Exam I Solutions
PHY2049
Summer 2011

(Ch. 21.53)

1. Two small, positively charged spheres have a combined charge of 50 μC. If each sphere is repelled from the other by an electrostatic force of 1N when the spheres are 2.0 m apart, what is the charge (in μC) on the sphere with the smallest charge?

(A) 11.6

Suppose that the charges on the two spheres are q_1 and q_2. Then $q_1+q_2 = 5x10^{-5}$ and $F = k\, q_1q_2/r^2$ means that $q_1q_2 = 4.44 \times 10^{-10}$. At this stage, note that if we define q_1 and q_2 as respectively x and y μC, then we have x+y = 50 and xy = 4000/9 = 444.44. There are several ways to solve these equations, most direct being the substitution. The answers for (x,y) are 11.6 μC and 38.4 μC.

(Ch. 22.5)

2. What is the magnitude (in pC) of a point charge whose electric field 50 cm away has a magnitude of 2V/m?

(A) 55.6 `

$E = kq/r^2$ which means that $q = Er^2/k = 5.56 \times 10^{-11}$ C

(Ch. 23.7)

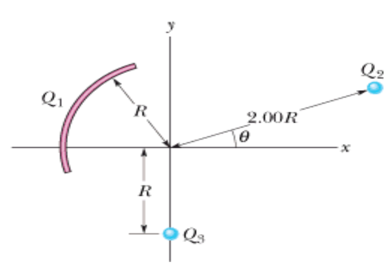
3. A proton is a distance d/2 directly above the center of a sqaure of side d. What is the magnitude of the electric flux (in nN.m^2/C) through the square?

(A) 3

It would be a sixth of the total flus through a cube which encloses the proton. The answer is $q/6\varepsilon_o$ = $3.01x10^{-9}$ Nm^2/C.

(Ch. 24.24)

4. what is the net electric potential (in mV) at the origin due to the circular arc of charge Q_1 = +7.21 pC and the two particles of charges $Q_2 = 4.00Q_1$ and $Q_3 = -2.00Q_1$? The arc's center of curvature is at the origin and its radius is R = 2.00 m; the angle indicated is θ = 20.0°. The potential is taken to be zero at infinity.



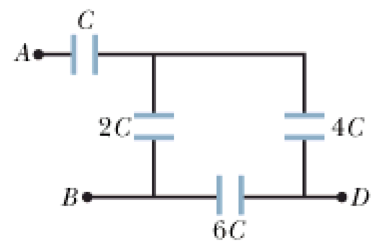
(A) 32.5

The potential is

$$V(r) = k\frac{Q_1}{R}\left[1 + \frac{4}{2} - 2\right] = k\frac{Q_1}{R}$$

(Ch. 25.68)

5. The capacitances of the four capacitors shown in Fig. 25-52 are given in terms of a certain quantity *C*. In ratio to C, what is the equivalent capacitance between points *A* and *B*? (*Hint:* First imagine that a battery is connected between those two points; then reduce the circuit to an equivalent capacitance.).



(A) 0.82

Here 4C and 6C are in series (= 12C/5), the combination is parallel to 2C (= 22C/5) and that combination is in series with 1C (= 22C/27). The equivalent capacitance ratio = 22/27 = 0.815

CQ 6. In the above problem, now consider a battery connected between points A and D. What fraction of the charge is stored on the 4C capacitor? Express your answer as a ratio to the charge stored on the C capacitor.

(A) 0.73

Now 2C and 6C are in series, the combo is parallel to 4C etc. The equivalent capacitance between A and D is 11C/13. That means that the charge in C, Q1 = 11CV/13. Where a V volt battery is attached between A and D. The potential drop across capacitor C is then 11V/13 and across 4C then, it is V – 11V/13 = 2V/13. The charge on 4C is 8CV/13. In ratio to the charge on Q_1*, that would be 8/11 = 0.73.*

(e1,sum10)

7. Two charges, q_1 = -1 C and q_2 = - 4 C, are placed along the x-axis a distance *L* apart with charge q_1 at the origin and q_2 at $x = L$. A third charge, q_3 = +4/9 C, is also placed along the x-axis such that there is no net Coulomb force on any of the charges. What is the position of this charge along the x axis in units of *L*, *i.e.,* what is *x/L*?

(A) 1/3

The force on charge q_1 *consists of repulsion due to charge* q_2 *and the force due to the unknown charge* q_3 *in the middle. Charge* q_3 *must be of opposite sign to cancel the repulsive force due to* q_2.

$$F_1 = kq_1\left[\frac{q_3}{x^2} - \frac{q_2}{L^2}\right]$$

$$\frac{x}{L} = \sqrt{\frac{q_3}{q_2}}$$

One can also get similar conditions by putting the forces on charge q_3 and q_2 to be zero. They are

$$F_2 = 0: \quad \frac{x}{L} = 1 - \sqrt{\frac{q_3}{q_1}}$$

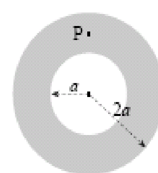
$$F_3 = 0: \quad \frac{x}{L} = \frac{1}{1 + \sqrt{\frac{q_2}{q_1}}}$$

Notice that the three answers for x/L are the same. This is guaranteed by the selection of the three charges.

The charges here are all magnitudes since their signs are already used in determining the canceling directions of the forces.

(s11)

8. The figure shows a uniformly charged, nonconducting spherical shell of inner radius a and outer radius 2a. If the electric field at the outer radius is E, what is the electric field at point P with radius r = 1.5a?



(A) 0.6E

It is a uniformly charged non-conducting sphere. Here the electric field can be finite inside (in a conducting sphere, the electric field vanishes and the charges reside at the surface). Suppose the total charge is Q. The the field just outside the shell is $E = kQ/(2a)^2$. The amount of charge enclosed by a Gaussian surface of radius r (between a < r < 2a) $q = Q\,(r^3-a^3)/[(2a)^3 - a^3]$. The electric field at P is:

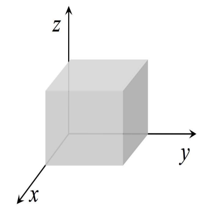
$$E_P = k\frac{q}{r^2} = k\frac{Q}{r^2}\frac{\left(\frac{r}{a}\right)^3 - 1}{7}$$

$$\frac{E_P}{E} = \frac{4}{7}\frac{\left(\frac{r}{a}\right)^3 - 1}{\left(\frac{r}{a}\right)^2}$$

(s10)

9. A non-uniform electric field given by ~E = (5.5ˆi − 2.1ˆj + (4.6z^2 − 3)ˆk) N/C pierces a cube

with sides 3 m, as shown in the figure. The cube has its rear corner at the origin. What is the total charge inside the cube?

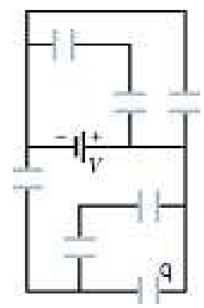


(A) +3.3 nC

A cube has sides with normals along the Cartesian axes. The total flux being the same on the sides with normals along x and y (except for their signs) will vanish. Only the sides up and down will contribute and they give, $\Phi = q/\varepsilon_o = 3^2\,[4.6x3^2]$ *==>* $q = 3.3$ *nC.*

(e2.3 f09)

10. In the circuit shown all capacitors are 6.0µF and the power supply is 12V. The charge (in µC) on the capacitor labeled q is:



(A) 29

Consider only the lower branch which is directly connected to the battery. It is parallel to the upper branch and has the potential difference of 12 V. In this branch, capacitor q is parallel to the two in series and the parallel combination is in series with another one. This last one being the one next to the battery will have all the charge of the equivalent capacitor of he lower branch, which is 18/5 µF. It s charge is therefore 12x18/5 = 43.2 µC. The potential difference across its two plates will be 43.2/6 = 7.2 V. The potential difference across the capacitor q is then (12 – 7.2) = 4.8 V. That goes to tell you that the charge in q must be 4.8 x 6 = 28.8 µC.

(e2.7, f10)

11. A copper wire and a nichrome wire of the same length and cross-section are connected in series across a large battery. If the resistivity of the copper wire is 1.7×10^{-8} Ω m and the resistivity of the nichrome wire is 1.1×10^{-6} Ω m, what is the power dissipated in the copper wire divided by the power dissipated in the nichrome wire?

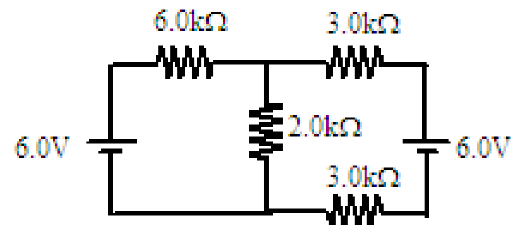
(A) 0.015

The wires are connected in series and have the same current. Power being I^2R*, and R being proportional to resistivity ρ, it follows that if the area and length are the same,* $P_{copper}/P_{nichrome}$ =

$\rho_{copper}/\rho_{nichrome} = 0.015$.

(e2.10, f09)

12. In the multi-loop circuit shown the current through the 2.0kΩ resistor is (in mA),



(A) 1.2

Kirchhoff's rules are designed to keep track of natural accounting and in fact lead to an answer that is different from one you might guess. Let's guess first: Two batteries are the same and connected to 8 Ohms in series (??). Hence the current in each circuit might be 3/4A and the total current is therefore 1.5A. Not so as you see clearly but why?

Now the right solution: Suppose that the currents in the left and right branches are I_1 and I_2, both in mA and going up. Through the middle branch, it must be (I_1+I_2) in mA and going down. The loop on the left gives $6 - 6I_1 - 2(I_1+I_2) = 0$. The loop on the right gives $6-6I_2 - 2(I_1+I_2) = 0$. Add the two equations and you get, $12 - 10(I_1+I_2) = 0$ or $(I_1+I_2) = 1.2A$.