

conservation of energy.

Total energy at  $x = \pm A$  = Total energy at arbitrary  $x$

$$\frac{1}{2} m v_A^2 + \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$k A^2 = m v^2 + k x^2$$

$$m v^2 = k (A^2 - x^2)$$

$$v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)} \quad (1)$$

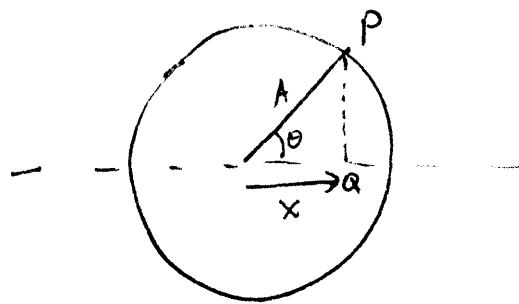
max at  $x=0$   
zero at  $x=\pm A$

$F = ma$  Newton's law

$F = -kx$  Hooke's law

$$m a = -kx$$

$$a = -\frac{kx}{m} \quad (2)$$



- P rotates at constant angular velocity  $\omega$
- Q is the projection of P onto the x axis
- Q undergoes simple harmonic motion

$$x = A \cos \theta$$

$$\theta = \omega t$$

$$\boxed{x = A \cos \omega t}$$

$$\cos \omega t = \frac{x}{A}$$

Along x axis,  $V = -V_t \cos(90^\circ - \theta)$

$$= -V_t \sin \theta$$

$$= -V_t \sin \omega t$$

$$\boxed{V = -\omega A \sin \omega t}$$

tangential speed for circular motion  
 $V_t = \omega A$

$$V^2 = \omega^2 A^2 \sin^2 \omega t$$

$$= \omega^2 A^2 (1 - \cos^2 \omega t)$$

$$= \omega^2 A^2 \left(1 - \frac{x^2}{A^2}\right)$$

$$= \omega^2 (A^2 - x^2)$$

$$V = \pm \omega \sqrt{A^2 - x^2}$$

Comparing with ①

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \text{ angular frequency}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

Point P rotates  $2\pi$  radians in  $T$  seconds.

$$\boxed{f = \frac{\omega}{2\pi}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$$

Using

$$F = ma = -kx$$

$$a = -\frac{k}{m}x = -\omega^2 x$$

$$\boxed{a = -\omega^2 A \cos \omega t}$$

Simple pendulum

$$F_t = -mg \sin \theta$$

$$\approx -mg \theta$$

$$= -mg \left( \frac{s}{L} \right)$$

$$= - \left( \frac{mg}{L} \right) s$$



$k_{\text{eff}}$

$$T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}}$$

$$= 2\pi \sqrt{\frac{m}{\frac{mg}{L}}}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

