Chapter 6: Conservation of Energy

Work

Kinetic Energy

Work done by gravitational force

GPE

Energy and Work: loose definitions

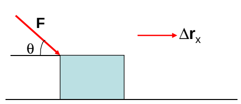
- **Very common terms, beware: everyday life meanings are not identical what these terms mean in physics**
- **Loose definitions in physics:**
  - A moving body is said to have **kinetic energy;** the faster the object moves, the more energy it has.
  - Something capable of increasing kinetic energy of an object is said to have **potential energy** stored.
  - In order to increase object's kinetic energy, one needs to apply a force and do some **work**.

Work

Force · Time = ?
Force · Distance = ?

Both are actually very much relevant:

- F·t characterizes the change in momentum (to be discussed later)
- F·Δr characterizes the change in energy, i.e.

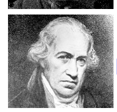


$$W = F\Delta r\cos\theta$$

For a constant force $\vec{F}$ acting over a displacement $\vec{\Delta r}$

Units

Work = Energy: Joule (J) $1\ \text{J} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$



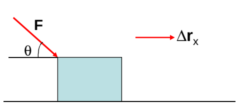
Power: Watt (W) $1\ \text{W} = \frac{\text{J}}{\text{s}} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$

1 horsepower = 746 Watt

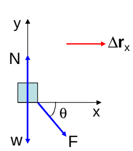
Energy = Power x time: 1 kWatt-hour = 3.6 MJ

Warning! Do not confuse Work, Watt, Weight

It is only the force in the direction of the displacement that does work.



An FBD for the box at left:



The work done by the force **F** is:

$$W_F = F_x\Delta r_x = (F\cos\theta)\Delta x$$

Example continued:

The work done by the normal force **N** is: $W_N = 0$

The normal force is perpendicular to the displacement.

The work done by gravity (**w**) is: $W_g = 0$

The force of gravity is perpendicular to the displacement.

The net work done on the box is:

$$W_{\text{net}} = W_F + W_N + W_g$$
$$= (F\cos\theta)\Delta x + 0 + 0$$
$$= (F\cos\theta)\Delta x$$

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In general, the work done by a force **F** is defined as

$$W = F\Delta r\cos\theta$$

where F is the magnitude of the force, Δr is the magnitude of the object's displacement, and θ is the angle between **F** and Δ**r** (drawn tail-to-tail).

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Why do we tell you to memorize this?

a) **Lot of mechanical problems can't be solved exactly but we still want to understand part of the problem.**
b) **Because these concepts help understand complex mechanical systems. You can understand why by studying the *history* a little**

First, some questions about billiards

http://www.youtube.com/watch?v=p_aOWYYnIHo
http://comp.uark.edu/~jgeabana/mol_dyn/KinThI.html

What did you see?

There are many physical systems with many particles where velocities change a lot. Gottfried Wilhelm Leibnitz (Newton's competitor) ~ 1680:

Sometimes even when velocities change, *something* stays the same:

Total *kinetic energy of all the particles didn't change in gas simulation:*

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2$$

"Conservation laws"

Statement about a physical quantity which doesn't change with time.

Energy, momentum…

Kinetic Energy

$K = \frac{1}{2}mv^2$ is an object's translational kinetic energy.

This is the energy an object has because of its state of motion.

It can be shown that, in general $W_{\text{net}} = \Delta K.$

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Example

The extinction of the dinosaurs and the majority of species on Earth in the Cretaceous Period (65 Myr ago) is thought to have been caused by an asteroid striking the Earth near the Yucatan Peninsula. The resulting ejecta caused widespread global climate change.

If the mass of the asteroid was 10^{16} kg (diameter in the range of 4-9 miles) and had a speed of 30.0 km/sec, what was the asteroid's kinetic energy?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(10^{16}\text{ kg}\right)\left(30\times10^3\text{ m/s}\right)^2$$
$$= 4.5\times10^{24}\text{ J}$$

This is equivalent to ~10^9 Megatons of TNT.

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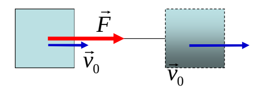
Accelerating free object (*const. a*!)

$$W = Fd\cos\theta = Fd = ma\cdot d$$



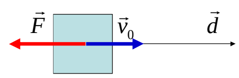
$$d = v_0 t + \frac{1}{2}at^2$$
$$v = v_0 + at$$

Kinetic energy: $K = \dfrac{mv^2}{2}$

$$W = ma\cdot d = ma\left(v_0 t + \frac{at^2}{2}\right) = ma\left(v_0\frac{v-v_0}{a} + \frac{a}{2}\left(\frac{v-v_0}{a}\right)^2\right) = \ldots = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$
$$= K - K_0$$

Decelerating free object

$$W = Fd\cos\theta = -Fd = -ma\cdot d$$



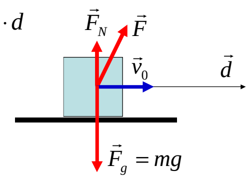
$$d = v_0 t - \frac{1}{2}at^2$$
$$v = v_0 - at$$

$$W = -ma\cdot d = -ma\left(v_0 t - \frac{at^2}{2}\right) = \ldots = \frac{mv^2}{2} - \frac{mv_0^2}{2} = K - K_0$$

Work can be negative,
if you act with a force against displacement

Sliding an object along a table

$$W = F_{net} d\cos\theta = F_{net,x} d = ma_x\cdot d$$



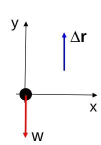
$$d = v_{x0}t - \frac{a_x t^2}{2}$$
$$v_x = v_{x0} - a_x t$$

$$W = \ldots = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

Work can be zero, if the force is normal to displacement

Example: A ball is tossed straight up. What is the work done by the force of gravity on the ball as it rises?

FBD for rising ball:



$$W_g = w\Delta y\cos 180°$$
$$= -mg\Delta y$$

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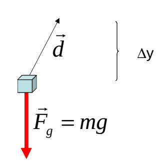
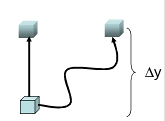
Work done *by* gravitational force

$$W_g = F_g d\cos\theta = -mg\Delta y$$



Work done by gravity
- **depends on the change in height only**
- **independent of the path!**



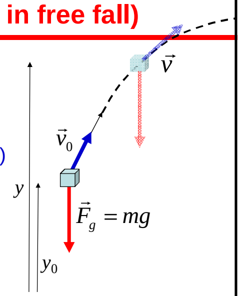
Work done by gravitational force alone (e.g. projectile in free fall)

$$W_g = -mg(y - y_0)$$

$$W_g = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$ (work-KE "theorem")

$$-mg(y - y_0) = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

$$\frac{mv^2}{2} + mgy = \frac{mv_0^2}{2} + mgy_0 = const$$



Sneak preview:
total energy = "potential energy" + KE is conserved

Gravitational Potential Energy Part 1

Objects have potential energy because of their location (or configuration).

There are potential energies associated with different (but not all!) forces. Such a force is called a **conservative force**.

In general $W_{cons} = -\Delta U$

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The change in gravitational potential energy (only near the surface of the Earth) is

$$W_g = -mg\Delta y \quad \Rightarrow \quad \Delta U_g = mg\Delta y$$

where Δy is the change in the object's vertical position with respect to some reference point that you are free to choose.

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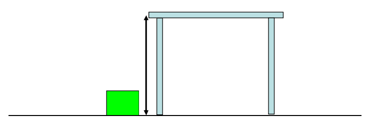
Example: What is the change in gravitational potential energy of the box if it is placed on the table? The table is 1.0 m tall and the mass of the box is 1.0 kg.



First: Choose the reference level at the floor. U = 0 here.

$$\Delta U_g = mg\Delta y = mg(y_f - y_i)$$
$$= (1.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m} - 0\text{ m}) = +9.8\text{ J}$$

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Example continued:

Now take the reference level (U = 0) to be on top of the table so that $y_i = -1.0$ m and $y_f = 0.0$ m.

$$\Delta U_g = mg\Delta y = mg(y_f - y_i)$$
$$= (1\text{ kg})(9.8\text{ m/s}^2)(0.0\text{m} - (-1.0\text{ m})) = +9.8\text{ J}$$

The results do not depend on the location of U = 0.

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Energy Conservation laws

Mechanical energy is $E = K + U$

Whenever nonconservative forces do no work, the mechanical energy of a system is conserved. That is $E_i = E_f$ or $\Delta K = -\Delta U$.

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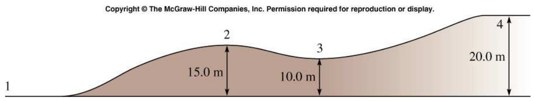
Example (text problem 6.28): A cart starts from position 4 with v = 15.0 m/s to the left. Find the speed of the cart at positions 1, 2, and 3. Ignore friction.



$$E_4 = E_3$$
$$U_4 + K_4 = U_3 + K_3$$
$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_3 + \frac{1}{2}mv_3^2$$
$$v_3 = \sqrt{v_4^2 + 2g(y_4 - y_3)} = 20.5\ \text{m/s}$$

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Example continued:

$$E_4 = E_2$$
$$U_4 + K_4 = U_2 + K_2$$
$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_2 + \frac{1}{2}mv_2^2$$
$$v_2 = \sqrt{v_4^2 + 2g(y_4 - y_2)} = 18.0\ \text{m/s}$$

Or use $E_3 = E_2$

$$E_4 = E_1$$
$$U_4 + K_4 = U_1 + K_1$$
$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_1 + \frac{1}{2}mv_1^2$$
$$v_1 = \sqrt{v_4^2 + 2g(y_4 - y_1)} = 24.8\ \text{m/s}$$

Or use $E_3 = E_1$

$E_2 = E_1$

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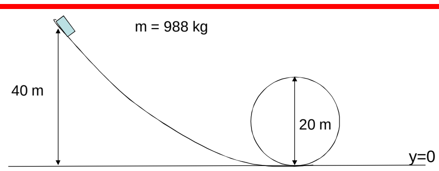
Example (text problem 6.84): A roller coaster car is about to roll down a track. Ignore friction and air resistance.



(a) At what speed does the car reach the top of the loop?

$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_f^2$$
$$v_f = \sqrt{2g(y_i - y_f)} = 19.8\ \text{m/s}$$

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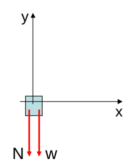
Example continued:

(b) What is the force exerted on the car by the track at the top of the loop?

FBD for the car:



Apply Newton's Second Law:

$$\sum F_y = -N - w = -ma_r = -m\frac{v^2}{r}$$
$$N + w = m\frac{v^2}{r}$$
$$N = m\frac{v^2}{r} - mg = 2.9\times 10^4\ \text{N}$$

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Example continued:

(c) From what minimum height above the bottom of the track can the car be released so that it does not lose contact with the track at the top of the loop?

Using conservation of mechanical energy:

$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_{\min}^2$$

Solve for the starting height $$y_i = y_f + \frac{v_{\min}^2}{2g}$$

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Example continued:

What is $v_{\min}$? $v = v_{\min}$ when N = 0. This means that

$$N + w = m\frac{v^2}{r}$$
$$w = mg = m\frac{v_{\min}^2}{r}$$
$$v_{\min} = \sqrt{gr}$$

The initial height must be

$$y_i = y_f + \frac{v_{\min}^2}{2g} = 2r + \frac{gr}{2g} = 2.5r = 25.0\ \text{m}$$

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