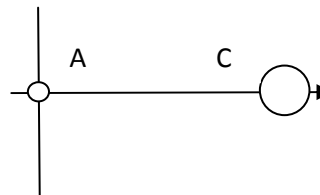Solutions Exam 1
Phy2049
Summer 2010

1. Two charges, q_1 = (-1, 1) C and q_2 = (-4, 9) C, are placed
along the x-axis a distance L apart with charge q_1 at the
origin and q_2 at x=L (see figure). A third charge, q_3 =
(+4/9, -9/16) C, is also placed along the x-axis such that there
is no net Coulomb force on any of the charges. What is
position of this charge along the x axis in units of L, i.e. what
is x/L?

A   C

Answer (1/3, ¼)

Solution: The force on charge 1 consists of repulsion due to charge 2 and the force due to the
unknown charge 3 in the middle. Charge 3 must be of opposite sign to cancel the force due to 2.

$$F_A = kq_1 \left[\frac{q_3}{x^2} - \frac{q_2}{L^2} \right] = 0$$
$$\frac{x}{L} = \sqrt{\left(\frac{q_3}{q_2} \right)}$$

The charges here are all magnitudes since their signs are already used in determining the
directions of the forces.

2. Suppose we have an insulating spherical ball of uniform charge density ρ and radius R. At
what radius or radii from the center of the sphere is the electric field strength reduced by a factor
of (16, 9) from the electric field strength at the surface?

Answer: (R/16 and 4R, R/9 and 3R)

The electric field due to a uniform distribution of charges in a sphere shaped insulator is given by

$$E = k\frac{\rho}{3}r \qquad r < R$$
$$= k\frac{\rho R^3}{3r^2} \qquad r > R$$

The electric field can be smaller at two locations, smaller than R where the field increases
linearly and at a position larger than R where the electric field decreases with the square of the
distance. The answer follows.

3.1 Two isolated conducting spheres are separated by a large distance. Sphere 1 has a radius of R
and an initial charge 3Q while sphere 2 has a radius of (3R, 2R) and an initial charge (7Q, 6Q). A
very thin copper wire is now connected to the spheres. Charge flows between the spheres until
they reach the same electrical potential. How much charge will be transferred from sphere 2 to
sphere 1? (Note that the charge transferred can be positive, negative or zero .)

Answer: (–Q/2, zero)

The total charge on the two balls remains the same. The final charges (Q'$_1$, Q'$_2$ on the two balls are determined by equal potentials

$$V = \frac{Q_1'}{R} = \frac{Q_2'}{mR} \qquad Q_1' + Q_2' = (m+1)Q_1' = Q_1 + Q_2 = (10Q, 9Q)$$

For the first set of numbers, m = 3, and Q$_1$' = 2.5Q, i.e. the charge transferred for 2 to 1 is <mark>-0.5Q</mark>.

4. A charged oil drop with a mass of 2 x 10^{-4} kg is held suspended by a downward electric field of 300N/C. The charge on the drop is: (g = 9.80 m/s^2)

Answer: -6.5 X 10^{-6} C

The charge sign has to be negative to get an upward force from a downward electric field. Given that q = mg/E = <mark>6.53 x 10^{-6}</mark> C

5.1. Positive charge +Q is uniformly distributed on the upper half of a semi-circular rod and negative charge -Q is uniformly distributed on the lower half. What is the direction of the electric Field at point P, the center of the semicircle, in degrees of angle measured from the positive x axis?
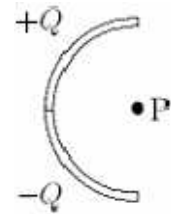
Answer: 270º

The total electric field is pointed downwards, the angle measured from the positive x axis is <mark>270º</mark>.

6. A particle with charge 3μC is placed at the origin. An identical particle, with the same charge, is placed 2m from the origin on the x axis, and a third identical particle, with the same charge, is placed 2m from the origin on the y axis. The magnitude of the force on the particle at the origin is:

Answer: 2.9 x 10^{-2} N

The total magnitude of the force as the vector sum of the forces due to the two other charges:
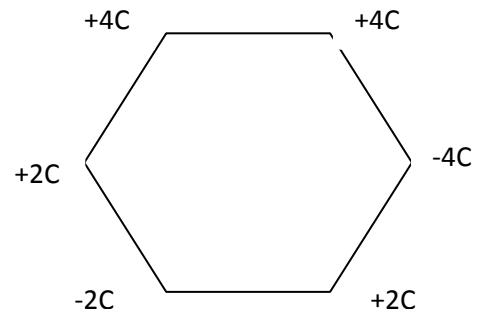
$F = \sqrt{2}k\frac{q^2}{d^2} =$ <mark>0.0286 N</mark>

7. What is the electric potential at the origin (center) of the hexagonal array of charged particles. The side length s = 20cm and q = 5 x 10^{-9}C.

Answer: 1350 V

The charges are all equidistant. The potential becomes

$V = \frac{k}{s}\Sigma\, q_i =$ <mark>1350 V</mark>

8. A point particle with charge q is at the center of a Gaussian surface in the form of a (octahedron, hexahedron). The electric flux through any one face of the cube is:

Answer: $(q/8\varepsilon_o, q/6\varepsilon_o)$

9. A 140 pF capacitor is charged to a potential difference of 60 V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to (48, 52.5, 56) V, what is the capacitance of this second capacitor?

Answer: (35, 20, 10) pF

In the beginning $Q = C_1 V_1$. In the end $V_2 = Q/(C_1 + C_2)$. Thus $C_2 = C_1(V_1/V_2 - 1)$.

10. Three identical light bulbs A, B and C are connected in series to a constant voltage source. A wire is then connected across B. What is the brightness of A relative to its former brightness?

Answer: 2.25

The power consumed in each bulb in the beginning is $P = V^2/9R$. Putting a wire across B shorts that bulb and tales it out. Then the power consumed by each bulb is $V^2/4R$. The intensity ratio for a bulb is $9/4 = 2.25$.

11. In the circuit shown, what is the current (in amps) flowing through the 18V battery?

Answer: 1

Take the current through the left arm as clockwise I_1 (going up through the 18V battery. In the middle the current is I_2 also going up. In the right branch it will be I_1+I_2 going down. Take the outside loop and the right loop. The equations for potential drop are

$18-6I_1-24(I_1+I_2) = 0$ $5I_1 + 4I_2 = 3$
$6 - 24(I_1+I_2) - 12 I_2 = 0$ $4I_1 + 6I_2 = 1$

Solve these equations. The answer is $I_1 = 1A$ and $I_2 = -1/2$ A.
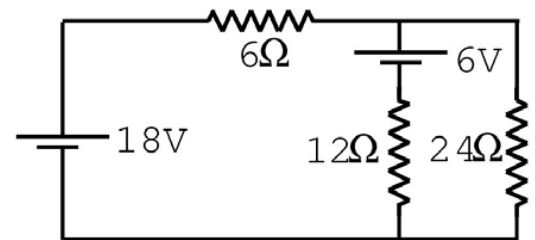
12. What is the equivalent resistance between points F and H? Each Resistor is 5 Ω.

Answer R/2 = 2.5Ω
There are 3 resistances in parallel, 2R, R and 2R.