

Phy 2049

Exam III Solutions

July 20, 2009

1. A ray of light from medium a, whose index of refraction is 1.89, enters medium b. The angle of incidence θ_a is 50° and the angle of refraction θ_b is 60° . What is the index of refraction of medium b?

- (1) 1.7 (2) 1.4 (3) 1.3 (4) 1.2 (5) 1.5

Snell's law $n_a \sin \theta_a = n_b \sin \theta_b$ or $n_b = 1.89 \sin 50 / \sin 60 = 1.67$

2. In the Figure $L_1 = 20$ mH, $L_2 = 30$ mH and $L_3 = 40$ mH.

The equivalent inductance (in mH) between the wire ends is:

- (a) 22 (b) 90 (c) 9.2 (d) 40 (e) 66

Here L_1 and L_2 are in series (equivalent inductance 50mH) and the combination is parallel to L_3 . The final equivalent inductance is $L = 50 \times 40 / (50 + 40) = 22$ mH.

3. The light from a professor's laser pointer can be considered as an electromagnetic plane wave. If the laser beam has 5 mW of power and illuminates a circular spot 2.5 mm in radius on a screen, what is the maximum electric field amplitude (E_m in N/C) at the screen?

- (1) 440 (2) 311 (3) 622 (4) 550 (5) none of these

The intensity of the light is $I = 5 \text{ mW} / (\pi (2.5 \text{ mm})^2) = 255 \text{ W/m}^2$. It is related to the maximum electric field amplitude via $I = E_m^2 / (2c\mu_0)$. $E_m = 438 \text{ V/m} = \text{N/C}$.

4. A horizontal beam of unpolarized light is incident upon a stack of 4 polarizers with axes of polarization, in order and measured clockwise from the vertical, at 30° , 75° , 120° and 180° . What is the ratio of the intensity of the transmitted beam to that of the incident beam?

- (1) 0.031 (2) 0.063 (3) 0.023 (4) 0.047 (5) 0.0063

The intensity of a transmitted beam is $I = I_o \cos^2 \theta$ where θ is the relative angle between the polarization of the incoming light and the polarization axis. In this problem, after the first polarizer the intensity will be $1/2$ (from an unpolarized source). But the successive polarizers will be at angles 45, 45 and 60. The final intensity ratio $I/I_o = 0.5 \times (1/2)^2 (1/2)^2 = (1/2)^5 = 0.03125$.

5. In the circuit shown, $L = 45$ mH, $R = 4.6 \Omega$ and $V = 12.0$ V. The switch S has been open for a long time then is suddenly closed at $t = 0$. At what value of t (in msec) will the current in the inductor reach 1.1 A?

- (1) 5.36 (2) 8.44 (3) 2.88 (4) 19.0 (5) None of these

The current is given by $I(t) = V/R [1 - \exp(-t/\tau)]$ where $\tau = L/R = 45/4.6 = 9.8$ ms. The final current in the inductor is $I_o = V/R = 2.61$ A. The time when the current in the inductor is 1.1 A has to be solved from the exponential expression.

$$t = -\tau \ln(1 - I/I_o) = 5.357 \text{ ms}$$

6. Refer to the previous problem. What is the total energy stored in the inductor a long time after the switch is closed?

- (1) 0.15 J (2) 0.048 J (3) 0.76 J (4) 0.2 J (5) None of these

$$E = \frac{1}{2} L I^2 = 0.153 \text{ J}$$

7. Light traveling horizontally enters a right prism through the hypotenuse, as shown in the Figure. The index of refraction of the prism is $n = 2$. At what angle is the light deflected from horizontal?

- (1) none of these (2) 26° (3) 19° (4) 45° (5) 36°

In the picture, light comes in horizontally and goes out at an angle below the horizontal. Application of Snell's law and some geometry leads to an answer = 67.34° . If the top vertex is μ , then the answer is given by $\sin^{-1}[n \sin\{\mu - \sin^{-1}(\sin\mu/n)\}]$. For $\mu = 50^\circ$ and $n = 2$, the answer is 67.34°

8 Two identical thin lenses of +6 cm focal lengths are separated by 20 cm. An object lies 11 cm from the first lens. The magnification of the 2 lens system is:

- (1) 9 (2) 21 (3) 1.2 (4) 7.5 (5) 4.6

For the first lense $q_1 = p_1 f_1 / (p_1 - f_1)$. Since $p_1 = 11$ cm and $f_1 = 6$ cm, we see that the first image is at 13.2 cm from the first lens and towards the second lens. The magnification of the first lens $m_1 = -q_1/p_1 = -1.2$. The image is real ($q_1 > 0$), larger and is inverted. The object is located at $p_2 = 20 - 13.2 = 6.8$ cm from the second lens. Thus $q_2 = p_2 f_2 / (p_2 - f_2) = 51$. The magnification is $m_2 = -q_2/p_2 = -51/6.8 = -7.5$. The image is straightened by the second lens, it is real and much larger. The overall magnification is $m = m_1 \times m_2 = 1.2 \times 7.5 = 9$.

9. A car radio uses an LC circuit and a variable capacitor to tune to different radio stations. The value of the capacitance to tune to a radio station of 1000kHz is C. What must its value be to tune to a station at 200kHz?

- (1) 25C (2) 20C (3) 15C (4) 10C (5) 5C

The resonance frequency is $\omega = 1/\sqrt{LC}$. To reduce the frequency by a factor of 5, the capacitance must increase by a factor of 25.

10. An object is placed 6 cm in front of a concave mirror of radius of curvature 20 cm. What is its magnification?

- (1) 2.5 (2) 1.43 (3) -2.5 (4) -1.43 (5) none of these.

$m = -q/p = f/(f-p) = 2.5$. The focal length is $f = R/2 = 10\text{cm}$.

11. The average intensity of light shining on a desk surface is 300 mW/m^2 . Assuming that the light is in the form of an electromagnetic plane wave, what is the maximum electric-field amplitude?

- (1) 15 V/m (2) 21V/m (3) 8.7 V/m (4) 10.63 (5) none of these

$I = E_m^2/2\mu_0c$. Solve for $E_m = 15\text{ V/m}$.

12. An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum works as follows: an LC oscillator produces a sinusoidal current in the antenna, which generates the wave, traveling outward at the speed of light. What is the wavelength (in meters) of the wave emitted by this system if $L = 0.323\ \mu\text{H}$ and $C = 45.0\text{pF}$?

- (1) 7.19 m (2) 1.14 m (3) 719 m (4) 114 m (5) None of these

The wavelength $\lambda = c/f = 2\pi c \sqrt{LC}$ where the lower case c is the speed of light and C is the capacitance. Plugging in the values, $\lambda = 7.19\text{ m}$.

13. A charged capacitor and an inductor are connected in series. At time $t = 0$ the current is zero, but the capacitor is charged. If T is the period of the resulting oscillations, the next time after $t = 0$ that the energy stored in the magnetic Field of the inductor is a maximum is:

- (1) $T/4$ (2) T (3) $T/2$ (4) none of these (5) $2T$

That would happen when the current is maximum. The first time it would happen at $t = T/4$