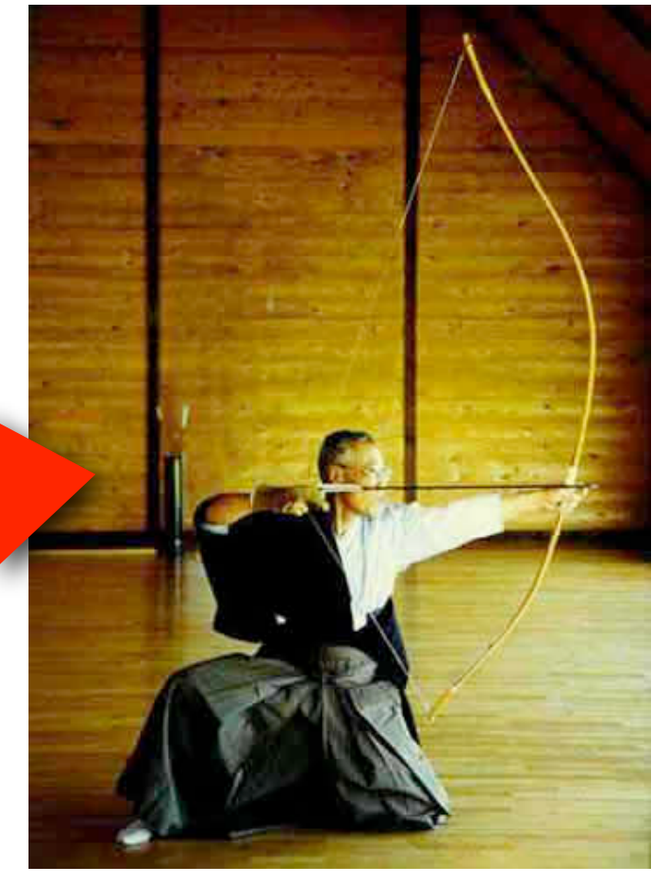
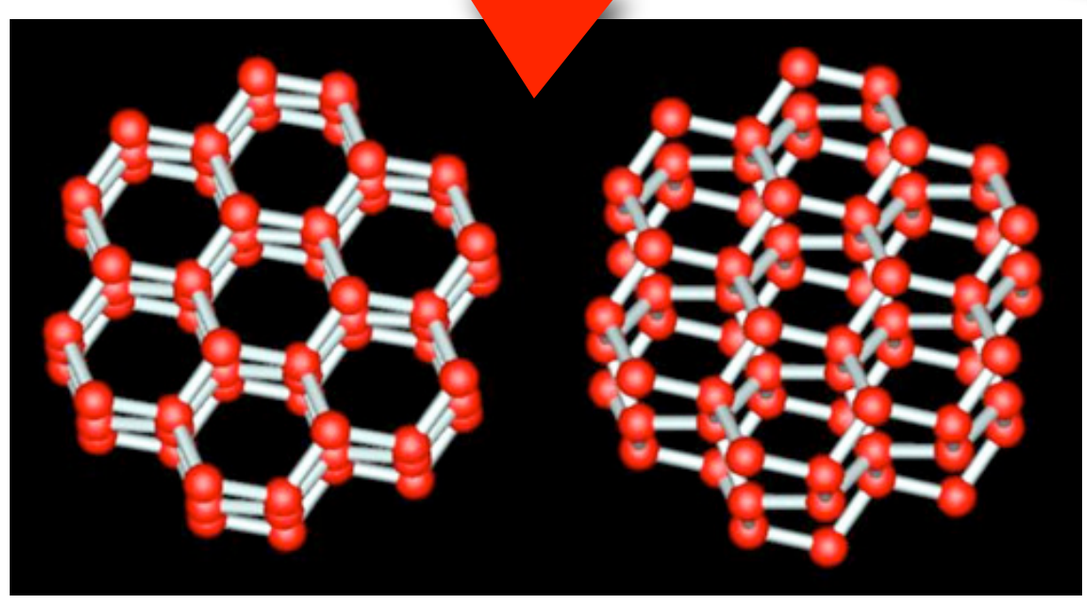
PHY2053: Lecture 13

Elastic Forces - Hooke's Law
Work done by elastic forces
Elastic Potential Energy
Power

Elastic Forces





Objectives

1) model the force produced by a compressed / extended spring

2) how does one model a system with multiple springs?

3) is the elastic force conservative?

4) if so, define potential energy of the spring

- include potential energy of the spring into energy conservation calculations

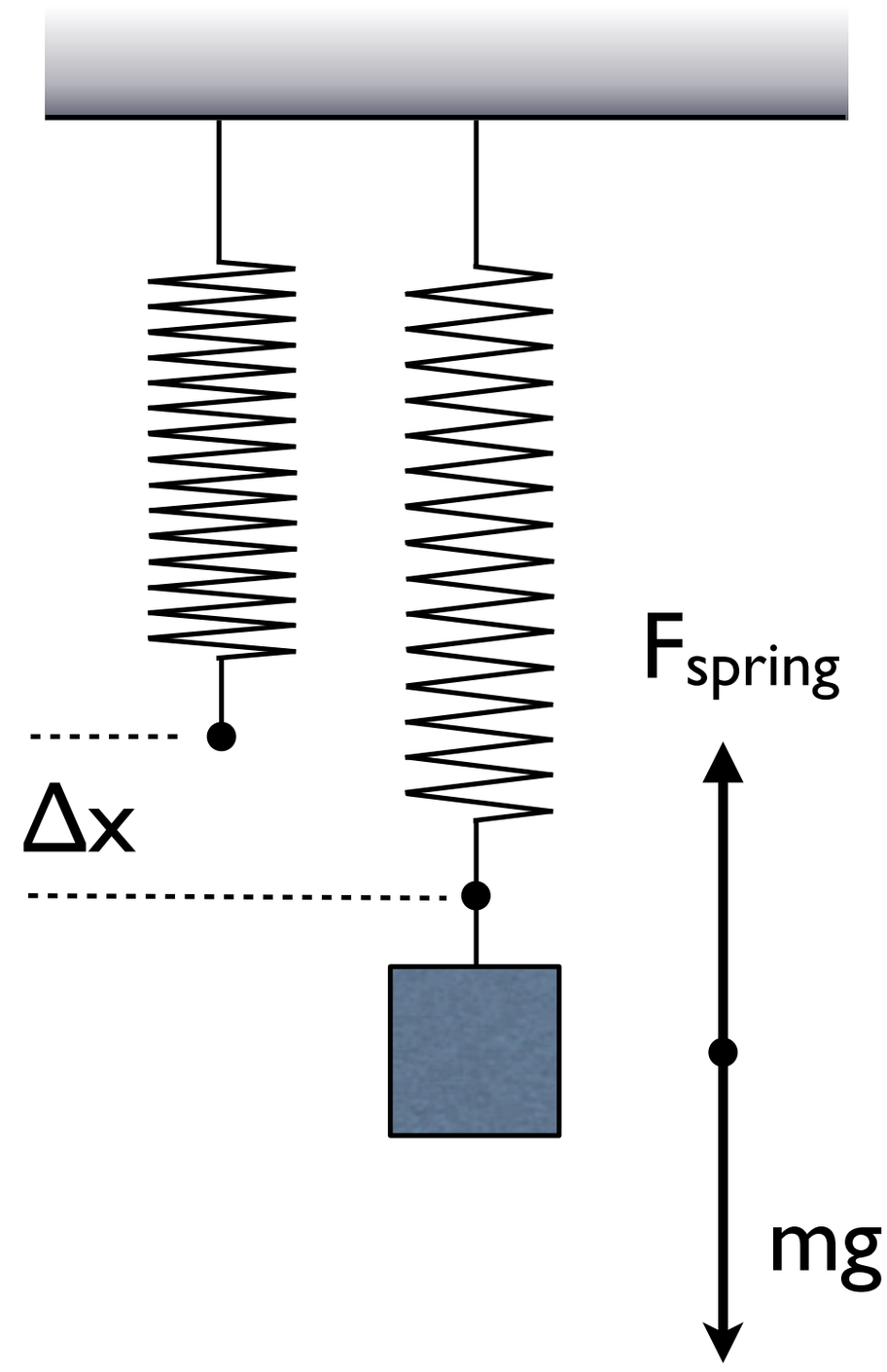
5) define concept of power

Hooke's Law

- Discovered by Robert Hooke
- c/a 1676 "Ut tensio, sic vis"
  "As the extension, so the force"
- Excellent description of many elastic systems

$$F_x = -k\,\Delta x$$

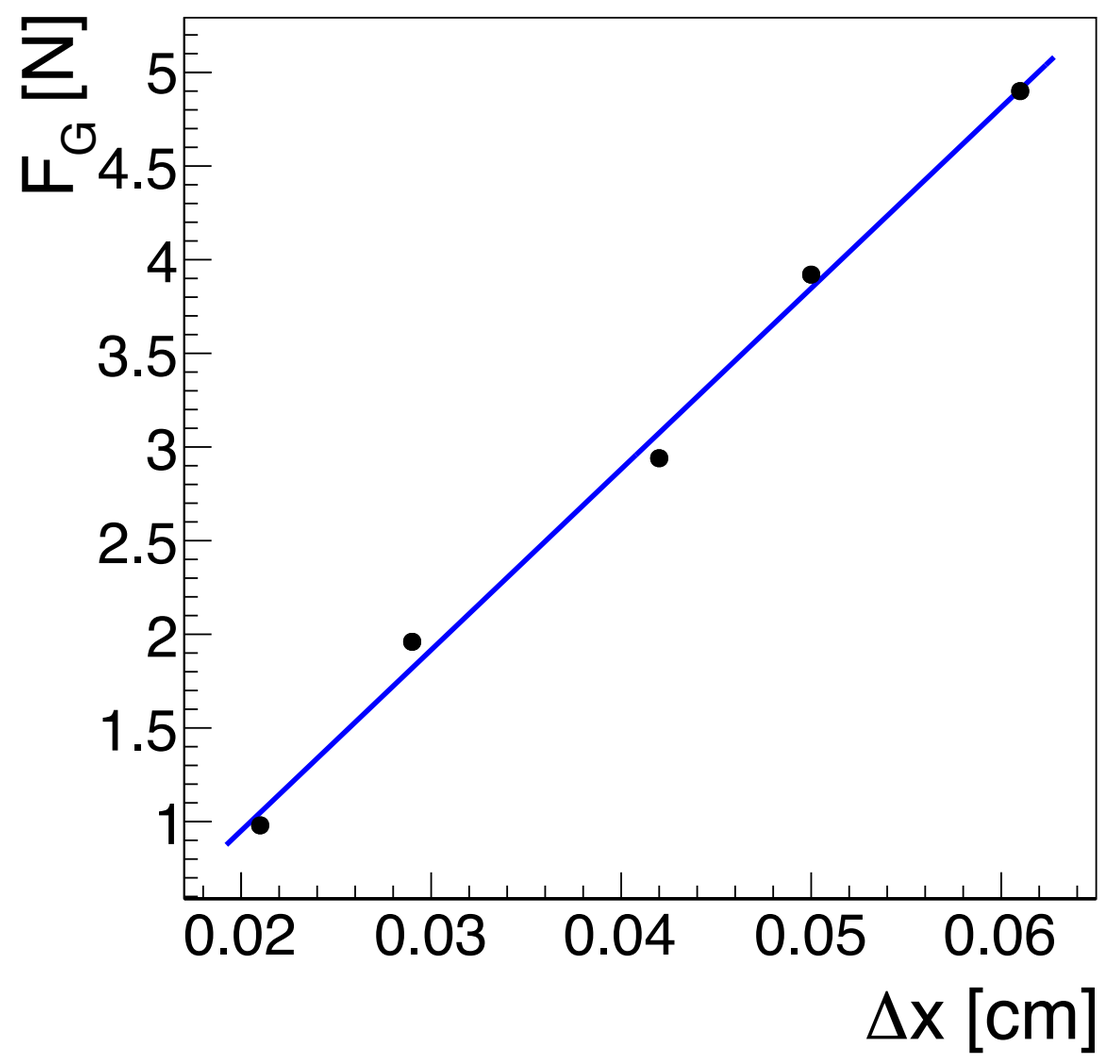
- Force is proportional to, and in the opposite direction of, the spring extension (compression)



Example #1: Real Spring

mass [g]	Δx [cm]	fitted Δx value [cm]
100	2.10	2.01
200	2.90	3.02
300	4.20	4.03
400	5.00	5.04
500	6.10	6.05

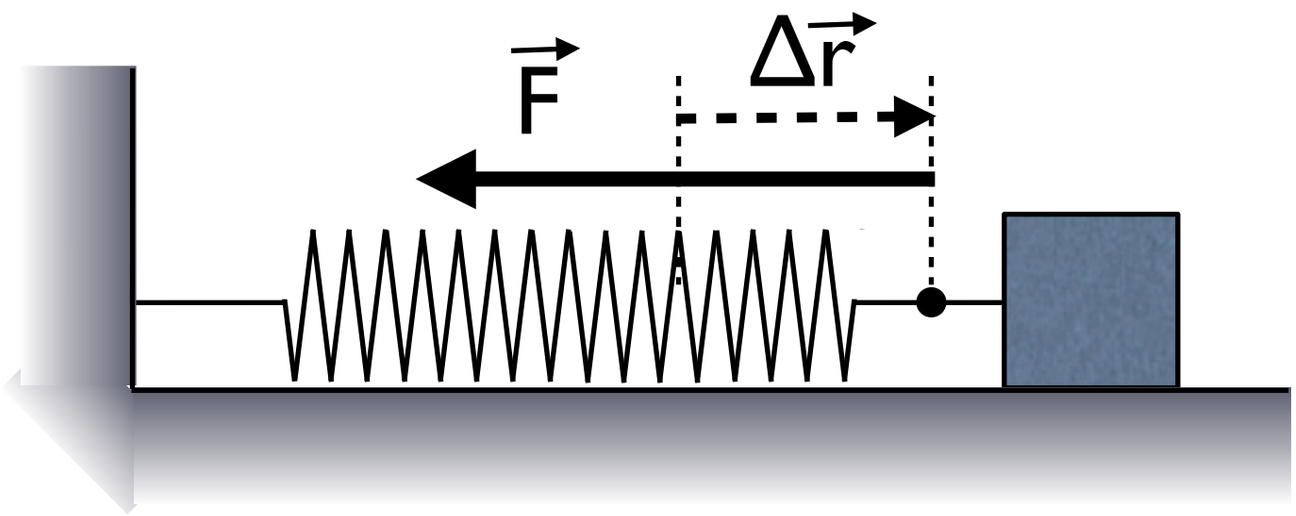
$k = 96.5 \pm 3.9$ N/m $\quad x_0 = 1.0 \pm 0.2$ cm



K = 97 N/m

x_0 = 1 cm

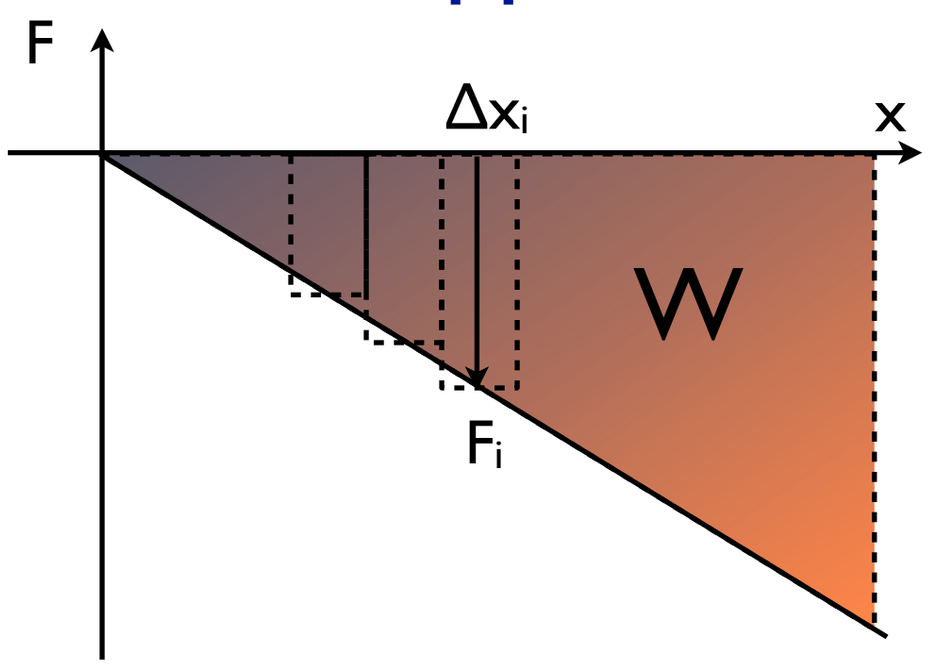
Work done by an



Recall the definition of work:

$$W = F\,\Delta r\,\cos(\theta)$$

- Force opposes deformation → work is always negative



Important: Force changes with extension (variable force)

$$W_{0\to x} = \sum_i F_i\,\Delta x_i = -\frac{1}{2}k\,x^2$$

[Convention: x = 0 for relaxed spring]

Elastic potential energy

- Elastic forces are conservative forces
- It makes sense to define "elastic" potential energy
- Same convention as for gravity:

$$\boxed{\begin{aligned}\Delta U_{elastic,x_1\rightarrow x_2} &= -W_{elastic} = \frac{1}{2}k\Delta(x^2) \\ &= \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\end{aligned}}$$

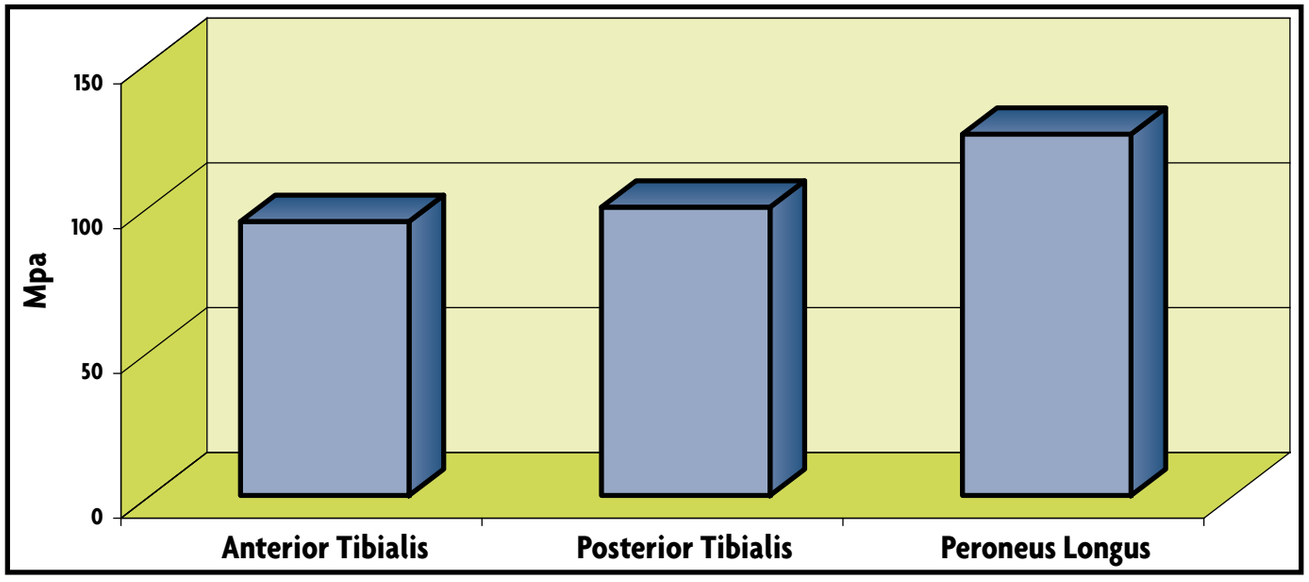
- In context of energy conservation:

$$\boxed{\begin{aligned}E_i &= K_i + U_{grav,i} + U_{elastic,i}\ [+W_{NC}] = \\ E_f &= K_f + U_{grav,f} + U_{elastic,f}\end{aligned}}$$

Example #3: Ballista

- A ballista uses torsion springs to store elastic potential energy, reportedly had range > 500 m
- 1/2 talent caliber [1 talent ~ 32 kg]
- What was the spring constant k if ballista had a 500m range? [$\Delta x \sim 2.0$ m]
- What was the tension in the retracting cable at full extension? [made of tendons]

Figure 3. Tendon strength in Newtons per cross-sectional area



Power

- Recall the unit "furlong"
- Related to farming - distance that a team of oxen could plough without resting
- 4 oxen would plough a furlong in half the time needed for a pair
- Power: how quickly is work getting done?
- Unit: 1 Watt = 1 Joule / second [1 kWh = 3.6×10^6 J]

$$P_{av} = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{F\Delta r\cos\theta}{\Delta t} = F\frac{\Delta r}{\Delta t}\cos\theta = F\,v_{av}\,\cos\theta$$

$$\boxed{P = F\,v\,\cos\theta}$$

Example #4: TGV Speed Record

The TGV high speed train holds a speed record of 550 km/h. The traction system can generate 19.6 MW of power. The limiting factor for the train's top speed is air drag.

Compute the force due to air drag at top speed.

Compare that to the weight of a killer whale (6 tons)

If the air drag force were maintained, what distance would be needed for the train to stop? The mass of the train is 380 tons. [in reality it took 70 km to stop]

Summary

- Elastic forces (springs) are modeled by Hooke's Law:

$$\boxed{F_x = -k\Delta x}$$

- Combining multiple springs [from Hooke's Law]:

$$\frac{1}{k_{serial}} = \frac{1}{k_1} + \frac{1}{k_2} + ... + \frac{1}{k_n}; \quad k_{parallel} = k_1 + k_2 + k_3 + ... + k_n$$

- Elastic forces are conservative, potential energy

$$\Delta U_{elastic, x_i \to x_f} = -W_{elastic} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

- Power - rate of work done over time:

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = Fv\cos\theta$$