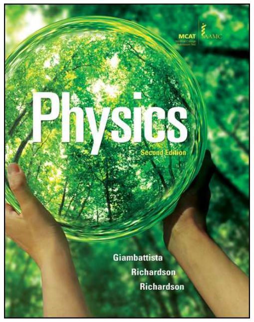
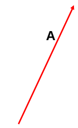
Chapter 3 Motion in a plane



Chapter 3: Motion in a Plane

- •Vector Addition
- •Velocity
- •Acceleration
- •Projectile motion
- •Relative Velocity

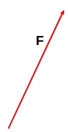
Vectors



A **vector** is a quantity that has both a **magnitude** and a **direction**. Position is an example of a vector quantity.

A **scalar** is a quantity with no direction. The mass of an object is an example of a scalar quantity.

Notation:



Vector: $\mathbf{F}$ or $\vec{\mathbf{F}}$

The magnitude of a vector: F or $|\mathbf{F}|$ or $|\vec{\mathbf{F}}|$.

The direction of vector might be "65° north of east"; "65° above the +x-axis"; or….

Scalar: m (not bold face; no arrow)

Vector Addition and Subtraction

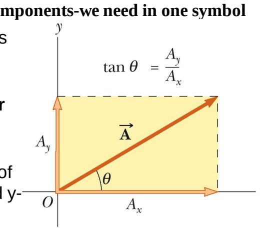
Vectors may be moved any way you please (to place them tip to tail) provided that you do not change their length nor rotate them.

If we lived in 1-Dimension, we would not need vectors, but… Would you like to be always standing in line, and the person in front of you only changed if he (or she) disappeared?

2-D Rules!

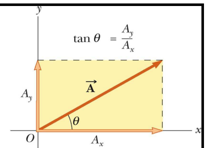
Vectors let us keep track of our way in any number of dimensions by carrying all the numbers-components-we need in one symbol

- Here is a vector that shows us going at an angle θ with the x axis
- It is useful to use **rectangular components** to manipulate vectors
  - These are the projections of the vector along the x- and y-axes A_x and A_y



A_x and A_y are scalars

Vector Components



- The x-component of a vector is the projection along the x-axis

$A_x = A\cos\theta \qquad \vec{\mathbf{A}}_x = A\cos\theta\,\vec{\mathbf{x}}$

- The y-component of a vector is the projection along the y-axis

$A_y = A\sin\theta \qquad \vec{\mathbf{A}}_y = A\sin\theta\,\vec{\mathbf{y}}$

Then, $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$ These equations are valid ***only if θ is measured with respect to the x-axis***

Graphically Adding Vectors

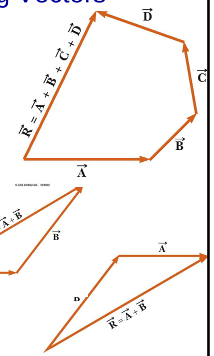
- When you add vectors, just put the tail of one on the head on the next…
- The resultant $\vec{\mathbf{R}}$ is drawn from the origin of the first vector to the end of the last vector
- Vectors obey the

commutative law of addition

The order in which the vectors are added doesn't affect the result

$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$

Note that vectors are unchanged by being moved as long as their direction or magnitude is not changed

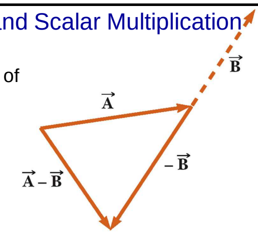


Vector Subtraction and Scalar Multiplication

- Special case of vector addition--add the negative of the subtracted vector

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$

- Continue with standard vector addition procedure



- The result of the multiplication or division of a vector by a scalar is a vector-the magnitude of the vector is multiplied or divided by the scalar

•If the scalar is positive, the direction of the result is the same as of the original vector

•If the scalar is negative, the direction of the result is opposite that of the original vector

Adding Vectors Algebraically

- Choose a coordinate system and sketch the vectors
- Find the x- and y-components of all the vectors

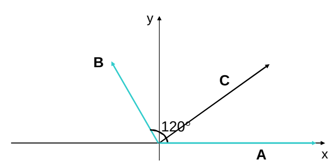
Add all the x-components $R_x = \sum A_x$

Add all the y-components $R_y = \sum A_y$

Use Pythagorean theorem find
The magnitude of the resultant: $R = \sqrt{R_x^2 + R_y^2}$

Use inverse tangent function
To find the direction of R: $\theta = \tan^{-1}\dfrac{R_y}{R_x}$

Example: Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A**+**B** (=**C**).



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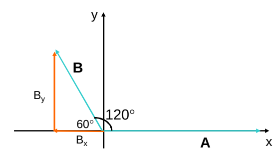
Example continued:



$B_x = B\cos120° = (3.00\,\text{m})\cos120° = -1.50\,\text{m}$

$B_y = B\sin120° = (3.00\,\text{m})\sin120° = 2.60\,\text{m}$

and A_x = 5.00 m and A_y = 0.00 m

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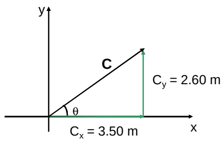
Example continued:

The components of **C**: $C_x = A_x + B_x = 5.00\ \text{m} + (-1.50\ \text{m}) = 3.50\ \text{m}$

$C_y = A_y + B_y = 0.00\ \text{m} + 2.60\ \text{m} = 2.60\ \text{m}$



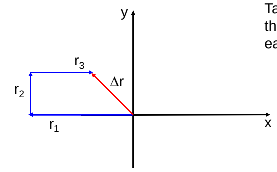
The length of C is:

$$C = |\mathbf{C}| = \sqrt{C_x^{\ 2} + C_y^{\ 2}} = \sqrt{(3.50\ \text{m})^2 + (2.60\ \text{m})^2} = 4.36\ \text{m}$$

The direction of C is: $\tan\theta = \frac{C_y}{C_x} = \frac{2.60\ \text{m}}{3.50\ \text{m}} = 0.7429$

$\theta = \tan^{-1}(0.7429) = 36.6°$ From the +x-axis

Example: Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.

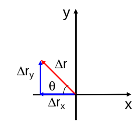


Take north to be in the +y direction and east to be along +x.

Example continued:

The displacement is $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. The initial position is the origin; what is $\mathbf{r}_f$?

The final position will be $\mathbf{r}_f = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$. The components are $r_{fx} = -r_1 + r_3 = -0.2$ miles and $r_{fy} = +r_2 = +0.2$ miles.



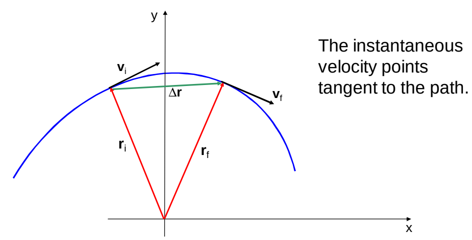
Using the figure, the magnitude and direction of the displacement are

$$|\Delta\mathbf{r}| = \sqrt{\Delta r_x^2 + \Delta r_y^2} = 0.283\ \text{miles}$$

$$\tan\theta = \frac{|\Delta r_y|}{|\Delta r_x|} = 1 \text{ and } \theta = 45° \quad \text{N of W.}$$

Motion, Velocity, Acceleration

A particle moves along the blue path as shown. At time t_1 its position is $\mathbf{r}_i$ and at time t_2 its position is $\mathbf{r}_f$.



The instantaneous velocity points tangent to the path.

$\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t}$ Points in the direction of Δ**r**

$$\text{Average velocity} = \mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t} \quad \left(\text{The x-component would be: } v_{av,x} = \frac{\Delta x}{\Delta t}\right)$$

$$\text{Instantaneous velocity} = \mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t}$$

This is represented by the slope of a line tangent to the curve on the graph of an object's position versus time.

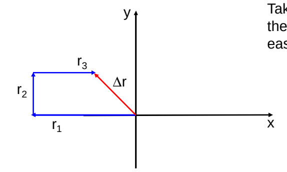
Example: Consider Margaret's walk to the store in the example on previous slides. If the first leg of her walk takes 10 minutes, the second takes 8 minutes, and the third 7 minutes, compute her average velocity and average speed during each leg and for the overall trip.

Use the definitions:

$$\text{Average velocity} = \mathbf{v}_{av} = \frac{\Delta \mathbf{r}}{\Delta t}$$

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of trip}}$$

Example: Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.

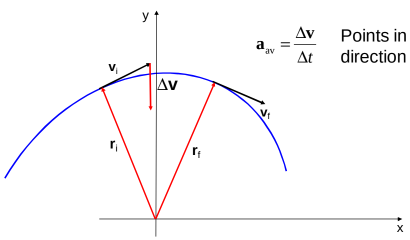


Take north to be in the +y direction and east to be along +x.

Example continued:

Leg	Δt (hours)	$\mathbf{v}_{av}$ (miles/hour)	Average speed (miles/hour)
1	0.167	3.00 (west)	3.00
2	0.133	1.50 (north)	1.50
3	0.117	2.56 (east)	2.56
Total trip	0.417	0.679 (45° N of W)	2.40

A particle moves along the blue path as shown. At time t_1 its position is $\mathbf{r}_0$ and at time t_2 its position is $\mathbf{r}_f$.



$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Points in the direction of Δ**v.**

The instantaneous acceleration can point in any direction.

A nonzero acceleration changes an object's state of motion.

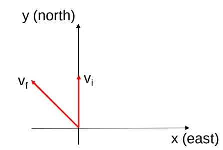
$$\text{Average acceleration} = \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

$$\text{Instantaneous acceleration} = \mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

These have interpretations similar to $\mathbf{v}_{av}$ and **v**.

Example (text problem 3.42): At the beginning of a 3 hour plane trip you are traveling due north at 192 km/hour. At the end, you are traveling 240 km/hour at 45° west of north.

(a) Draw the initial and final velocity vectors.



Example continued:

(b) Find $\Delta\mathbf{v}$.

The components are

$$\Delta v_x = v_{fx} - v_{ix} = -v_f \sin 45° - 0 = -170 \text{ km/hr}$$
$$\Delta v_y = v_{fy} - v_{iy} = +v_f \cos 45° - v_i = -22.3 \text{ km/hr}$$

The magnitude and direction are:

$$|\Delta\mathbf{v}| = \sqrt{\Delta v_x^{\ 2} + \Delta v_y^{\ 2}} = 171 \text{ km/hr}$$

$$\tan\varphi = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.1312 \Rightarrow \phi = \tan^{-1}(0.1312) = 7.5°$$ South of west

Example continued:

(c) What is $\mathbf{a}_{av}$ during the trip?

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

$$a_{x,av} = \frac{\Delta v_x}{\Delta t} = \frac{-170 \text{ km/hr}}{3 \text{ hr}} = -56.7 \text{ km/hr}^2$$
$$a_{y,av} = \frac{\Delta v_y}{\Delta t} = \frac{-22.3 \text{ km/hr}}{3 \text{ hr}} = -7.43 \text{ km/hr}^2$$

The magnitude and direction are:

$$|\mathbf{a}_{av}| = \sqrt{a_{x,av}^2 + a_{y,av}^2} = 57.2 \text{ km/hr}^2$$

$$\tan\phi = \frac{|a_{y,av}|}{|a_{x,av}|} = 0.1310 \Rightarrow \phi = \tan^{-1}(0.1310) = 7.5°$$ South of west