

initial momentum =  $\vec{p}_i$

final momentum =  $\vec{p}_f$

change in momentum  $\Delta \vec{p} = \vec{p}_f - \vec{p}_i$

Definition  $\vec{p}_i = m \vec{v}_i$

$\vec{p}_f = m \vec{v}_f$

$$\Delta \vec{p} = m (\vec{v}_f - \vec{v}_i) = m \Delta \vec{v}$$

Divide by  $\Delta t$

$$\frac{\Delta \vec{p}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t}$$

For constant force  $\vec{F} = m \vec{a} = m \frac{\Delta \vec{v}}{\Delta t}$

$$= \frac{\Delta \vec{p}}{\Delta t}$$

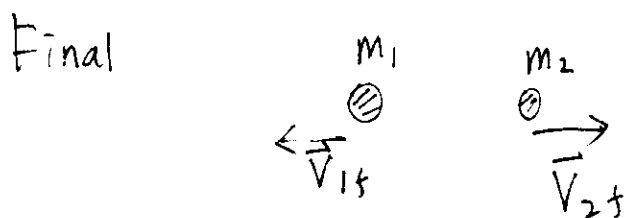
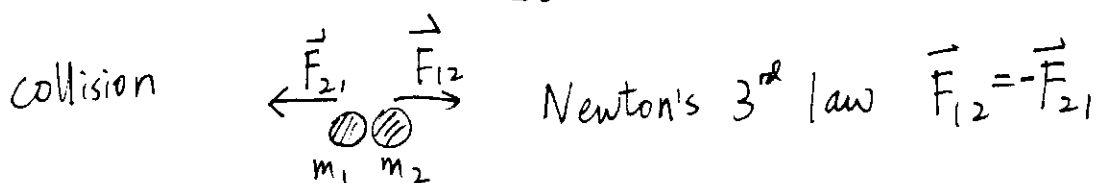
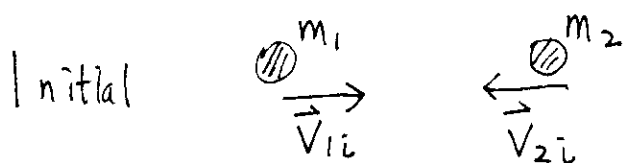
# Conservation of momentum

$$\Delta \vec{p} = \vec{F}_{av} \Delta t$$

$$\text{If } \vec{F}_{av} = 0 \Rightarrow \Delta \vec{p} = 0 \Rightarrow \vec{p}_i = \vec{p}_f$$

(same as Newton's 2<sup>nd</sup> law)

2 masses colliding



$$\Delta \vec{p}_1 = m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i} = \vec{F}_{21} \Delta t \quad \textcircled{1}$$

$$\Delta \vec{p}_2 = m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i} = \vec{F}_{12} \Delta t = -\vec{F}_{21} \Delta t \quad \textcircled{2}$$

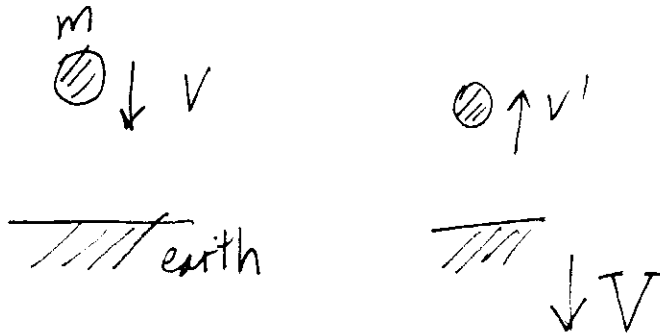
①+②

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 = m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i} + m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i} = 0$$

change in momentum of system  $m_1$  &  $m_2 = 0$   
total momentum is conserved.

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

# Ball bounce



Momentum conservation

$$m(-v) + M(0) = m(v') + M(-V)$$

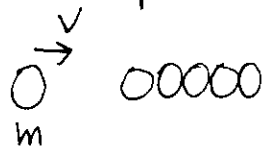
$\uparrow$  mass of ball       $\uparrow$  mass of earth

$$MV = m(v' + v)$$

$$V = \frac{m}{M}(v' + v)$$

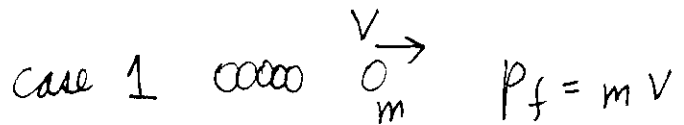
$\approx 0$  because  $m \ll M$

Momentum Balls experiment



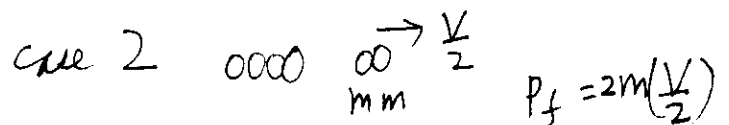
$$P_i = mv$$

$$KE_i = \frac{1}{2}mv^2$$



$$P_f = mv$$

$$KE_f = \frac{1}{2}mv^2$$



$$P_f = 2m\left(\frac{v}{2}\right)$$

$$KE_f = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2$$

$$= \frac{1}{4}mv^2$$


$$\neq KE_i$$

Inelastic collisions



$$P_i = mV + m(-V) = 0$$

$$P_f = 0$$

 inelastic  $KE_f = 0$



$$P_i = mV + 2m(-V) = -mV$$

stick  $P_f = 3m(V_f)$

$$P_i = P_f$$

$$-mV = 3mV_f$$

$$V_f = -\frac{V}{3}$$

$$KE_f = \frac{1}{2}(3m)\left(-\frac{V}{3}\right)^2$$

$$\neq 0$$