2-d Motion: Constant Acceleration

- Kinematic Equations of Motion (Vector Form)

Acceleration Vector (constant)
$$\vec{a} = a_x\hat{x} + a_y\hat{y}$$
Velocity Vector (function of t)
$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$
Position Vector (function of t)
$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Warning! These equations are only valid if the acceleration is constant.

The velocity vector and position vector are a function of the time t.

Velocity Vector at time t = 0.
$$\vec{v}_0 = v_{x0}\hat{x} + v_{y0}\hat{y}$$
Position Vector at time t = 0.
$$\vec{r}_0 = x_0\hat{x} + y_0\hat{y}$$

The components of the acceleration vector, a_x and a_y, are constants.
The components of the velocity vector at t = 0, v_{x0} and v_{y0}, are constants.
The components of the position vector at t = 0, x_0 and y_0, are constants.

2-d Motion: Constant Acceleration

- Kinematic Equations of Motion (Component Form)

$a_x =$ constant
$$v_x(t) = v_{x0} + a_x t$$
$$x(t) = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2$$

$a_y =$ constant
$$v_y(t) = v_{y0} + a_y t$$
$$y(t) = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$$

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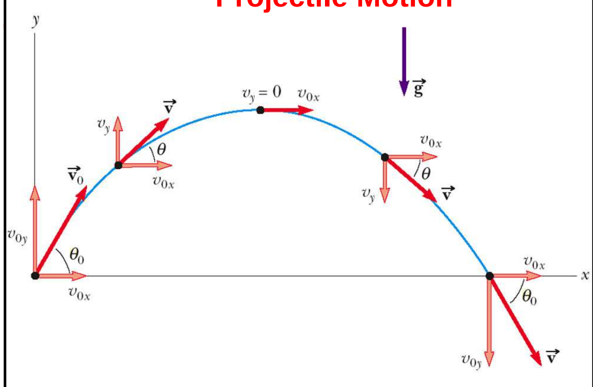
The components of the acceleration vector, a_x and a_y, are constants.
The components of the velocity vector at t = 0, v_{x0} and v_{y0}, are constants.
The components of the position vector at t = 0, x_0 and y_0, are constants.

- Ancillary Equations

Valid at any time t
$$v_x^2(t) - v_{xo}^2 = 2a_x(x(t) - x_0)$$
$$v_y^2(t) - v_{yo}^2 = 2a_y(y(t) - y_0)$$

Projectile Motion



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Rules of Projectile Motion

The x- and y-directions of motion completely independent
The x-direction is uniform motion: $a_x = 0$
The y-direction is free fall: $a_y = -g$
The initial velocity V_0 can be broken down into its x- and y-components
$$v_{Ox} = v_O\cos\theta_O \qquad v_{Oy} = v_O\sin\theta_O$$

- x-direction--$a_x = 0$
$$v_{xo} = v_o\cos\theta_o = v_x = \text{constant}$$
$x = v_{xo}t$ This is the only operative equation in the x-direction since there is uniform velocity in that direction

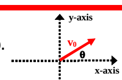
- y-direction--free fall: $a = -g$
  - –take the positive direction as upward
  - –uniformly accelerated motion, so the motion equations all hold

- Velocity at any time $v = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}\frac{v_y}{v_x}$

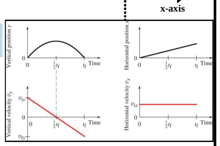
Example: Projectile Motion

- Near the Surface of the Earth (h = 0)

In this case, $a_x= 0$ and $a_y= -g$, $v_{x0} = v_0\cos\theta$, $v_{y0} = v_0\sin\theta$, $x_0 = 0$, $y_0 = 0$.



$$v_x(t) = v_0\cos\theta \qquad v_y(t) = v_0\sin\theta - gt$$
$$x(t) = (v_0\cos\theta)t \qquad y(t) = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

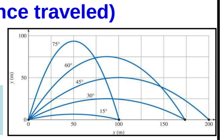


- Maximum Height H

The time, t_{max}, that the projective reaches its maximum height occurs when $v_y(t_{max}) = 0$. Hence,
$$t_{max} = \frac{v_0\sin\theta}{g} \qquad H = y(t_{max}) = \frac{(v_0\sin\theta)^2}{2g}$$

- Range R (maximum horizontal distance traveled)

The time, t_f, that it takes the projective reach the ground occurs when $y(t_f) = 0$. Hence,
$$0 = y(t_f) = (v_0\sin\theta)t_f - \tfrac{1}{2}gt_f^2 \qquad t_f = \frac{2v_0\sin\theta}{g}$$
$$R = x(t_f) = (v_0\cos\theta)t_f = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin 2\theta}{g}$$



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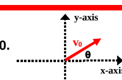

$$v_x(t) = v_0\cos\theta \qquad v_y(t) = v_0\sin\theta - gt$$
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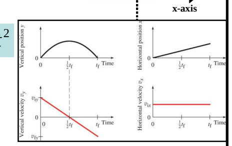

- Maximum Height H

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- Range R (maximum horizontal distance traveled)

For a fixed v_0 the largest R occurs when $\theta = 45°$!

The time, t_f, that it takes the projective reach the ground occurs when $y(t_f) = 0$. Hence,
$$0 = y(t_f) = (v_0\sin\theta)t_f - \tfrac{1}{2}gt_f^2 \qquad t_f = \frac{2v_0\sin\theta}{g}$$
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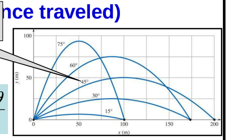


Problem-Solving Strategy

Select a coordinate system and sketch the path of the projectile--Include initial and final positions, velocities, and accelerations

Resolve the initial velocity into x- and y-components

Treat the horizontal and vertical motions independently

- **Horizontal motion:** Use techniques for problems with constant velocity
- **Vertical motion:** Use techniques for problems with constant acceleration

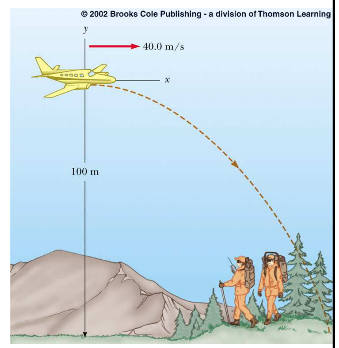
Some Special Cases of Projectile Motion

Object may be fired horizontally

The initial velocity is all in the x-direction

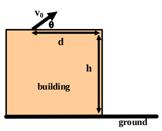
- $v_o = v_x$ and $v_y = 0$

All the general rules of projectile motion apply



Exam 1 Fall 2012: Problem 11

- Near the surface of the Earth a projectile is fired from the top of a building at a height h above the ground at an angle θ relative to the horizontal and at a distance d from the edge of the building as shown in the figure. If θ = 20° and d = 20 m, what is the minimum initial speed, v_0, of the projectile such that it will make it off the building and reach the ground? Ignoring air resistance.



$$d = R = \frac{v_0^2 \sin 2\theta}{g}$$

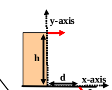
Answer: 17.5 m/s
% Right: 35%

$$v_0 = \sqrt{\frac{dg}{\sin 2\theta}} = \sqrt{\frac{(20m)(9.8m/s^2)}{\sin(2\times 20^\circ)}} \approx 17.46 m/s \approx 17.5 m/s$$

Exam 1 Spring 2012: Problem 12

- A beanbag is thrown horizontally from a dorm room window a height h above the ground. It hits the ground a horizontal distance d = h/2 from the dorm directly below the window from which it was thrown. Ignoring air resistance, find the direction of the beanbag's velocity just before impact.

Answer: 76.0° below the horizontal
% Right: 22%



$$v_x(t) = v_0 \qquad v_y(t) = -gt$$

$$x(t) = v_0 t \qquad y(t) = h - \tfrac{1}{2} g t^2$$

Let t_h be the time the beanbag hits the ground.

$$y(t_h) = 0 = h - \tfrac{1}{2} g t_h^2 \qquad t_h^2 = \frac{2h}{g}$$

$$x(t_h) = d = v_0 t_h \qquad v_0 = \frac{d}{t_h}$$

$$\tan\theta = \frac{|v_y(t_h)|}{|v_x(t_h)|} = \frac{g t_h}{v_0} = \frac{g t_h^2}{d} = \frac{2h}{d} = 4$$

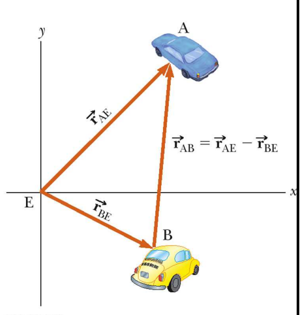
$$\theta \approx 76^\circ$$

Relative Position Vector

Position of car A relative to car B is given by the vector subtraction equation

- $\vec{r}_{AE}$ is the position of car A as measured by E
- $\vec{r}_{BE}$ is the position of car B as measured by E
- $\vec{r}_{AB}$ is the position of car A as measured by car B

$$\vec{r}_{AB} = \vec{r}_{AE} - \vec{r}_{EB}$$



Relative Velocity Notation

The pattern of subscripts can be useful in solving relative velocity problems

Assume the following notation:

- E is an observer, stationary with respect to the earth
- A and B are two moving cars

The rate of change of the displacements gives the relationship for the velocities

$$\vec{v}_{AB} = \vec{v}_{AE} - \vec{v}_{EB}$$