.2×10^{-7} Tm
(5) -6.5×10^{-8} Tm



Answer:

The magnetic filed integral over a loop is related to the total current going through the loop. The current is counted as positive if it is in the same direction as the right hand thumb points, when fingers are curled around the loop in the direction of integration.

$$\oint B\,ds = \mu_0 I_{TOT} = 4\pi \cdot 10^{-7} \cdot (-1+2-3+3) = 4\pi \cdot 10^{-7}\ \text{Tm}$$

Problem 18

A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a N-turn circular wire loop of radius r. The magnitude of the field varies with time according to $B = B_0\, e^{-t/\tau}$, where B_0 and τ are constants. Find an expression for the EMF magnitude in the loop as a function of time.

(1) $\dfrac{\pi r^2 B_0 N}{\tau}\, e^{-t/\tau}$ (2) $\dfrac{\pi r^2 B_0 N}{\tau}\,(1 - e^{-t/\tau})$ (3) $\pi r^2 B_0 N \tau\, e^{-t/\tau}$ (4) $\pi r^2 B_0 N \tau\,(1 - e^{-t/\tau})$ (5) $\pi r^2 B_0 N \tau$

Answer:

$$|E| = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d}{dt} B\pi r^2 N\right| = \left|\frac{d}{dt} B_0 e^{-t/\tau}\pi r^2 N\right| = \left|B_0 \pi r^2 N \frac{d}{dt} e^{-t/\tau}\right| = B_0 \pi r^2 N \frac{1}{\tau} e^{-t/\tau}$$

Problem 19

One of the experiments at the Large Hadron Collider is using a superconducting solenoid of 6 m in diameter and 12 m in length. Once energized, the solenoid has a 4-T magnetic field inside. Calculate total the stored energy in the magnetic field inside the solenoid.

(1) 2 GJ (2) 300 MJ (3) 10 MJ (4) 500 kJ (5) 40 kJ

Answer:

The total energy U is energy density $u = \frac{1}{2\mu_0}B^2$

times the inner volume of the solenoid $\pi r^2 L$:

$$U = \left(\frac{1}{2\mu_0}B^2\right)\left(\pi r^2 L\right) = \left(\frac{4^2}{2\cdot 3.14\cdot 10^{-7}}\right)\cdot\left(3.14\cdot 3^2\cdot 12\right) = 2\cdot 10^9 \text{ J}$$

Problem 20

A fuse of zero resistance, an inductor with inductance L=5 H, a battery with *emf* $\mathcal{E}$=10 V, and a switch are connected in series and make one loop. If the current through the fuse reaches the maximum allowed $I_{\max} = 3$ A, the fuse "blows" and thereafter has infinite resistance. The switch is initially open and closes at time $t = 0$ s. How long will it take since closing the switch till the fuse blows?

(1) 1.5 s (2) 0.06 s (3) 17 s (4) 6 s (5) 0.17 s

Answer:

The sum of difference potentials over all elements in the closed loop of any circuit is zero. Right after closing the switch, the sum of difference potentials over all elements in the circuit is:

$$\Delta V_{battery} + \Delta V_{inductor} + \Delta V_{fuse} + \Delta V_{switch} = 0$$

$$E + \left(-L\frac{dI}{dt}\right) + 0 + 0 = 0$$

$$\frac{dI}{dt} = \frac{E}{L}$$

$$I = \frac{E}{L}t$$

From where, it takes 1.5 s to reach 3A current.