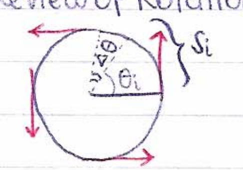
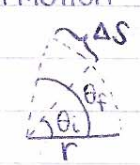
Class Notes: Lecture 19 27 October 2009

Review of Rotational Motion





$$\Delta\theta = \frac{\Delta S}{r}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{r} \frac{\Delta S}{\Delta t}$$

affected by limit

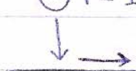
Unwrap the Circle: ←——←——←——

$\leftarrow V_t$ direction always changing
magnitude ↑ if it's spun faster (↓ if slower)

$\rightarrow a_t$ for decreasing V_t

$$\left.\begin{aligned} \Delta s &= \Delta\theta r \\ \omega &= V_t/r \\ \alpha &= a_t/r \end{aligned}\right\} \quad \begin{aligned} \omega &= \omega_i + \alpha t \\ \omega^2 &= \omega_i^2 + 2\alpha\Delta\theta \\ \Delta\theta &= \omega_i t + \left(\tfrac{1}{2}\right)\alpha t^2 \end{aligned}$$

CQ 1: $r = 1.2cm$



$V_i = 18 \frac{rad}{s}$

$a = 1.9 \frac{rad}{s^2}$

Ans: Use $\omega^2 = \omega_i^2 + 2\alpha\Delta x$

CQ2: Δx?

Ans: $\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

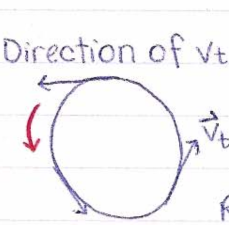
$0 = (18^2) + 2\underbrace{(-1.9)}_{\text{slowing down = negative!}}\Delta\theta$

$\Delta\theta = 85.3 rad$

$C = 2\pi r$

$\Delta S = r\Delta\theta = 1.02m$

Direction of v_t and ω; a_t and α



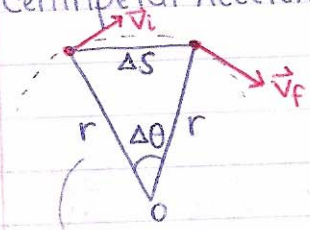
Right hand rule

ω out to you if ↺, ⊥ to page
ω into page if ↻, ⊥ to page

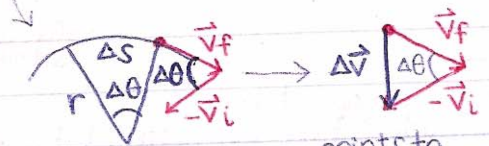
ω, α in same direction if speeding up
V_t, a_t opposite if slowing down

$\vec{a}_t$ quantifies Δ mag of $\vec{v}_t$, but $\vec{v}_t$ also changes

Centripetal Acceleration



$$|a_c| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t} \qquad \to \Delta\vec{v} = \vec{v}_f - \vec{v}_i$$



$$\Delta\vec{v} = \vec{v}_f + (-\vec{v}_i)$$

points to center of circle

$$\Delta\theta = \frac{\Delta s}{r}$$

when $\Delta\theta \to 0$, the dif between arc & straight line = negligible

$$\Delta\theta = |\Delta\vec{v}| / v_t$$

$$|\Delta\vec{v}| = \Delta\theta\, v_t \quad \therefore \quad |a_c| = \lim_{\Delta t \to 0} \underbrace{\frac{v_t \Delta\theta}{\Delta t}}_{=\omega}$$

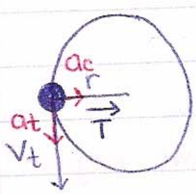
$$= v_t\,\omega$$

$$= v_t\left(\frac{v_t}{r}\right)$$

$$a_c = \frac{v_t^2}{r}$$

Also, $\omega = \frac{v_t}{r}$; $v_t = \frac{\omega}{r}$

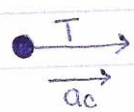
So: $a_c = \omega^2 r$



Total a = vector sum of $\underbrace{a_c,\ a_t}_{\text{perpendicular}}$

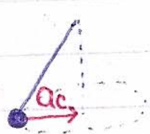
$$a = \sqrt{a_t^2 + a_c^2}$$

Free body diagram:



$$T = m(a_c) = m\frac{v_t^2}{r}$$

CQ3:



CQ4: Not for points, #24