Waves

A **wave** is a **disturbance** that **propagates energy** through a medium without net mass transport.

Ocean waves provide example of **transverse** waves in which if we focus on a small volume of water, at a particular location, we would see that it moves (mostly) up and down as the wave moves by.

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html
Sound waves propagating through the atmosphere provide example of a **longitudinal** wave.

Consider a speaker oscillating at 440 Hz (middle C). For one half cycle it pushes the air in front of it, compressing the air, thereby increasing the pressure just in front of it. This pressurized volume of air pushes on the adjacent volume of air pressurizing it, and so on.

The half cycle has launched a disturbance of pressurized air propagating out from the speaker. Before this disturbance has traveled very far, the diaphragm quickly draws back, in the second half of the cycle now rarifying the air in front of it.
This train of pressurized/rarified regions propagate through the air.

Here the air molecules slosh back and forth along the direction of the wave motion but not very far as the waves move to the right.

We can plot these pressure variations versus position and immediately see the similarity with other waves to recognize that these have a common description.
A convenient way to represent waves is to draw **wave fronts**.

Wave fronts are points of constant phase on a wave, often chosen as the crests of the propagating wave (moving with it).

A point source radiates its wave energy radially outwards **in all directions**.
Since waves transport energy from one place to another and energy is conserved the radiation pattern (shape of the wavefronts) is important.

To be concrete we consider a spherical speaker that launches sound waves radially outwards in all directions.

The speaker radiates sound energy with a power (energy radiated/time) $P$.

If we draw a spherical shell of radius $r$ around the speaker then (ignoring energy losses to friction in the air) since the shell intercepts all the radiated power, the total power intercepted is the emitted power $P$.

Something that will be true for any radius shell that we draw (independent of $r$).
But a sound detector (like an ear) doesn’t typically surround a source. Rather it has a typically (much) smaller area that intercepts only a portion of any passing waves.

This makes it sensible to define for waves the **intensity** which is the \(\text{power crossing}/(\text{unit area})\) (with the area perpendicular to the waves direction of propagation):

\[
I = \frac{P}{A}
\]

For a **spherically radiating source** the intensity at distance \(r\) from the source is,

\[
I = \frac{P}{4\pi r^2} \left( \frac{\text{source power}}{\text{area of shell of radius } r} \right)
\]

The intensity falls off with the distance from the source squared.
Suppose that the source radiates with \( P = 10 \text{ mW} \).

Then at 1 meter the intensity is
\[
I = \frac{P}{4\pi r^2} = \frac{10 \times 10^{-3} \text{ W}}{4\pi (1 \text{ m})^2} = 8.0 \times 10^{-4} \frac{\text{W}}{\text{m}^2}
\]
while
\[
I = \frac{P}{4\pi r^2} = \frac{10 \times 10^{-3} \text{ W}}{4\pi (10 \text{ m})^2} = 8.0 \times 10^{-6} \frac{\text{W}}{\text{m}^2}
\]

If the detector is an ear with an area of
\( 1 \text{ cm}^2 = 10^{-4} \text{ m}^2 \) then the power it intercepts, \( P' \), is
\[
P' = I \cdot A_{\text{ear}}
\]

So at 1 m,
\[
P' = 8.0 \times 10^{-4} \frac{\text{W}}{\text{m}^2} (10^{-4} \text{ m}^2) = 8.0 \times 10^{-8} \text{ W}
\]
and at 10 m,
\[
P' = 8.0 \times 10^{-6} \frac{\text{W}}{\text{m}^2} (10^{-4} \text{ m}^2) = 8.0 \times 10^{-10} \text{ W}
\]
Note that this fall off of the intensity with the inverse square distance from a spherically radiating source is really a consequence of the conservation of energy.

\[ I = \frac{P}{4\pi r^2} \]

As the waves spread out spherically they distribute the same power (the emitted source power) over an ever increasing area (the surface area of increasing radius shells).

If instead of spherical waves we had **plane waves** the sound could go much further without reduction of the intensity.
The demo in the lobby with the two large parabolic reflectors does just that.

If you whisper in your softest voice into one reflector it converts the hemi-spherical waves emanating from your mouth into plane waves.

Despite the long distance across the lobby to the other dish the plane waves get there, little affected by the distance.

At the second dish the plane waves are reconverted to spherical waves that focus down to an ear, where what you said can be heard.
To develop some of the characteristic features of waves we consider waves induced on a string by a vibrating rod.

The rod here vibrates with a constant frequency (cycles/s) inducing transverse waves in the string that propagate to the right.

Any particular point (P) on the string executes simple harmonic oscillations, up and down with an amplitude $A$ (the maximum displacement), a period $T$ (time per cycle) and a frequency $f$ (cycles/s).
The period of this oscillation is set by the period with which the rod vibrates.

Since frequency and period are inversely related \( (f = 1/T) \) the frequency with which point P oscillates is also set by the frequency with which the rod oscillates.

So \( f \) (and thus \( T \)) are established by the source (generally true).

Now consider two adjacent crests of the resulting wave. The distance between them is defined as the wavelength \( \lambda \).
The **time taken** for the wave to move by **one wavelength** is clearly one cycle which gives the wave velocity as:

\[ v = \frac{\lambda}{T} \]

Since

\[ f = \frac{1}{T} \]

This can also be expressed as,

\[ v = \lambda f \]

The wave velocity is given by the wavelength times the frequency.
Now in contrast to the frequency or period which is set by the source. The wave velocity is set by properties of the medium.

If the tension in the string is $F$ and its linear density (mass/length) is $\mu$ then the wave speed down the string is given by (GRR p. 397)

$$v = \sqrt{\frac{F}{\mu}}$$

$$= \sqrt{\frac{\text{N}}{\text{kg/m}}} = \sqrt{\frac{\text{kg m/s}^2}{\text{kg/m}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}}$$
Example

A 100 cm long taught string has a mass of 6.00 g. A wave is found to travel at 10.0 m/s on this string. A second string has the same length and tension but half the mass of the first. What will the speed of the wave be on the second string?

\[ v_1 = \sqrt{\frac{F}{\mu_1}} \quad v_2 = \sqrt{\frac{F}{\mu_2}} \]

Divide

\[ \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\mu_1}{\mu_2}} \]

\[ v_2 = v_1 \sqrt{\frac{\mu_1}{\mu_2}} \]
\[
\frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}}
\]

\[v_2 = v_1 \sqrt{\frac{\mu_1}{\mu_2}}\]

But told that

\[m_2 = \frac{m_1}{2} \quad \rightarrow \quad \frac{m_2}{L} = \frac{m_1}{L_2} \quad \rightarrow \quad \mu_2 = \frac{\mu_1}{2}\]

Then

\[v_2 = v_1 \sqrt{\frac{\mu_1}{\mu_1/2}} = v_1 \sqrt{2}\]

Also, given that

\[v_1 = 10 \frac{m}{s}\]

So,

\[v_2 = (10.0 \frac{m}{s}) \sqrt{2} = 14.1 \frac{m}{s}\]
Example

A transverse wave on a taught string has amplitude $A$, wavelength $\lambda$ and speed $v$.

A point of the string only moves in the transverse direction. Call its maximum speed $v_{T_{\text{max}}}$.

In terms of $A$ and $\lambda$ what is $\frac{v_{T_{\text{max}}}}{v}$?
Since each point of the string undergoes simple harmonic oscillations the transverse displacement of a point is,

\[ y = A \cos \omega t \]

And its transverse speed is,

\[ v_T = -\omega A \sin \omega t \]

The sine function repeatedly oscillates between ±1 so

\[ v_{T_{\text{max}}} = \omega A \]
But \( \omega = 2\pi f \)  so,

\[ v_{T_{\text{max}}} = \omega A = 2\pi f A \]

The wave speed is \( v = \lambda f \)

So

\[ \frac{v_{T_{\text{max}}}}{v} = \frac{2\pi f A}{\lambda f} = 2\pi \frac{A}{\lambda} \]
For the simple harmonic oscillator the acceleration of the mass has maximum magnitude at point:

\[ F = -kx \]

\[ x(t) \]

\[ \pm x_{\text{max}} \]
The piston in an internal combustion engine executes approximately simple harmonic motion. When the engine is running at 3000 rpm the period for the piston’s motion is?

A) 0.00033 s

B) 0.020 s

C) 50 s

D) 3000 s
Reflection of waves at boundaries

Two cases: boundary fixed or free

Fixed end case

Reflection inverted

Reflection not inverted
Wave interference

Consider two single pulse waves on the same rope propagating in opposite directions.

They pass through each other and continue on completely unaffected by the “collision”.

During the time when they occupy the same segment of rope they obey the principle of superposition.

That is the net displacement at each point becomes simply the sum of the individual displacements at each point.
The superposition principle is a **signed summation** so that if one of the pulses has a negative displacement then the **resultant wave** (the sum) can be smaller than one or both waves as they cross.

If the two waves are mirror images of each other then as they cross the net amplitude is zero.