## Announcements

1. HW6 due March 4.
2. Midterm1:
if you want to look at your scantron, see Prof. Chan before end of today
3. Solutions to even number questions in textbook
starting Chapter 6, numerical values will be posted in the HW solutions page (after HW6 is due).

## Average Angular Speed

- The average angular speed, $\omega$, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$
\omega_{\mathrm{av}}=\frac{\theta_{\mathrm{f}}-\theta_{\mathrm{i}}}{\mathrm{t}_{\mathrm{f}}-\mathrm{t}_{\mathrm{i}}}=\frac{\Delta \theta}{\Delta \mathrm{t}}
$$



Unit: rad/s

## The Radian

- The radian is a unit of angular measure
- The radian can be defined as the arc length s along a circle divided by the radius $r$
- 

$$
\theta=\frac{\mathrm{s}}{\mathrm{r}}
$$



## Average Angular Acceleration

- The average angular acceleration $\alpha$ of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$
\alpha_{\mathrm{av}}=\frac{\omega_{\mathrm{f}}-\omega_{\mathrm{i}}}{\mathrm{t}_{\mathrm{f}}-\mathrm{t}_{\mathrm{i}}}=\frac{\Delta \omega}{\Delta \mathrm{t}}
$$

Unit: rad/s ${ }^{2}$

- Displacements

$$
\Delta s=\Delta \theta r
$$

- Speeds
$v_{t}=\omega r$
- Accelerations

$$
\mathrm{a}_{\mathrm{t}}=\alpha \mathrm{r}
$$

- Every point on the rotating object has the same angular motion
- Not every point on the rotating object has the same linear motion


## Relationship Between Angular and Linear Quantities

## Analogies Between Linear and Rotational Motion

| Linear Motion with $a$ Constant <br> (Variables: $\boldsymbol{x}$ and $\boldsymbol{v}$ ) | Rotational Motion about a Fixed <br> Axis with $\boldsymbol{\alpha}$ Constant (Variables: $\boldsymbol{\theta}$ and $\omega$ ) |  |
| :---: | :---: | :---: |
| $v=v_{i}+a t$ | $\omega=\omega_{i}+\alpha t$ | $[7.7]$ |
| $\Delta x=v_{i} t+\frac{1}{2} a t^{2}$ | $\Delta \theta=\omega_{i} t+\frac{1}{2} \alpha t^{2}$ | $[7.8]$ |
| $v^{2}=v_{i}^{2}+2 a \Delta x$ | $\omega^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta$ | $[7.9]$ |

A coin with a diameter of 2.40 cm is dropped on edge onto a horizontal surface. The coin starts out with an initial angular speed of $18.0 \mathrm{rad} / \mathrm{s}$ and rolls in a straight line without slipping. If the rotation slows with an angular acceleration of magnitude $1.90 \mathrm{rad} / \mathrm{s}^{2}$, how far does the coin roll before coming to rest?


Newton's Second Law says that the centripetal acceleration is accompanied by a force
$\mathrm{F}_{\mathrm{C}}=\mathrm{ma}_{\mathrm{C}}$
$\mathrm{F}_{\mathrm{C}}$ stands for any force that keeps an object
following a circular path

- Tension in a string
- Gravity
- Force of friction


## Centripetal Acceleration

- Centripetal refers to "center-seeking"
- The direction of the velocity changes, the speed remains constant
- The acceleration is directed toward the center of the circle of motion

(b)


## Centripetal Acceleration

- The magnitude of the centripetal acceleration is given by

$$
a_{c}=\frac{v_{t}^{2}}{r}
$$

- This direction is toward the center of the circle
- The angular velocity and the linear velocity are related ( $\mathrm{v}_{\mathrm{t}}=\omega \mathrm{r}$ )
- The centripetal acceleration can also be related to the angular velocity

$$
a_{c}=\omega^{2} r
$$



- General equation $F_{C}=m a_{C}=\frac{m v^{2}}{r}$
- If the force vanishes, the object will move in a straight line tangent to the circle of motion


## Level Curves

- Friction is the force that produces the centripetal acceleration
- Can find the frictional force, $\mu$, or v

- A component of the normal force adds to the frictional force to allow higher speeds
$\tan \theta=\frac{\mathrm{v}^{2}}{\mathrm{rg}}$
or $\mathrm{a}_{\mathrm{c}}=\mathrm{g} \tan \theta$

- Look at the forces at the top of the circle
- The minimum speed at the top of the circle can be found

$$
v_{\text {top }}=\sqrt{g R}
$$


(a)
.

## Banked Curves



## Total Acceleration

- The tangential component of the acceleration is due to changing speed
- The centripetal component of the acceleration is due to changing direction
- Total acceleration can be found from these components

$$
a=\sqrt{a_{t}^{2}+a_{c}^{2}}
$$

