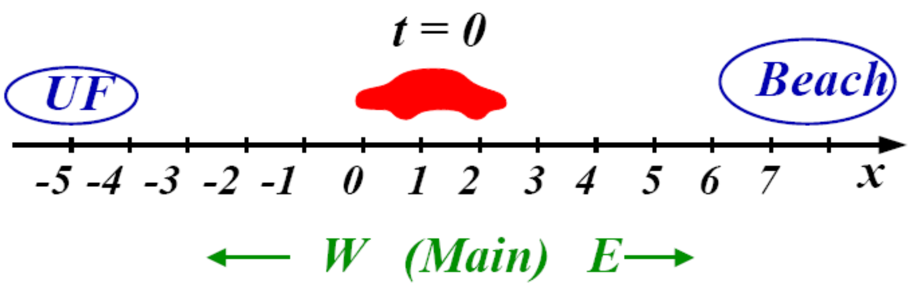
1-d Motion: Position & Displacement

- **The x-axis:**

We locate objects by specifying their position along an axis (in this case x-axis). The positive direction of an axis is in the direction of increasing numbers. The opposite is the negative direction.

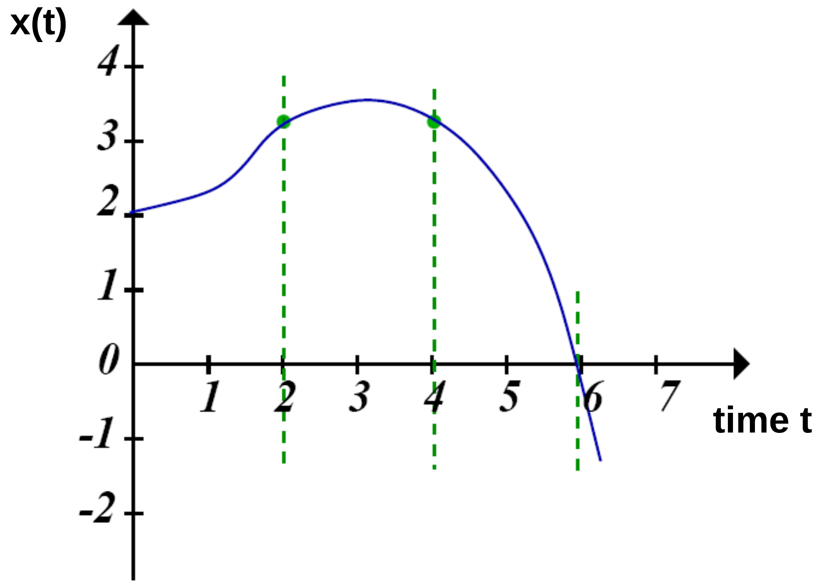


- **Displacement:**

The change from position x_1 to position x_2 is called the displacement, Δx.

$$\Delta x = x_2 - x_1$$

The displacement has both a magnitude, $|\Delta x|$, and a direction (positive or negative).



- **Graphical Technique:**

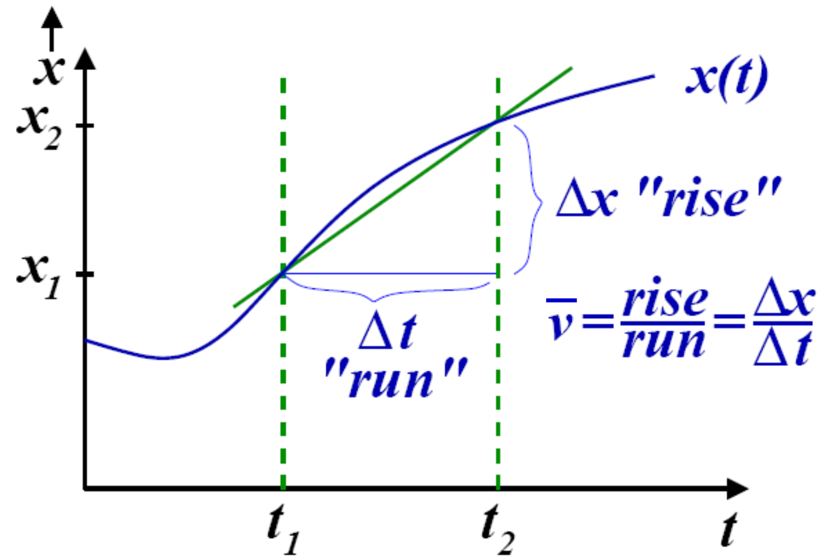
A convenient way to describe the motion of a particle is to plot the position x as a function of time t (*i.e.* x(t)).

1-d Motion: Average Velocity

$$\bar{v} = v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{x_2 - x_1}{t_2 - t_1}$$

- **Average Velocity**

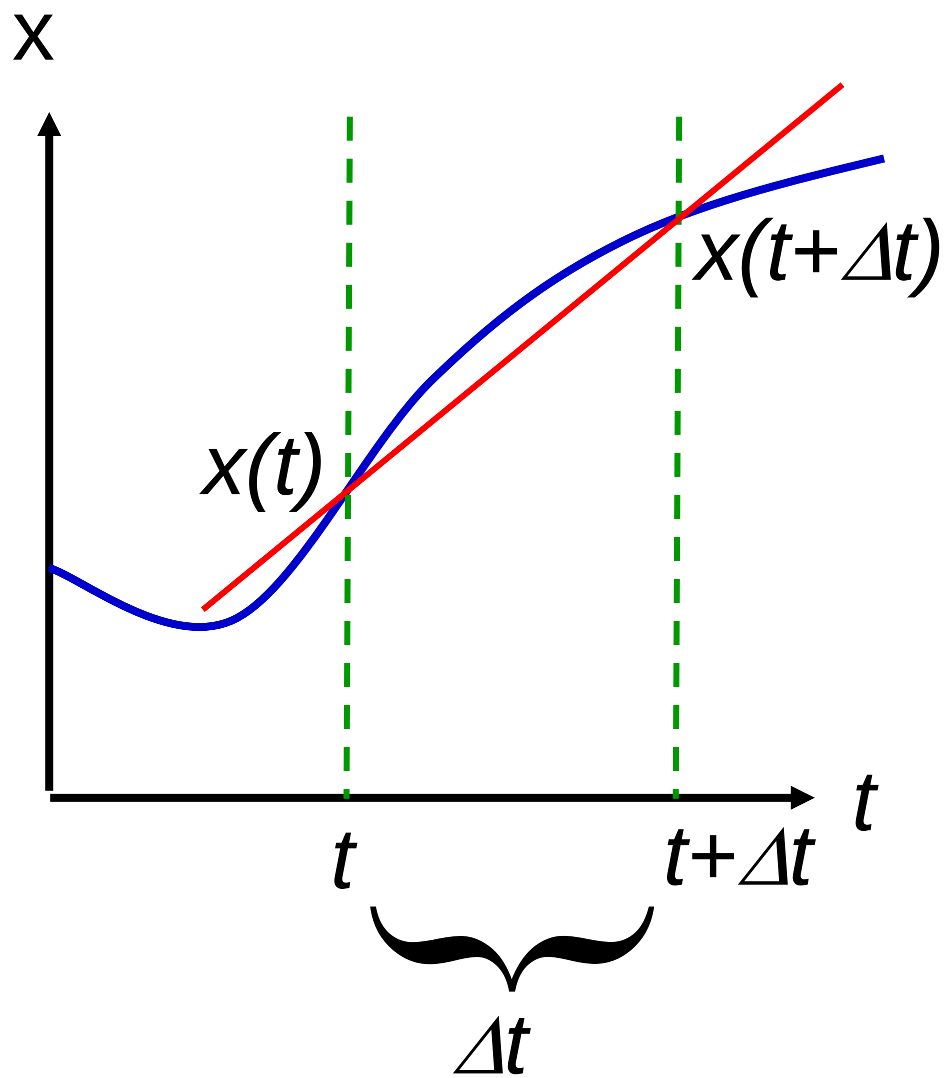
The average velocity is defined to be the displacement, Δx, that occurred during a particular interval of time, Δt (*i.e.* $v_{ave} = \Delta x/\Delta t$).



- **Average Speed**

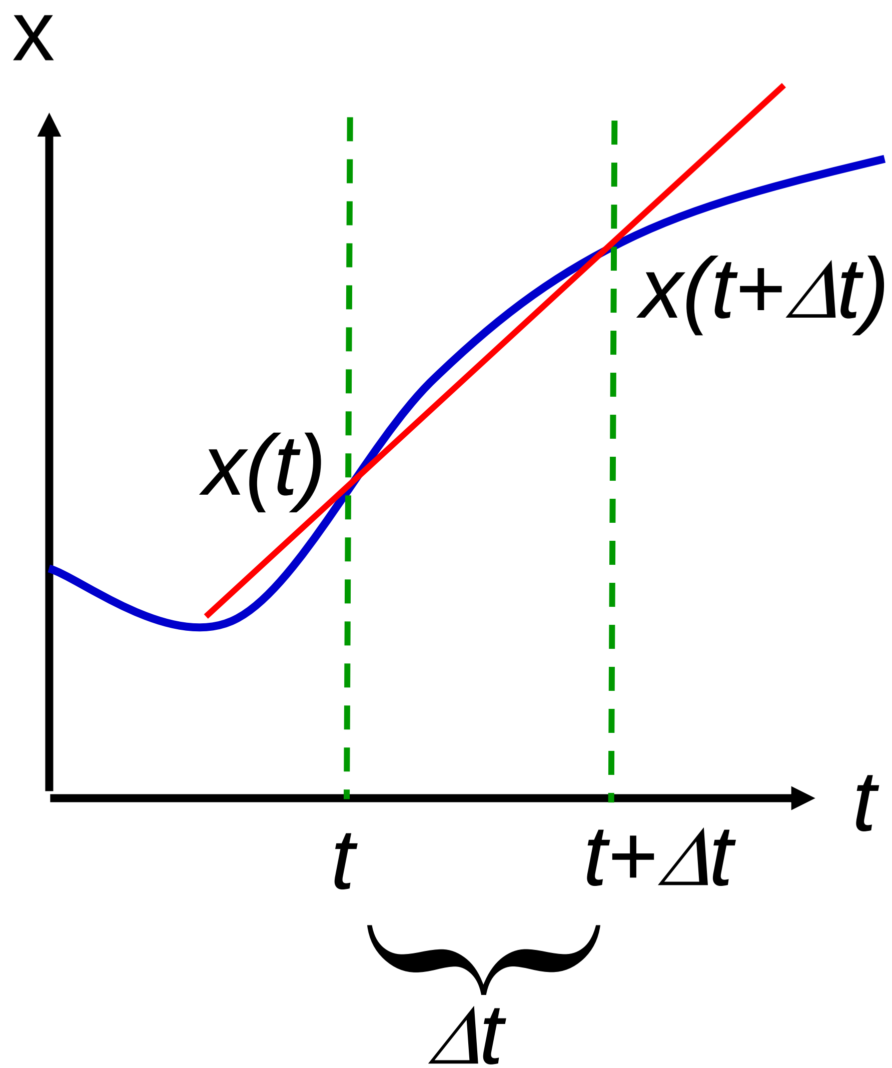
The average speed is defined to be the magnitude of total distance covered during a particular interval of time, Δt (*i.e.* s_{ave} = (total distance)/Δt).

1-d Motion: Instantaneous Velocity



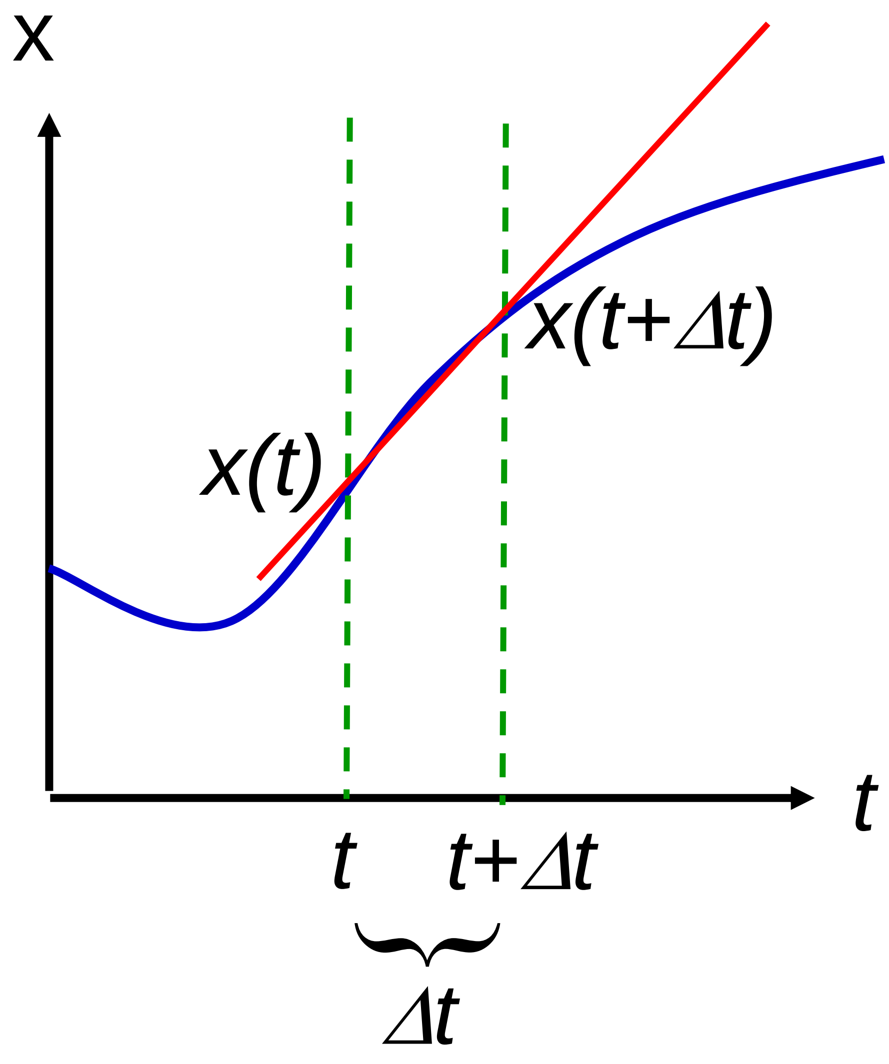
shrink Δt

1-d Motion: Instantaneous Velocity



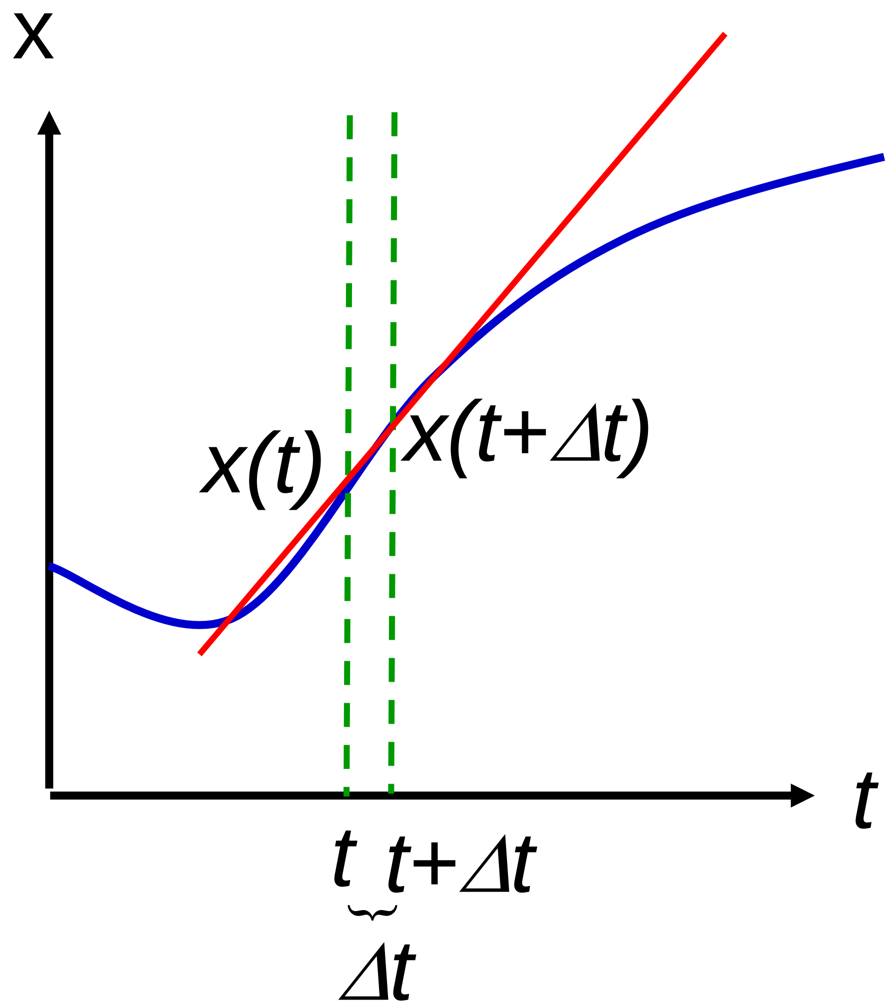
shrink Δt

1-d Motion: Instantaneous Velocity



shrink Δt

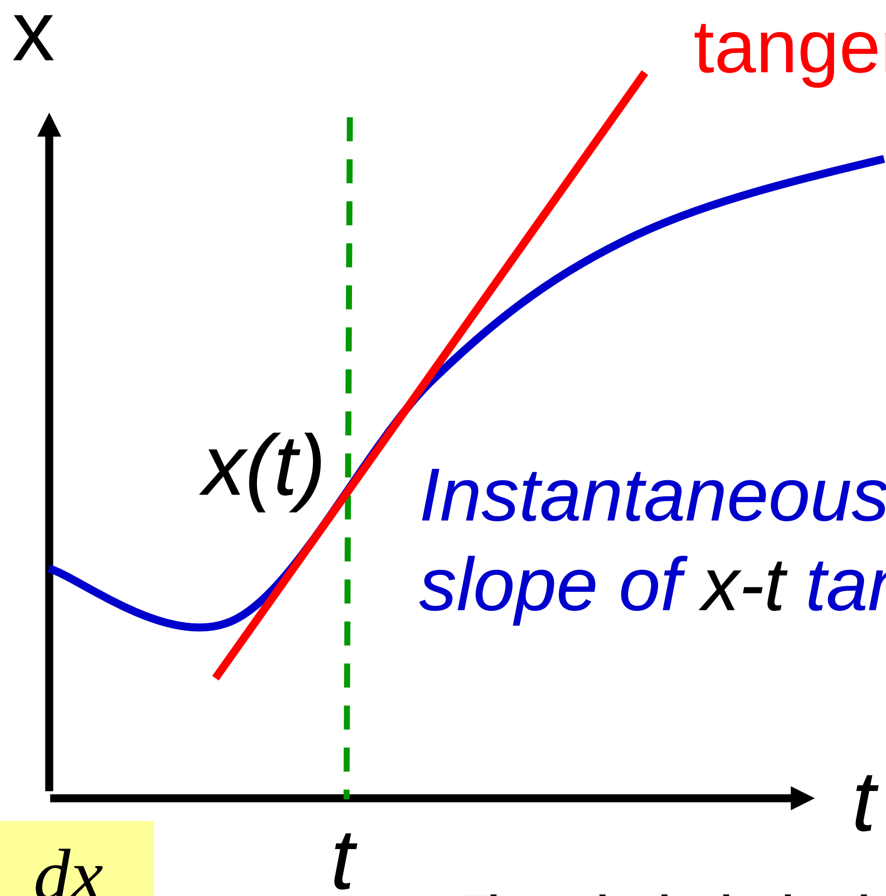
1-d Motion: Instantaneous Velocity



shrink Δt

1-d Motion: Instantaneous Velocity

tangent line at t



Instantaneous velocity v(t) is slope of x-t tangent line at t

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The velocity is the derivative of x(t) with respect to t.

1-d Motion: Acceleration

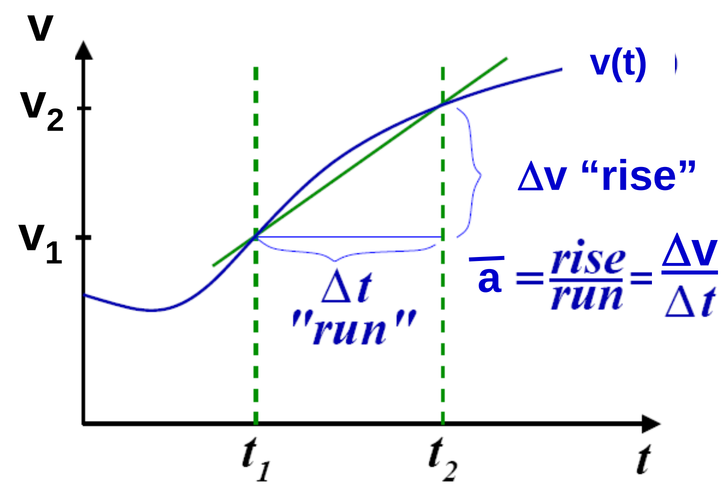
$$\bar{a} = a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{v_2 - v_1}{t_2 - t_1}$$

- **Acceleration**

When a particles velocity changes, the particle is said to undergo acceleration (*i.e.* accelerate).

- **Average Acceleration**

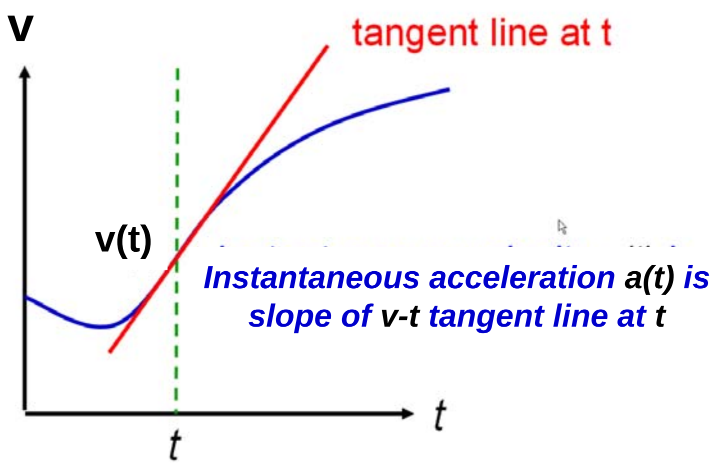
The average acceleration is defined to be the change in velocity, Δv, that occurred during a particular interval of time, Δt (*i.e.* $a_{ave} = \Delta v/\Delta t$).



- **Instantaneous Acceleration**

The acceleration is the derivative of v(t) with respect to t.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$



Equations of Motion: a = constant

- **Special case!**

  (constant acceleration)

$$a(t) = a$$

- **v is a linear function of t**

$$v(t) = v_0 + at$$

v at t = 0

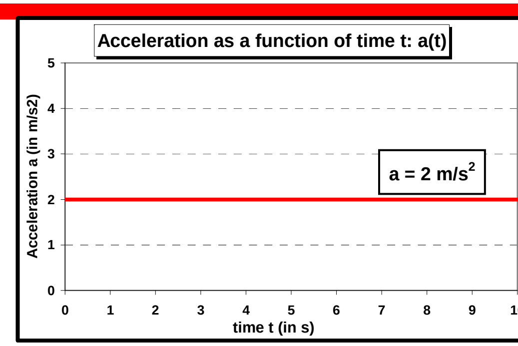
- **x is a quadratic function of t**

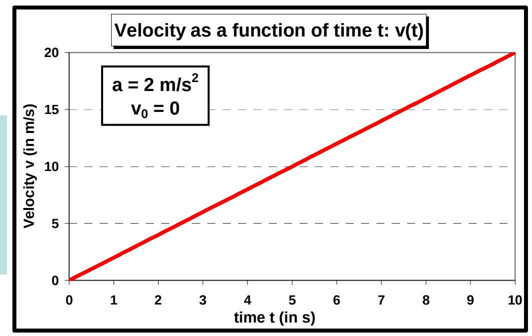
$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

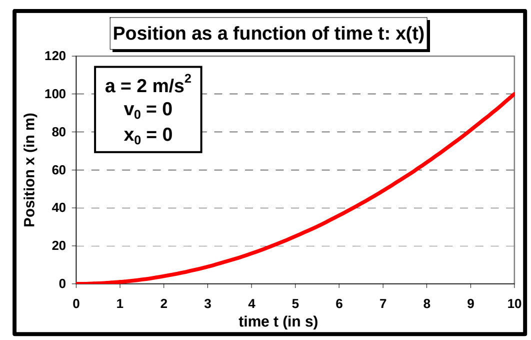
x at t =0

- **Note also that**

$$v^2 = v_0^2 + 2a(x - x_0)$$



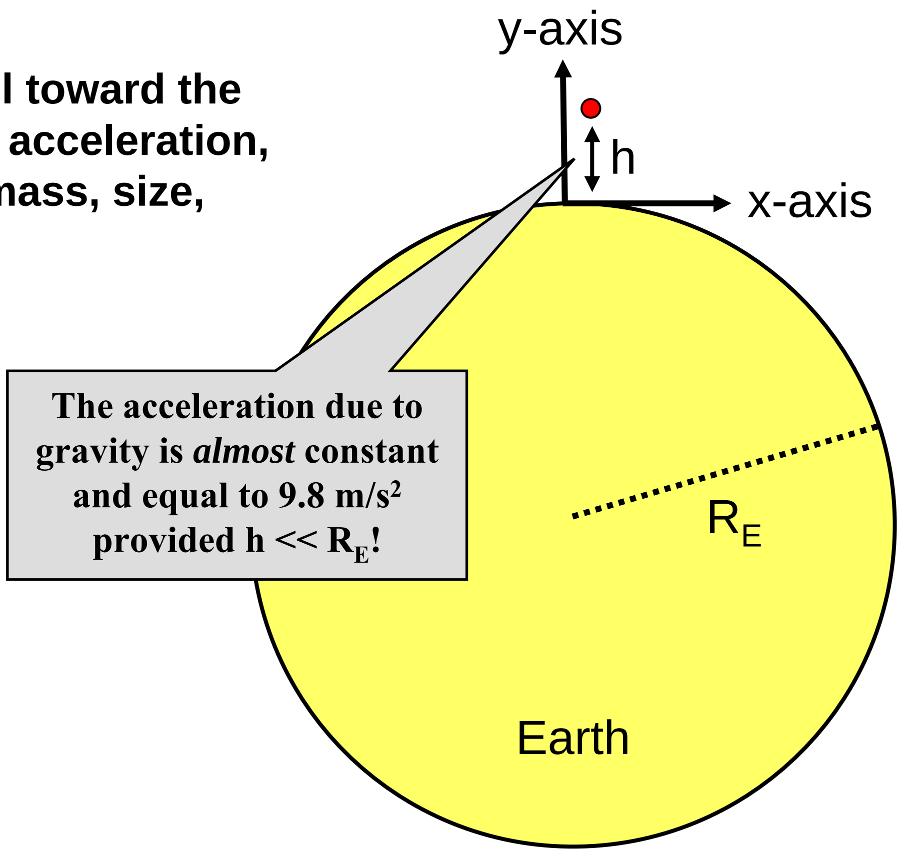




Acceleration Due to Gravity

- ## Experimental Result

Near the surface of the Earth all objects fall toward the center of the Earth with the same constant acceleration, g ≈ 9.8 m/s^2, (*in a vacuum*) independent of mass, size, shape, etc.



- ## Equations of Motion

$$a_y = -g \approx -9.8 m/s^2$$

$$v_y(t) = v_{y0} - gt$$

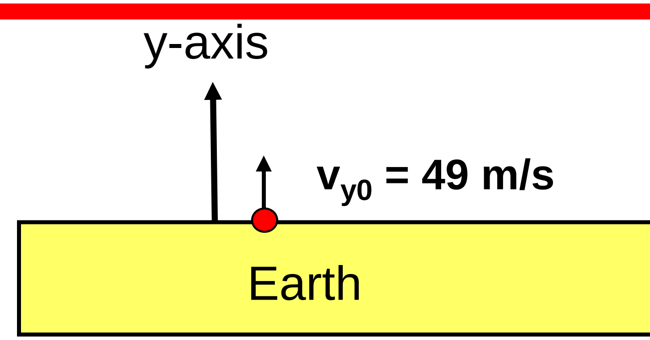
$$y(t) = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

Equations of Motion: Example Problem

• Example Problem

A ball is tossed up along the y-axis (in a vacuum on the Earth's surface) with an initial speed of 49 m/s.



How long does the ball take to reach its maximum height?

$$t = \frac{v_{y0}}{g} = \frac{49m/s}{9.8m/s^2} = 5s$$

What is the ball's maximum height?

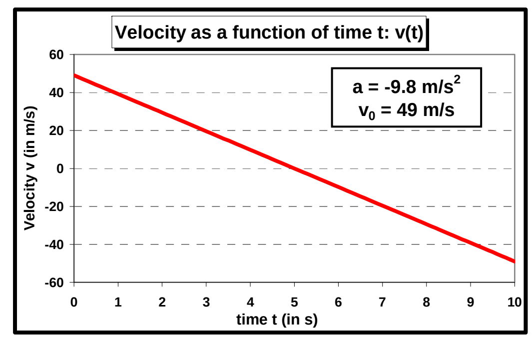
$$h = \frac{v_{y0}^2}{2g} = \frac{(49m/s)^2}{2(9.8m/s^2)} = 122.5m$$

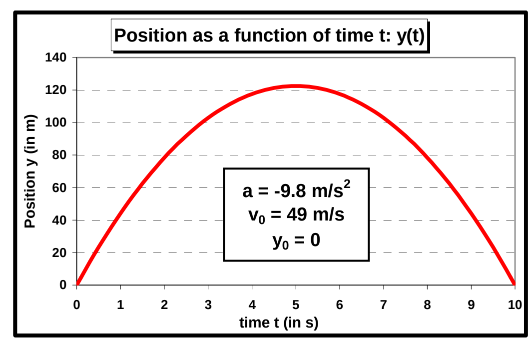
How long does it take for the ball to get back to its starting point?

$$t = \frac{2v_{y0}}{g} = 10s$$

What is the velocity of the ball when it gets back to its starting point?

$$v_y = -v_{y0} = -49m/s$$

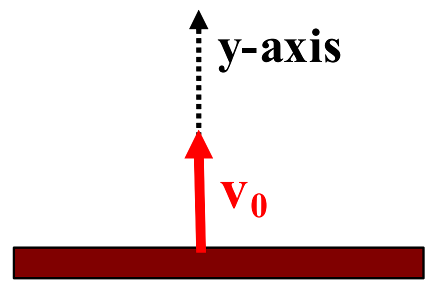




Final Exam Fall 2010: Problem 9

- **Near the surface of the Earth a startled armadillo leaps vertically upward at time t = 0, at time t = 0.5 s it is a height of 0.98 m above the ground. At what time does it land back on the ground?**

Answer: 0.9 s
% Right: 47%



$$y(t) = v_o t - \tfrac{1}{2} g t^2 = t(v_o - \tfrac{1}{2} g t)$$

Let t_f be the time the armadillo lands back on the ground.

$$y(t_f) = 0 = t_f (v_o - \tfrac{1}{2} g t_f)$$

$$t_f = \frac{2 v_o}{g}$$

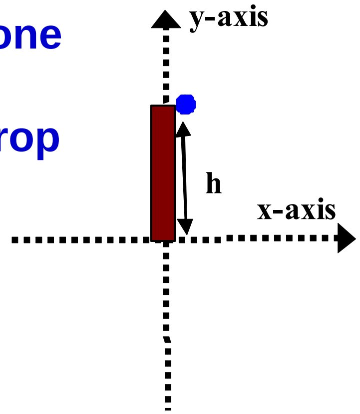
Let t_h be the time the armadillo is at the height h.

$$y(t_h) = h = v_o t_h - \tfrac{1}{2} g t_h^2 \quad \longrightarrow \quad v_o = \frac{h + \tfrac{1}{2} g t_h^2}{t_h}$$

$$t_f = \frac{2 v_o}{g} = \frac{2(h + \tfrac{1}{2} g t_h^2)}{g t_h} = \frac{2h}{g t_h} + t_h = \frac{2(0.98m)}{(9.8m/s^2)(0.5s)} + (0.5s) = (0.4s) + (0.5s) = 0.9s$$

Example Problem: 1d Motion

- **A suspension bridge is 60.0 m above the level base of a gorge. A stone is thrown or dropped off the bridge. Ignore air resistance. At the location of the bridge g has been measured to be 9.83 m/s^2. If you drop the stone how long does it take for it to fall to the base of the gorge?**



In this case, $a_x = 0$ and $a_y = -g$, $v_{x0} = 0$, $v_{y0} = 0$, $x_0 = 0$, $y_0 = h$. Hence,

$$v_x(t) = 0 \qquad v_y(t) = -gt$$

$$x(t) = 0 \qquad y(t) = h - \tfrac{1}{2}gt^2$$

The time, t_f, that the it takes the stone to reach the ground occurs when $y(t_f) = 0$. Hence,

$$y(t_f) = 0 = h - \tfrac{1}{2}gt_f^2 \qquad t_f = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(60m)}{(9.83m/s^2)}} \approx 3.494s$$

- **If you throw the stone straight down with a speed of 20.0 m/s, how long before it hits the ground?**

In this case, $a_x = 0$ and $a_y = -g$, $v_{x0} = 0$, $v_{y0} = -v_0$, $x_0 = 0$, $y_0 = h$. Hence,

Have to use the Quadratic Formula!

$$y(t) = h - v_0 t - \tfrac{1}{2}gt^2 \qquad y(t_f) = 0 = h - v_0 t_f - \tfrac{1}{2}gt_f^2$$

$$t_f = \frac{-v_0 \pm \sqrt{(v_o)^2 + 2gh}}{g} = \frac{-20m/s + \sqrt{(-20m/s)^2 + 2(9.83m/s^2)(60m)}}{9.83m/s^2} \approx 2.01s$$

Take the positive root!

Final Exam Fall 2011: Problem 7

- Near the surface of the Earth a suspension bridge is a height H above the level base of a gorge. Two identical stones are simultaneously thrown from the bridge. One stone is thrown straight down with speed v_0 and the other is thrown straight up a the same speed v_0. Ignore air resistance. If one of the stones lands at the bottom of the gorge 2 seconds before the other, what is the speed v_0 (in m/s)?

Answer: 9.8

% Right: 36%

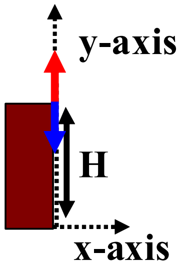
$$y_{up}(t) = H + v_o t - \tfrac{1}{2}gt^2 \qquad y_{down}(t) = H - v_o t - \tfrac{1}{2}gt^2$$

$$y_{up}(t_{up}) = 0 = H + v_o t_{up} - \tfrac{1}{2}gt_{up}^2 \qquad y_{down}(t_{down}) = 0 = H - v_o t_{down} - \tfrac{1}{2}gt_{down}^2$$

$$v_o t_{up} = \tfrac{1}{2}gt_{up}^2 - H$$

$$v_o t_{down} = -\tfrac{1}{2}gt_{down}^2 + H$$

$$v_0(t_{up} + t_{down}) = \tfrac{1}{2}g(t_{up}^2 - t_{down}^2)$$



$$v_0 = \tfrac{1}{2}g\frac{(t_{up}^2 - t_{down}^2)}{(t_{up} + t_{down})} = \tfrac{1}{2}g(t_{up} - t_{down}) = \tfrac{1}{2}(9.8m/s^2)(2s) = 9.8m/s$$

Exam 1 Fall 2010: Problem 4

- **A motorist drives along a straight road at a constant speed of 80 m/s. Just as she passes a parked motorcycle police officer, the officer takes off after her at a constant acceleration. If the officer maintains this constant value of acceleration, what is the speed of the police officer when he reaches the motorist?**

Answer: 160 m/s
% Right: 18%

$$a_{car} = 0 \qquad a_{cop} = a$$

$$v_{car}(t) = v_0 \qquad v_{cop}(t) = at$$

$$x_{car}(t) = v_0 t \qquad x_{cop}(t) = \tfrac{1}{2}at^2$$

Let t_c be the time it takes for officer to reach the motorist.

$$x_{car}(t_c) = x_{cop}(t_c)$$

$$v_0 t_c = \tfrac{1}{2}at_c^2 \qquad t_c = \frac{2v_0}{a}$$

$$v_{cop}(t_c) = at_c = a\left(\frac{2v_0}{a}\right) = 2v_0$$

$$= 2(80m/s) = 160m/s$$