Chapter 8

Rotational Equilibrium and Rotational dynamics

Torque and Equilibrium

- First Condition of Equilibrium
  - The net external force must be zero

$$\Sigma \vec{\mathbf{F}} = 0 \; or$$

$$\Sigma \vec{\mathbf{F}}_x = 0 \quad and \quad \Sigma \vec{\mathbf{F}}_y = 0$$

  - This is a statement of translational equilibrium

- The Second Condition of Equilibrium states

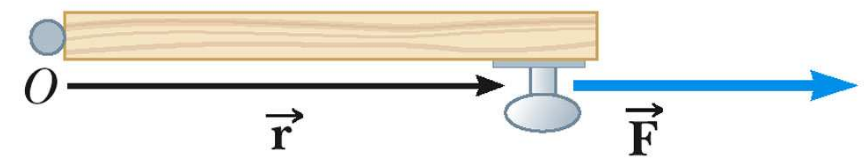
$$\Sigma \vec{\tau} = 0$$

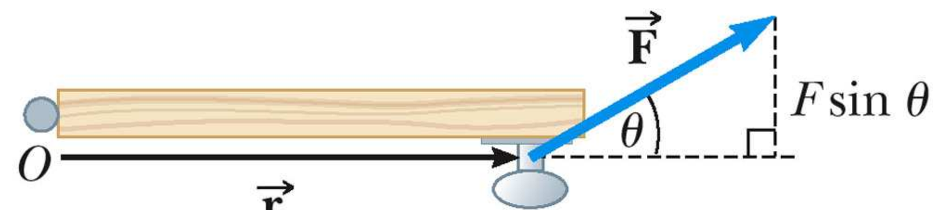
  - The net external torque must be zero

A hobbit house

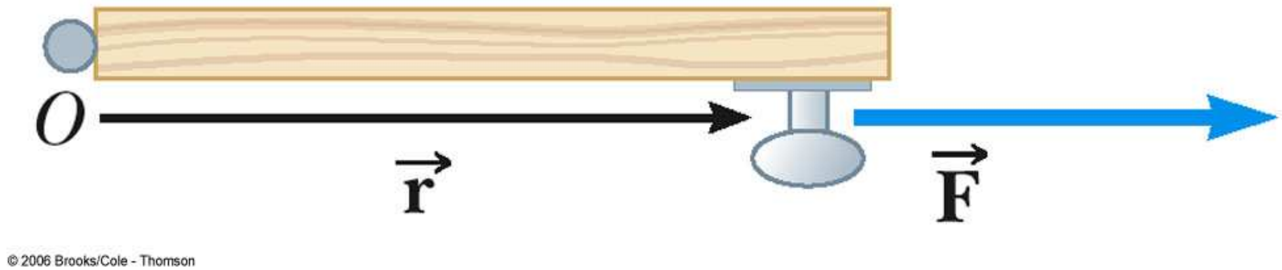
Three Factors affect torque

- The *magnitude* of the force
- The *position* of the application of the force
- The *angle* at which the force is applied

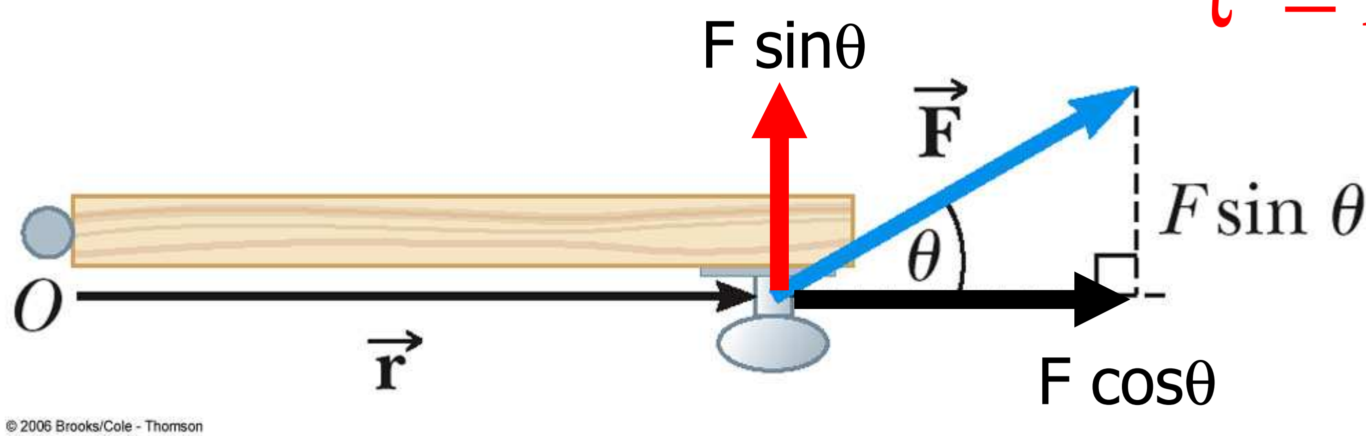




$$\tau = r F \sin \theta$$



$$\tau = r\,F\sin\theta$$



Torque and Equilibrium

- First Condition of Equilibrium
  - The net external force must be zero

$$\Sigma\vec{\mathbf{F}} = 0 \; or$$

$$\Sigma\vec{\mathbf{F}}_x = 0 \quad and \quad \Sigma\vec{\mathbf{F}}_y = 0$$

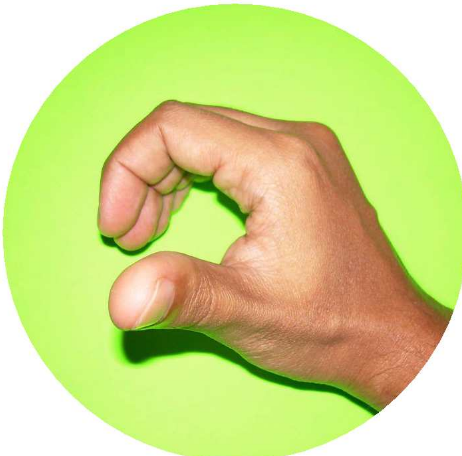
  - This is a statement of translational equilibrium

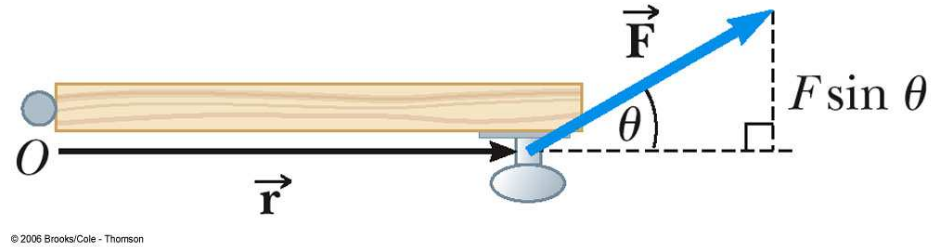
- The Second Condition of Equilibrium states

$$\Sigma\vec{\tau} = 0$$

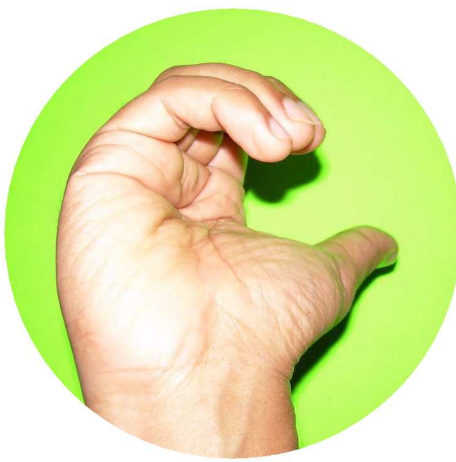
  - The net external torque must be zero

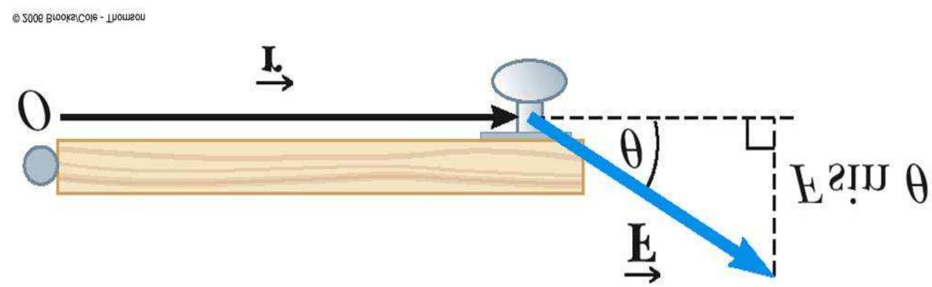
Torque direction: Right hand rule again





Force turns it in the counterclockwise direction



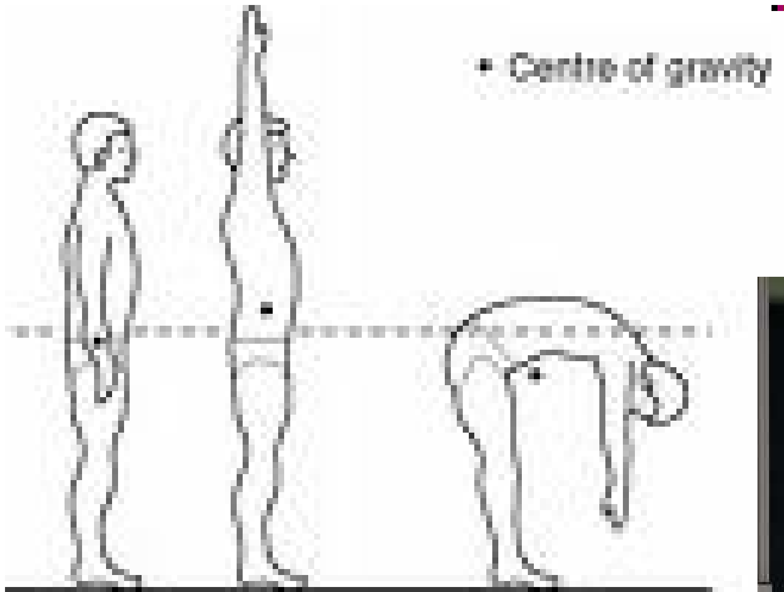


Force turns it in the clockwise direction

Center of Gravity

- In finding the torque produced by the force of gravity, all of the weight of the object can be considered to be concentrated at a single point

Center of gravity



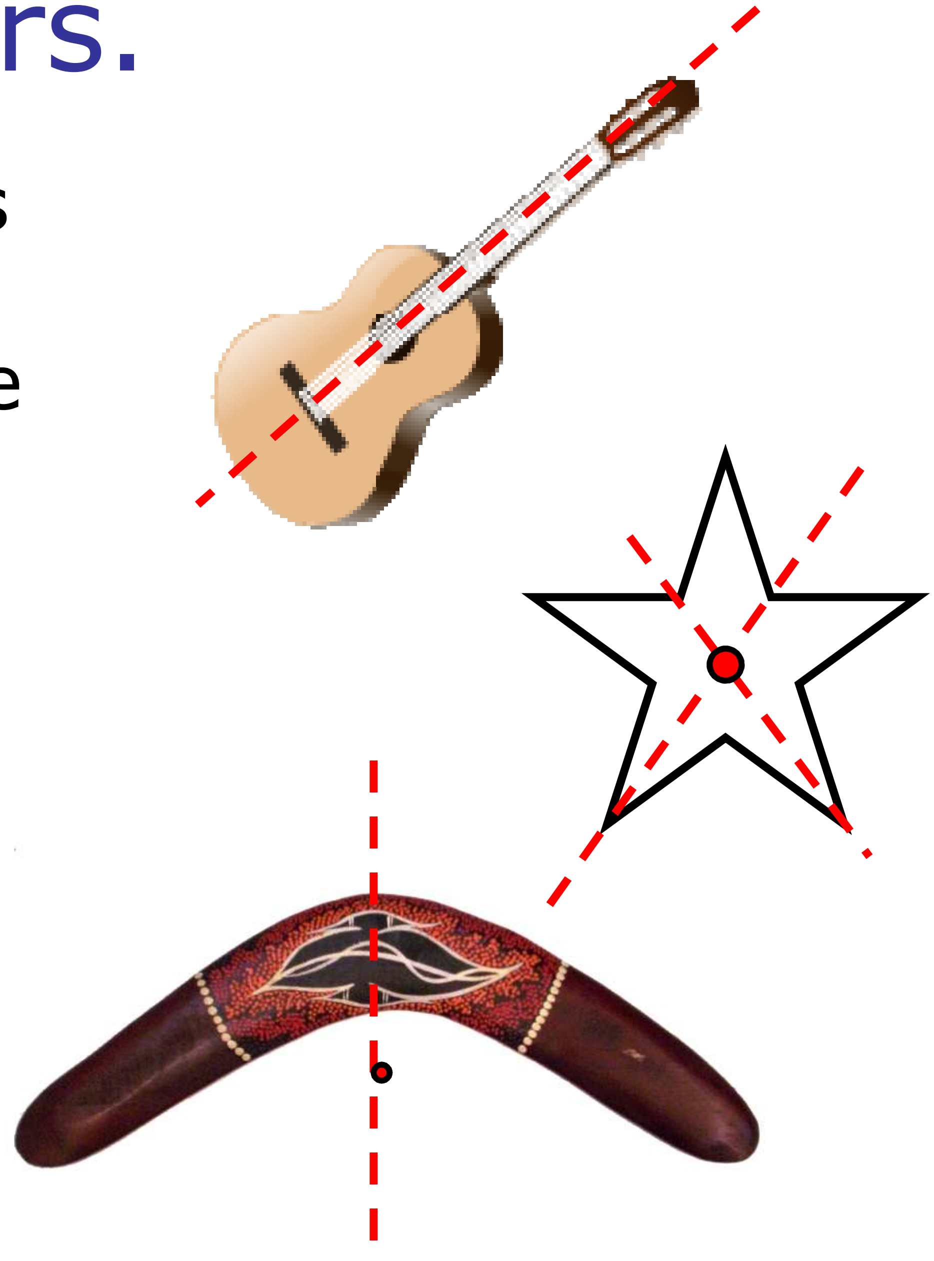
$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} \quad and \quad y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

A few pointers.

- If a body has a symmetry and it has a uniform density then the **cg** is on the line of symmetry.

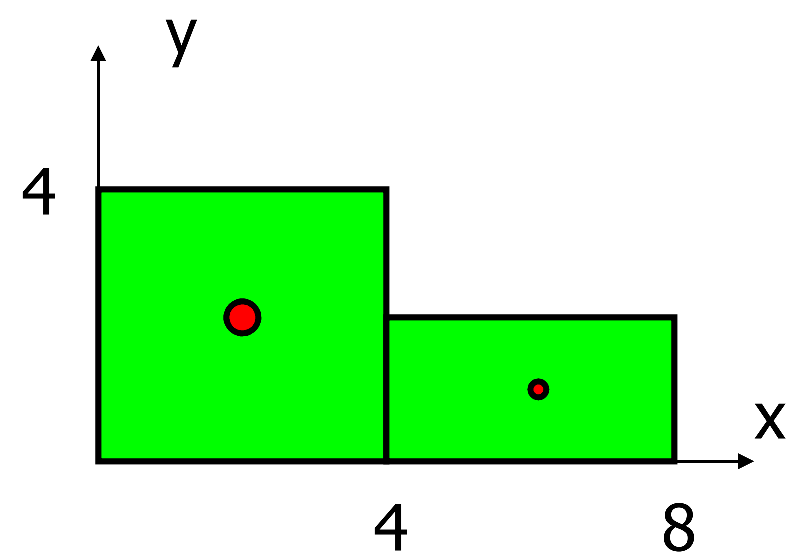
- The center of symmetry coincides with the **cg**.

- The **cg** might be outside the object



Example

- Find the cg of a 4x8 uniform sheet of plywood with the upper right quadrant removed.



$$m_1 = 2M; (x_1, y_2) = (2,2)$$

$$m_2 = M; (x_2, y_2) = (6,1)$$

$$x_{cg} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2M \cdot 2 + M \cdot 6}{2M + M} = \frac{10 \cdot M}{6 \cdot M} = \frac{10}{3} \ ft$$

$$y_{cg} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{2M \cdot 2 + M \cdot 1}{2M + M} = \frac{5 \cdot M}{3 \cdot M} = \frac{5}{3} \ ft$$

Torque and Equilibrium

- First Condition of Equilibrium
  - The net external force must be zero

$$\Sigma\vec{\mathbf{F}} = 0 \; or$$

$$\Sigma\vec{\mathbf{F}}_x = 0 \quad and \quad \Sigma\vec{\mathbf{F}}_y = 0$$

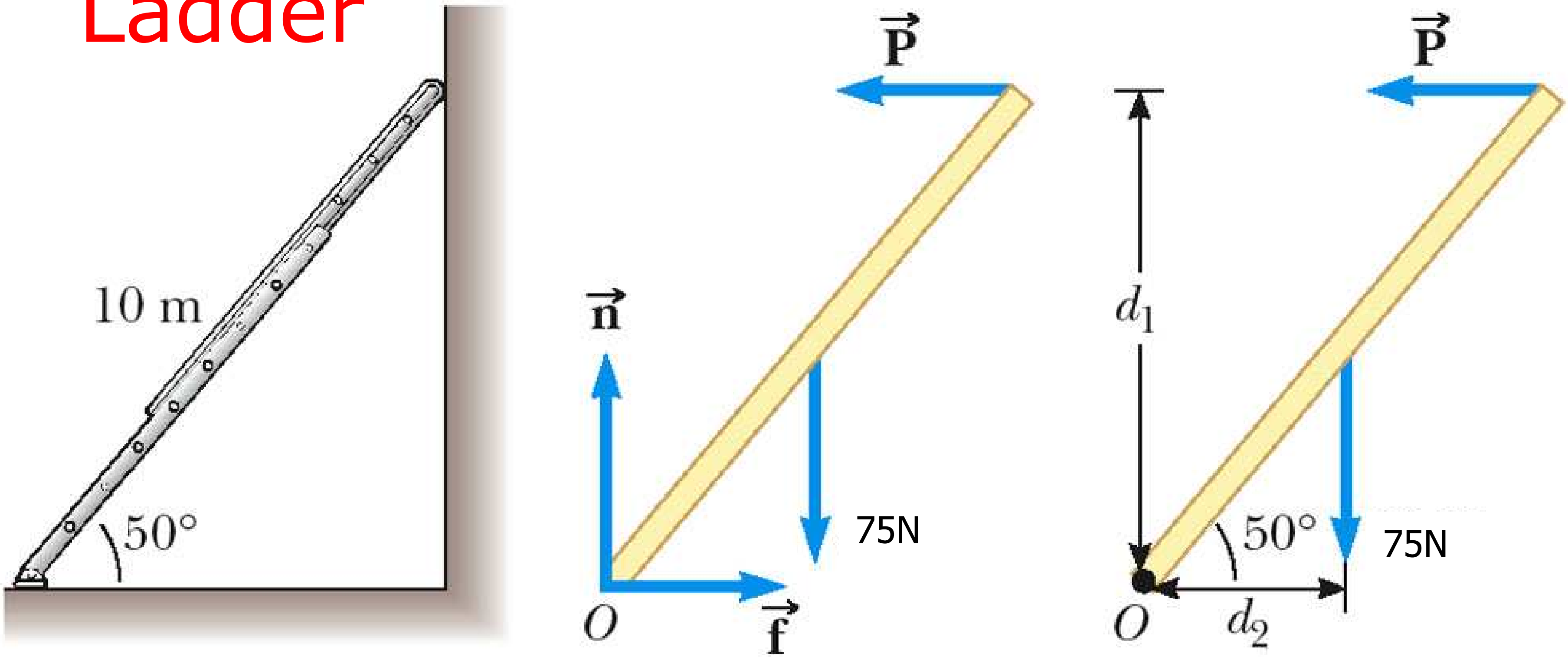
  - This is a statement of translational equilibrium

•The Second Condition of Equilibrium states

$$\Sigma\vec{\tau} = 0$$

- The net external torque must be zero

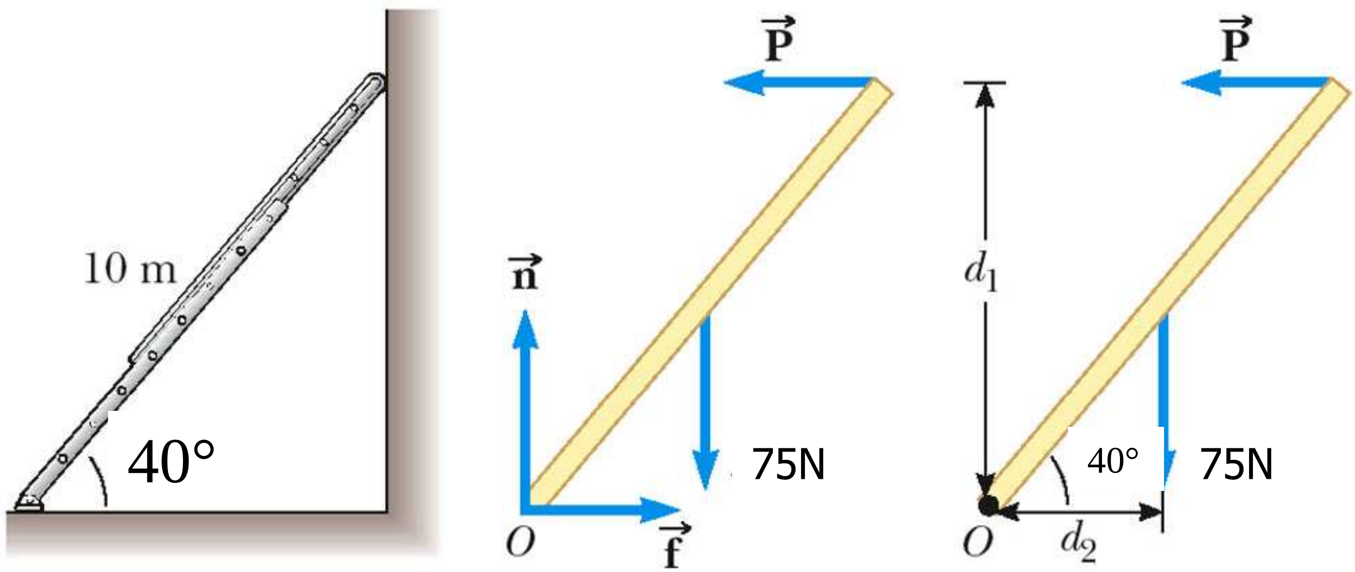
Example of a Free Body Diagram-- Ladder



- free body diagram shows normal force and force of static friction acting on the ladder at the ground

In-class quiz 18-1

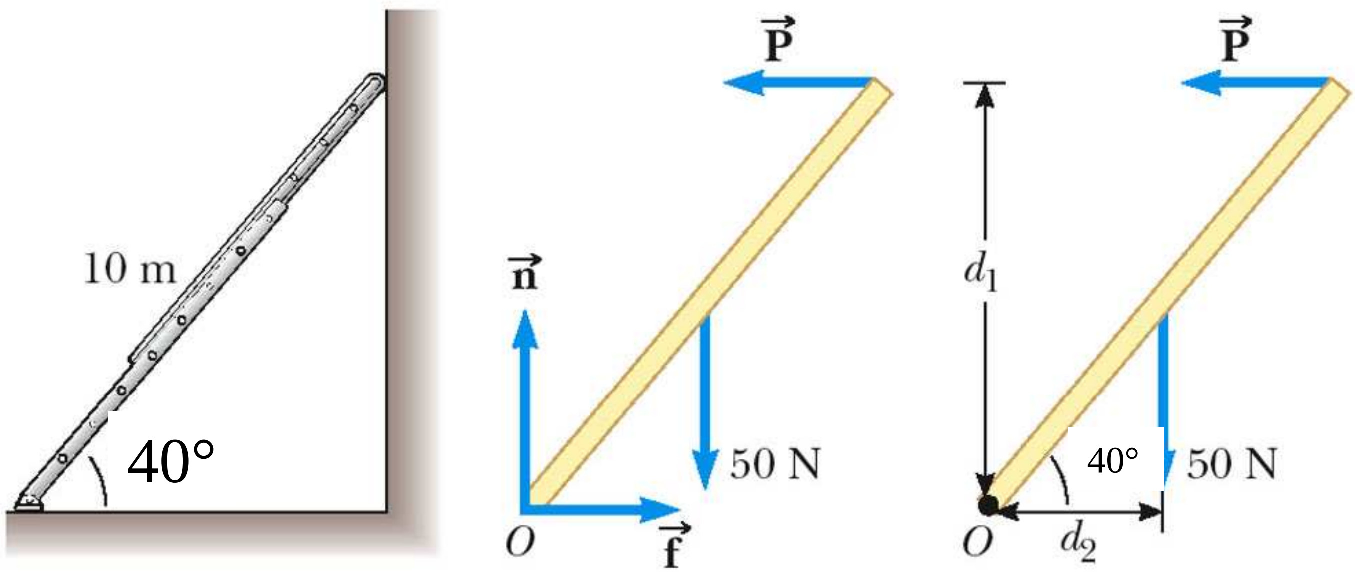
Find the force P of the wall on the top of the 10 meter ladder that weights 75 N



A. 50 N
B. 25N
C. 30 N
D. 21 N
✓ E. 45 N

In-class quiz 18-1

Find the force P of the wall on the top of the 10 meter ladder that weights 50 N



A. 50 N

B. 25N

✓ C. 30 N

D. 21 N

E. 45 N

A 100-N uniform ladder, 8.0 m long, rests against a smooth vertical wall. The coefficient of static friction between ladder and floor is 0.40. What minimum angle can the ladder make with the floor before it slips?

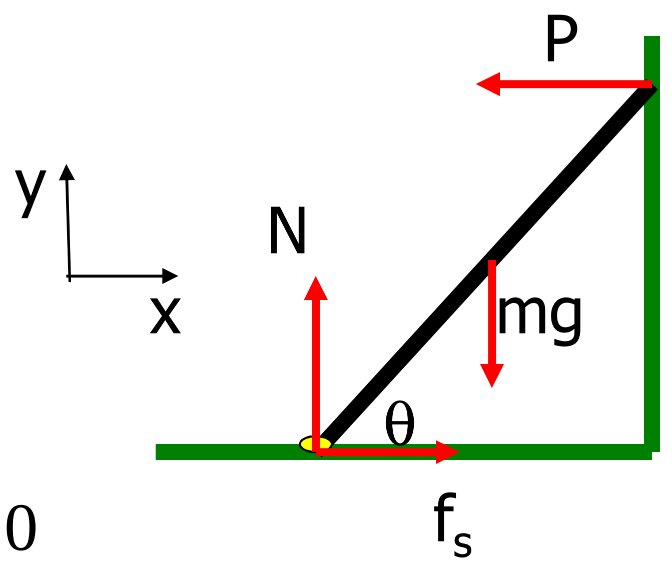
A. 42^o
B. 22^o
C. 18^o
D. 51^o
E. 39^o

A 100-N uniform ladder, 8.0 m long, rests against a smooth vertical wall. The coefficient of static friction between ladder and floor is 0.62. What minimum angle can the ladder make with the floor before it slips?

A. 42^o
B. 22^o
C. 18^o
D. 51^o
E. 39^o

Example: a ladder against a wall

- What minimum angle can the ladder make with the floor before it slips?



$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau = 0$$

$$f_s - P = 0$$

$$n - mg = 0$$

$$PL\sin\theta - mg\frac{L}{2}\cos\theta = 0$$

$$f_s = \mu_s n = \mu_s mg$$

$$\mu_s mgL\sin\theta - mg\frac{L}{2}\cos\theta = 0$$

$$\mu_s \sin\theta - \frac{1}{2}\cos\theta = 0$$

$$\tan\theta = \frac{1}{2\mu_s}$$

Torque and Angular Acceleration

Newton's Second Law for a Rotating Object

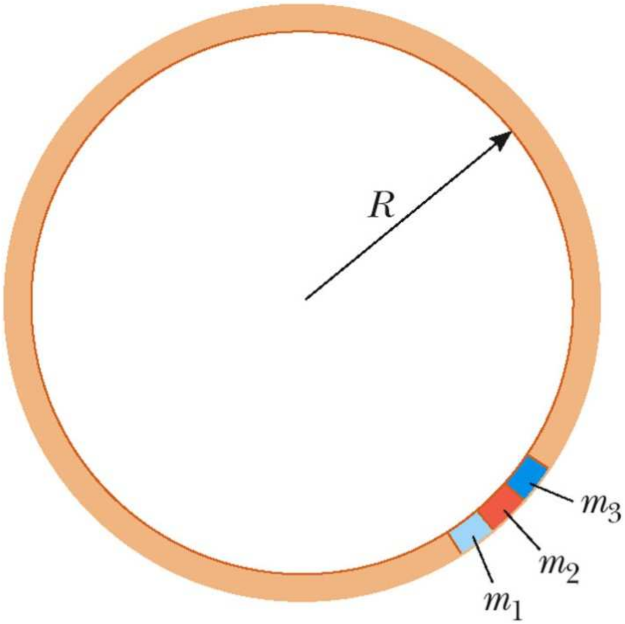
$$\Sigma\tau = I\alpha$$

analogous to $\Sigma \mathbf{F} = m\mathbf{a}$

I = *moment of inertia*

For Uniform Ring $I = \Sigma m_i r_i^2 = MR^2$

moment of inertia depends on quantity of matter *and* its distribution *and* location of axis of rotation



Other Moments of Inertia