Exam II Solutions
Phy 2049
Summer 2010

1. At a certain position outside Gainesville, the magnetic field of the Earth is $39\mu T$, horizontal to the surface, and directed due North. If the magnitude of the total field is exactly zero 8 cm above a long straight, horizontal wire that carries a constant current i, what is the magnitude and direction of the current?

(1) 16 A, west to east (2) 8 A, north to south (3) 16 A, east to west (4) 4.2 A, west to east (5) 8 A, south to north

The question was rephrased to,

At a certain position outside Gainesville, the magnetic field of the earth is horizontal to the surface, pointed due north, and is 39μT in magnitude. If the magnetic field 8 cm above a current carrying horizontal wire is zero, what are the magnitude and the direction of the current?

The current carrying wire has to produce a field of 39μT, pointed due south, at a distance of 8 cm above it. The current has to be directed west to east.

$$B = \frac{\mu_o i}{2\pi r} => i = \frac{2\pi r}{\mu_o} B = \boxed{15.6\ A}$$

2. A wire of length L carries a current i. If the wire is bent into a circular coil of N turns and placed into a uniform magnetic field B, what is the magnitude of the maximum possible torque on the loop?

(1) $L^2iB/4\pi N$ (2) $LiB\pi/2N$ (3) $2L^2iB/\pi^2N^2$ (4) $3LiB/\pi N$ (5) $L^2\pi iB/4N^2$

The maximum torque on a set of N coils is NIAB. The area $A = \pi r^2$. But the total length of wire used $2\pi rN = L$. It follows that $r = L/2\pi N$. The answer follows on substitution.

3. A conducting wire is formed into two long, semi-infinite straight sections connected by a quarter circle of radius R, as shown in the figure. What is the magnitude and direction of the magnetic field at the center of the quarter circle?

(1) $(\mu_o i/4\pi R)(2 + \pi/2)$, out of the page   (2) $\mu_o i/4\pi R$, out of the page (3) $\mu_o i/2\pi R$, into the page (4) $\mu_o i/4R$, into the page   (5) none of these.
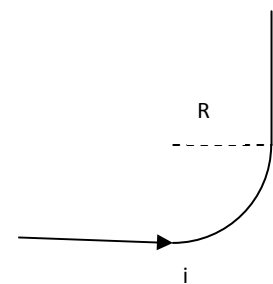
The magnetic fields, due to all elements are in the same direction, they add out of the page. The field due to a half infinite wire is $\mu_o i/4\pi R$. Magnetic field due to a

R

i

Problem 3

quarter circle is $\mu_o i/8R$. Two wires and a quarter circle added give you answer (1).

4. A hollow hemispherical "bowl" of radius R is placed in a *uniform* magnetic field of magnitude B which is parallel to its axis. What is the magnetic flux Φ_B through the bowl?

(1) $\pi R^2 B$ (2) $2\pi R^2 B$ (3) $4\pi R^2 B$ (4) $6\pi R^2 B$ (5) none of these
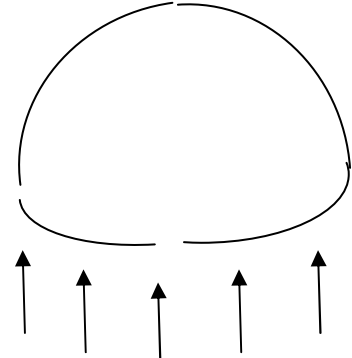
I was very pleased with the response to this question.

Strictly speaking there is no exception to the statement that through any closed surface, the magnetic flux is zero. I had revised the question to asking for the flux through the top surface of the bowl in picture. Here the subtlety is that while the total surface area of the hemispherical bowl is $2\pi R^2$, the flux is defined as B.dA. The correct answer is the first answer.

Problem 4

5. In the circuit shown, $L = 56$ mH, $R = 4.\,6\Omega$ and $V = 12.0$ V. The switch S has been open for a long time, then is suddenly closed at $t = 0$. At what value of t *(in msec)* will the current in the inductor reach 1.1 A? (Figure from 2049 exam III, July 2008).

(1) 6.67   (2) 10.5   (3) 2.88   (4) 19.0   (5) None of these
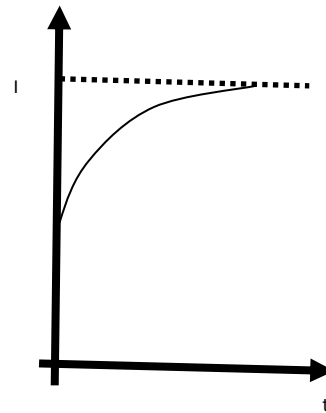
The circuit is a bit subtle. There are two resistors, in parallel and one branch has an inductor. When the switch is closed, the current through the branch including the inductor will be zero. All current (V/R) will pass through the other branch. Once the Faraday effect has taken its time, the inductor will short out, both branches will have the current with the total current being 2V/R, as if the two resistors in parallel were all there was in the circuit.

The equation describing this evolution is the equation for the inductor branch,

$$V = L\frac{di}{dt} + iR$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$\tau = {}^L\!/_R$$

For the record: $t = -\left(\frac{L}{R}\right)\ln\left(1 - \frac{iR}{V}\right) = $ 6.67 ms.

CQ

6. Refer to the previous problem. What is the total energy stored in the inductor a long time after the switch is closed?

(1) 0.19 J   (2) 0.048 J   (3) 0.76 J   (4) 0.034 J   (5) None of these.

$u = 1/2Li^2$ with $I = V/R = 12/4.6 = 2.61A$ and $u = 0.19J$.

7. A small light bulb is placed 3 cm inside a solid cube of Lucite (n=1.50) and turned on. What is the radius (in cm) of the illuminated circle that would be seen by an observer standing above the Lucite block?

(1) 2.68   (2) 3.00   (3) 3.35   (4) 4.50   (5) 3.42

$r = htan(arcsin(\frac{1}{n}) = 2.68$ cm.

8. An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum works as follows: an LC oscillator produces a sinusoidal current in the antenna, which generates the wave, traveling outward at the speed of light. What is the wavelength (in meters) of the wave emitted by this system if $L = 0.323 \mu H$ and $C = 45.0$ pF?

(1) 7.19 m   (2) 1.14 m   (3) 719 m   (4) 114 m   (5) None of these

$$f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$
$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 7.19 \text{ m}.$$

9. A ray is incident on one face of a triangular glass prism in air. The angle of incidence θ is chosen so that the emerging ray also makes the same angle θ with the normal to the other face, as shown. If the apex angle of the prism is φ = 68.0∘, and the index of refraction of the prism is n = 1.60, what is θ (in degrees)?

(1) 64   (2) 52   (3) 34   (4) 68   (5) 88
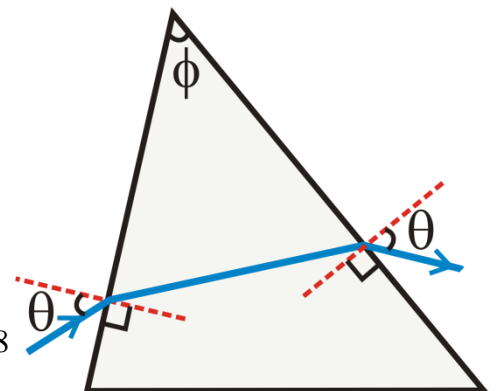
Since the angle of incidence on left and the angle of exit on right are the same θ, it follows that the angle of refraction at the first surface and the angle of incidence at the second surface must also be the same, each therefore = Φ/2 = 34°. From Snell's law then, θ = arcsin [1.6 sin 34] = 63.47°.

10. The light from a professor's laser pointer can be considered as an electromagnetic plane wave. If the laser beam has 5 mW of power and illuminates a circular spot 2.5 mm in radius on a screen, what is the maximum electric field amplitude (E_m in N/C) at the screen?

(1) 440   (2) 311   (3) 622   (4) 550   (5) none of these

Here is one for a formula: $S = I = P/A = E_m^2/2\mu_o c$. Or $E_m = \sqrt{[2\mu_o cP/\pi r^2]} = $ 438.18 N/C

11. A car radio uses an LC circuit and a variable capacitor to tune to different radio stations. The value of the capacitance to tune to a radio station of 1000 kHz is C. What must its value be to tune to a station at 500 kHz?

(1) 4C   (2) 20C   (3) 15C   (4) 10C   (5) 25C

The resonance frequency $\omega = 1/\sqrt{LC}$. If we halve the frequency, we must increase the capacitance by a factor of 4.

12. A laser beam with intensity 10^6 W/m^2 and wavelength 632.8 nm is aimed vertically upward. What is the maximum radius in nm of a spherical particle of graphite (density 2100 kg/m^3) that can be supported by the laser beam against gravity ($g = 9.8$ m/s^2)? Assume that the particle is totally absorbing.

(1) 120   (2) 490   (3) 1190   (4) 230   (5) 190

Here is another one for a formula. If the particle is totally absorbing, the pressure is $P = I/c$ and the upward force is $F_r = PA$. This has to be balanced by gravity whereby the downward force is $F_g = \rho g\, 4\pi r^3/3$. By equating the two forces, we get the minimum radius as $R = 3I/4cg\rho = $ 118.7 nm.