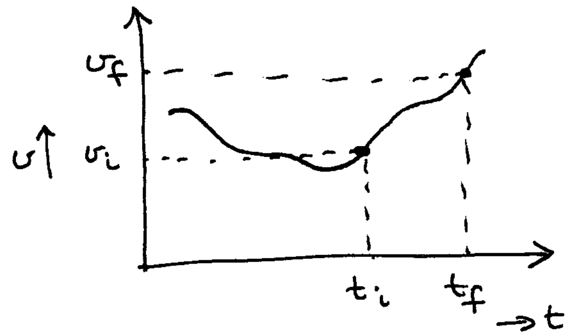
Average acceleration

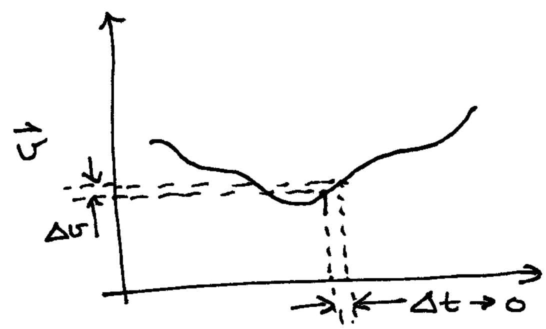
$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$



Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

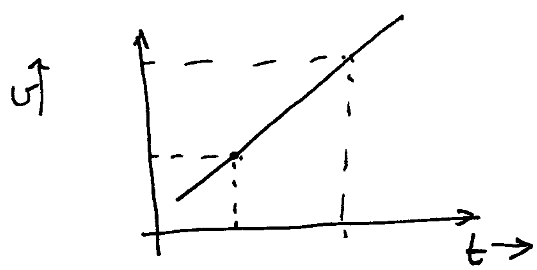
$\Rightarrow$ a ~~at~~ a time t is the slope of the tangent to the v vs. t curve at that time.



We will deal mainly with constant acceleration

When acceleration is constant:

$a = \bar{a}$ and v varies linearly with t.



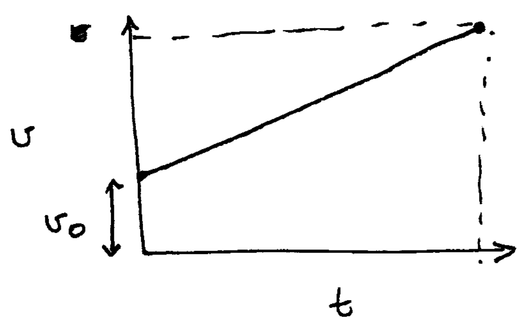
DEMO OF CART ON INCLINED PLANE SHOWING CONSTANT ACCELERATION.

Kinematic equations (constant acceleration).

$$a = \bar{a} = \frac{v_f - v_i}{t_f - t_i}, \quad \text{let} \quad v_i = v_0,\ t_i = 0,\ v_f = v,\ t_f = t$$

$$\Rightarrow a = \frac{v - v_0}{t} \Rightarrow \boxed{v = v_0 + at} \rightarrow (1)$$

$\Rightarrow$



slope of the line $= a$

intercept $= v_0$

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{t}$$

since v varies linearly from v_0 to v,

$$v_{av} = \frac{v_0 + v}{2} \Rightarrow \frac{\Delta x}{t} = \frac{v_0 + v}{2} \Rightarrow \boxed{\Delta x = \left(\frac{v_0 + v}{2}\right)t} \rightarrow (2)$$

Using (1) & (2)

$$\Delta x = \left(\frac{v_0 + v_0 + at}{2}\right)t \Rightarrow \boxed{\Delta x = v_0 t + \frac{1}{2}at^2} \rightarrow (3)$$

from (1), $t = \dfrac{v - v_0}{a}$

~~Use~~ ~~in~~ Substitute for t in 3

$$\Rightarrow \quad \Delta x = v_0\left(\frac{v-v_0}{a}\right) + \frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2$$

$$= \frac{v_0 v}{a} - \frac{v_0^2}{a} + \frac{1}{2a}\left(v^2 + v_0^2 - 2vv_0\right)$$

$$= \cancel{\frac{v_0 v}{a}} - \frac{v_0^2}{a} + \frac{v^2}{2a} + \frac{v_0^2}{2a} - \cancel{\frac{v v_0}{a}}$$

$$= \frac{v^2}{2a} - \frac{v_0^2}{2a}$$

$$\Rightarrow \boxed{v^2 = v_0^2 + 2a\Delta x} \longrightarrow \quad 4$$

<u>free fall</u>

for objects moving vertically under gravity near the surface of the earth with no other force acting on the ~~obj~~ object, $\boxed{a = g = -9.8\ m/s^2}$

DEMO - BOTTLE OF WATER FALLING UNDER GRAVITY

The stopping distance problem in detail.

Case I.

$v_0 = 20$ mph

$v = 0 \rightarrow$ (because car comes to a stop)

$\Delta x = 10$ ft

$v^2 = v_0^2 + 2a\Delta x$

$\Rightarrow 0 = 20^2 + 2a.(0.0019)$ $\qquad$ [10 ft = 0.0019 miles]

$\Rightarrow a = -105263.16$ mile/hour^2

Case II

Same road conditions and braking

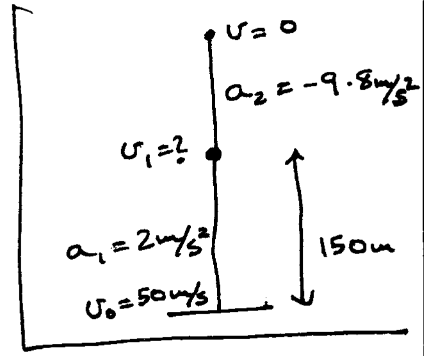
$\Rightarrow a = -105263.16$ mile/hour^2

$v_0 = 40$ mph

$v = 0$

$\Rightarrow 0 = 40^2 + 2a\Delta x$

$\Rightarrow \Delta x = 0.0076$ miles $=$ $\underline{\underline{40 \text{ feet}}}$



Problem 2-49

Stage 1 $(\Delta y)_1 = 150\,m$

Initial velocity $= v_i = v_0 = 50\,m/s$

acceleration $= a_1 = +2\ m/s^2$

final velocity $= v_1 = ?$ (unknown)

Stage 2 $(\Delta y)_2 = ?$

Initial velocity = final velocity of stage 1 $= v_1$

final velocity $= 0$

$a_2 = g = -9.8\ m/s^2$

From stage 1

$$\underbrace{v_1^2 = v_0^2 + 2a_1(\Delta y)_1}_{\text{Kinematic equation (4)}} \Rightarrow v_1 = \sqrt{50^2 + 2\times 2\times 150}$$

$$= 55.68\ m/s$$

From stage 2

(Kinematic eq^n. (4), final velocity of stage 2 = 0)

$$0^2 = v_1^2 + 2a_2(\Delta y)_2$$

$$= (55.68)^2 - 2\times 9.8\times (\Delta y)_2$$

$$\Rightarrow (\Delta y)_2 = 158.16\ m$$

Total height reached by rocket $= (\Delta y)_1 + (\Delta y)_2$

$$= 308.1\ m$$

time for stage 1

$v_0 = 50\ m/s$, $v_1 = 55.68\ m/s$, $a_1 = 2\ m/s^2$

Use Kinematic equation ① ($v = v_0 + at$)

$\Rightarrow t_1 = \frac{v_1 - v_0}{a_1} \Rightarrow t_1 = \frac{55.68 - 50}{2} = 2.84\ s$

time for stage 2

$v_1 = 55.68\ m/s$, $v = 0\ m/s$, $a_2 = -9.8\ m/s^2$

Use Kinematic equation ①

$\Rightarrow t_2 = \frac{0 - v_1}{a_2} = \frac{-55.68}{-9.8} = 5.68\ s$

time to fall back down to the ground

$\Delta y = -308.1\ m$ (notice the negative sign because displacement is doconcoard).

$a = -9.8\ m/s^2$

Initial velocity $= 0$

Use kinematic equation ③ ($\Delta x = v_0 t + \frac{1}{2}at^2$)

$\Rightarrow -308.1 = 0 \times t - \frac{1}{2} \times 9.8 \times t^2 = -4.9\ t^2$

$\Rightarrow t = 7.93\ s$

total time $= t_1 + t_2 + t = 2.84 + 5.68 + 7.93$
$= 16.4\ s$