PHY 2049
Summer 2011
Final exam

1. In the circuit shown all capacitors are $4.0 \mu \mathrm{~F}$ and the power supply is 12 V . The potential drop (in V ) across the capacitor labeled q is:
(A) 4.8
(B) 12
(C) 3.6
(D) 7.2
(E) 20

The lower branch is directly across the battery and therefore is subject to the full potential difference. Its equivalent capacitance is $2.4 \mu C$. The charge on the first capacitor is then
 $2.4 \times 12=28.8 \mu \mathrm{C}$. The potential difference across the first capacitor is $28.8 / 4=7.2 \mathrm{~V}$. The potential differnce across $q$ then is $12-7.2=4.8 \mathrm{~V}$.
2. In the multi-loop circuit shown, the current through the $2.0 \mathrm{k} \Omega$ resistor is (in mA)
(A) 1.2
(B) 0.4
(C) 2.4
(D) 0.8

Not enough information
Suppose that the currents in the left and right branches are $I_{1}$ and $I_{2}$, both in $m A$ and going up. Through the middle branch, it must be $\left(I_{1}+I_{2}\right)$ in $m A$ and going down. The loop on the left gives $6-6 I_{1}-2\left(I_{1}+I_{2}\right)=0$. The loop on the right gives $6-6 I_{2}-2\left(I_{1}+I_{2}\right)=0$. Add the two equations and you get, $12-10\left(I_{1}+I_{2}\right)=0$ or $\left(I_{1}+I_{2}\right)=1.2 \mathrm{~A}$.
3. A non-uniform electric field given by $\sim E=\left(5.5^{\wedge} i-2.1^{\wedge} j+\left(4.6 z^{2}-3\right)^{\wedge} k\right)$ $\mathrm{N} / \mathrm{C}$ pierces a cube with sides 3 m , as shown in the figure. The cube has its rear corner at the origin. What is the total charge inside the cube?
(A) +3.3 nC
(B) -3.3 nC
(C) +1.5 nC
(D) -1.5 nC
(E) None of these


A cube has sides with normals along the Cartesian axes. The total flux being the same on the sides with normals along $x$ and $y$ (except for their signs) will vanish. Only the sides up and down will contribute and they give, $\Phi=q / \varepsilon_{o}=3^{2}\left[4.6 x 3^{2}\right]==>q=3.3 \mathrm{nC}$.
4. The capacitances of the four capacitors shown in Fig. 25-52 are given in terms of a certain quantity $C$. In ratio to C , what is the equivalent capacitance between points $A$ and $B$ ? (Hint: First imagine that a battery is connected between those two points; then reduce the circuit to an equivalent capacitance.).

(A) 0.82
(B) 0.41
(C) 13
(D) 0.63
(E) None of these

Here $4 C$ and $6 C$ are in series $(=12 C / 5)$, the combination is parallel to $2 C(=22 C / 5)$ and that combination is in series with $1 C(=22 C / 27)$. The equivalent capacitance ratio $=22 / 27=0.815 C$.
5. CQ 4. In the above problem, now consider a battery connected between points A and D . What fraction of the charge is stored on the 2 C capacitor? Express your answer as a ratio to the charge stored on the C capacitor.
(A) None of these.
(B) 1.0
(C) 0.52
(D) 4.0
(E) 0.73

Now 2C and 6C are in series, the combo is parallel to 4C etc. The equivalent capacitance between $A$ and $D$ is $11 C / 13$. That means that the charge in $C, Q 1=11 C V / 13$. Where a $V$ volt battery is attached between $A$ and $D$. The potential drop across capacitor $C$ is then $11 V / 13$ and across the rest, e.g. $4 C$ then, it is $V-11 V / 13=2 V / 13$. The charge on $2 C$ is the same as the charge on the equivalent capcaitor $3 / 2 C$ or $3 C V / 13$. In ratio to the charge on $Q_{1}$, that would be $3 / 11=0.27$.
6. A long straight wire carries a current $\mathrm{i}_{1}=30 \mathrm{~A}$ and a rectangular loop carries a current $\mathrm{i}_{2}=20 \mathrm{~A}$. Taking $\mathrm{a}=1.00 \mathrm{~cm}, \mathrm{~b}=8.00 \mathrm{~cm}$ and $\mathrm{L}=30.00 \mathrm{~cm}$, what is the net force on the loop due to the wire $i_{1}$ ?
(A) 3.2 mN
(B) 1.6 mN
(C) 0.8 mN
(D) 0.4 mN
(E) 0.2 mN

Here the magnetic field due to the wire is into the plane of the paper and decreases
 with distance from the wire. The forces on the right and left branches of the loop are outwards, to the right at the right wire and to the left at the left branch. They cancel even though their calculation must incorporate the distance dependence of the field. At the top and the bottom wires the field is constant, into the plane of the paper and the force is attractive for the top wire and bigger since it is closer. The net force on the loop is the difference between the top and the bottom wires and net attractive.

$$
F=i_{2} L \frac{\mu_{o}}{2 \pi} i_{1}\left[\frac{1}{a}-\frac{1}{a+b}\right]=i_{2} L \frac{\mu_{o}}{2 \pi} i_{1} \frac{b}{a(a+b)}
$$

$=3.2 \mathrm{mN}$.
7. Unpolarized light is sent into the system of three polarizing sheets, where the polarizing directions of the first and third sheets are at angles $\theta_{1}=30^{\circ}$ (counterclockwise) and $\theta_{3}=30^{\circ}$ (clockwise) with respect to the y axis. What fraction of the initial light intensity emerges from the system if the middle sheet has its axis lined along the $y$-axis?

(A) 0.28
(B) 0.13
(C) 0.5 (D) 0.75
(E) 0.92

Here the answer is

$$
I=I_{0} \frac{1}{2} \cos ^{4} 30=0.28
$$

8. CQ. In the problem above, put the optical axis of the middle sheet at some arbitrary angle $\theta$, measured clockwise from the y axis. At what angle $\theta$ (in degrees) is the emergent light of maximum intensity?
(A) 0
(B) 30
(C) 90
(D) 45
(E) 60

The formal answer can be expressed in terms of angle $\theta$, i.e.

$$
I=I_{0} \frac{1}{2} \cos ^{2}(\theta+30) \cos ^{2}(\theta-30) \quad \alpha \quad\left(4 \cos ^{2} \theta-1\right)^{2}
$$

which has to be maximized now with respect to $\theta$. The answer is $\theta=0$
9. A metal rod is forced to move with constant velocity along two parallel metal rails, which are connected with a strip of metal (see figure). A magnetic field of 0.5 T points out of the page. The separation between the rails is $\mathrm{L}=30 \mathrm{~cm}$, the rod has resistance $40 \Omega$, and velocity $70 \mathrm{~cm} / \mathrm{s}$. What force (in mN ) must be applied to the rod to keep it moving at the constant
 velocity?
(A) 0.4
(B) 0.5
(C) 0.3
(D) 0.2
(E) 0.1

This problem is a test of whether you believe in Newton's laws. The rod's motion creates an emf and a current $I=B L v / R$. Lenz's law says that the current is bottom to top in bar and clockwise in the loop. Therefore there will be a force on the bar to the right $F=I L B=(B L)^{2} v / R=0.394 \mathrm{mN}$.

You must apply a force of that magnitude directed to left which will let the bar move with a constant velocity.
10. A ray is incident on one face of a triangular glass prism in air. The angle of incidence $\theta$ is chosen so that the emerging ray also makes the same angle $\theta$ with the normal to the other face, as shown. If the apex angle of the prism is $\varphi=72^{\circ}$ and $\theta=50$, what is the index of refraction of the prism.?
(A) 1.3 (B) 1.1
(C) 1.6
(D) 2
(E) 2.2


Since the entrance and exit angles, measured with surface normals are the same, the refracted angles must also be same and each must be $1 / 2$ of the vertex angle $\varphi$. Therefore $n=\sin \theta / \sin (\varphi / 2)=1.3$.
11. An object is 20 cm to the left of a lens of focal length +10 cm . A second lens, of focal length +12.5 cm , is 30 cm to the right of the first lens. The distance between the original object and the final image is:
(1) 0 cm
(2) 50 cm
(3) 100 cm
(4) 28 cm
(5) infinity
12. A concave mirror forms a real image that is twice the size of the object. If the object is 20 cm
from the mirror, the radius of curvature of the mirror must be about:
(1) 27 cm
(2) 20 cm
(3) 13 cm
(4) 40 cm
(5) 80 cm
13. The image of an erect candle, formed using a convex mirror is always:
(1) virtual, erect, and smaller than the candle
(2) virtual, erect, and larger than the candle
(3) virtual, inverted, and smaller than the candle
(4) virtual, inverted, and larger than the candle
(5) real, inverted, and larger than the candle
14. A line in the spectrum of a certain element is a doublet with wavelengths 581.6 and 583.0 nm . Calculate the minimum number of lines needed in a grating that will resolve this doublet in the second-order spectrum
(1) 208
(2) 166
(3) 350
(4) 83
(5) 450
15. Two waves of light, whose wavelength $\lambda$ is 600 nm , are initially in phase and travel in air. One of the waves then travels through a plastic sheet which is $4 \mu \mathrm{~m}$ thick with an index of refraction of 1.5 , while the other wave travels through another plastic sheet which is $3.5 \mu \mathrm{~m}$ thick with an index of refraction of 1.6. What multiple of $\lambda$ gives the phase difference of the two waves, after they have emerged from the plastic sheets?
(A) 0.67
(B) 0.094
(C) 0.13
(D) 0.4
(E) 0.83

