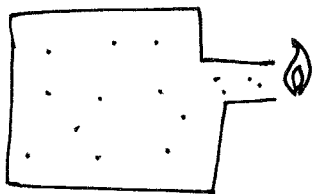


Milk jug rocket

$\Delta m = \text{mass of } \frac{1}{2} \text{ gas}$

$M = \text{mass of milk jug.}$

$$(M + \Delta m) \cdot 0 = MV + \Delta m \cdot v$$

$$\Rightarrow V = -\frac{\Delta m}{M} \cdot v$$

for large V , v has to be extremely large since $\Delta m \ll M$. That's why there was an explosion when I ignited the gas.

Glancing collisions

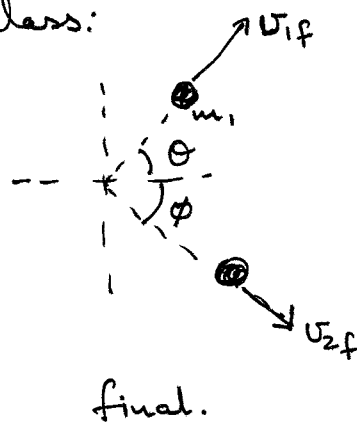
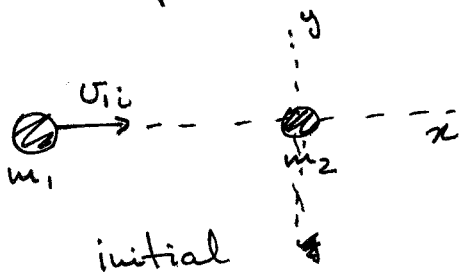
$$\vec{P}_i = \vec{P}_f \quad (\text{conservation of momentum})$$

Since momentum is a vector if $\vec{P}_i = \vec{P}_f$

then $P_{ix} = P_{fx}$ & $P_{iy} = P_{fy}$

See formula in lecture 17 slides.

For the problem shown in class:



$$P_{ix} = m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1i} + 0 = m_1 v_{1i}$$

$$P_{iy} = ~~m_1 v_{1iy}~~ + m_2 v_{2iy} = 0 + 0 = 0$$

$$P_{fx} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi$$

$$P_{fy} = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f} \sin \theta - m_2 v_{2f} \sin \phi$$

$$P_{ix} = P_{fx} \Rightarrow m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi = m_1 v_{1i}$$

$$P_{iy} = P_{fy} \Rightarrow m_1 v_{1f} \sin \theta - m_2 v_{2f} \sin \phi = 0$$

KE is conserved for all elastic collisions including glancing collisions. See equation in lecture slides.

REMEMBER

~~$v_{1i} + v_{1f} = v_{2i} + v_{2f}$~~ ~~is not applicable~~
 is NOT applicable because this equation true ONLY for 1D elastic collisions.

Rotations → see lecture slides

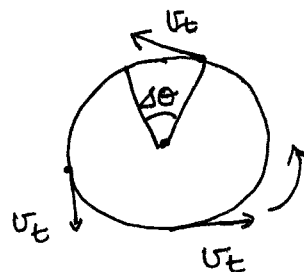
$\Delta \theta = \frac{\Delta s}{r}$, $\Delta \theta$ has units radians but is dimensionless.

Angular velocity

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{1}{r} \frac{\Delta s}{\Delta t} = \frac{v_t}{r} \text{ where}$$

v_t is the tangential velocity.

if ω changes: $\Delta \omega = \frac{\Delta v_t}{r}$



$$\text{Angular acceleration } \alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{1}{r} \frac{\Delta v_t}{\Delta t} = \frac{a_t}{r}$$

$\Rightarrow \alpha = \frac{a_t}{r}$, where a_t is the tangential acceleration.

$\begin{aligned} \Delta s &= r \Delta \theta \\ v_t &= r \omega \\ a_t &= r \alpha \end{aligned}$	→	I
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"Kinematic" equations for angular quantities

$$v_t = v_{ti} + a_t t$$

$$\Rightarrow \frac{v_t}{r} = \frac{v_{ti}}{r} + \frac{a_t}{r} t$$

$$\Rightarrow \omega = \omega_i + \alpha t \rightarrow \text{II}$$

Similarly

$$\Delta \theta = \omega_i t + \frac{1}{2} \alpha t^2 \rightarrow \text{III}$$

$$\omega^2 = \omega_i^2 + 2\alpha \Delta \theta \rightarrow \text{IV}$$