Last time

The total work W done on an object is equal to its change in kinetic energy

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2 = K - K_0 = \Delta K$$

For a constant force being applied to an object,

$$W = F\Delta r \cos\theta$$

Recap: Mechanical Energy E

The mechanical energy of a body is the sum of the kinetic energy and the potential energy of the body.

E=K+U

- Characterize the instant state of the body
- U is the sum of all potential energy associated with conservative forces
- The change of the mechanical energy is:

$$\Delta E = \Delta K + \Delta U$$

RECAP

Conservative Force ⇒ Potential energy U

Potential Energy ⇒ Conservative Force

Mechanical Energy E=K+U

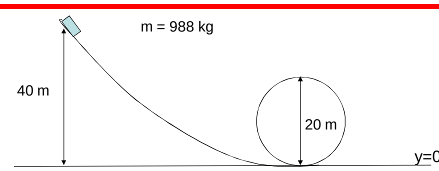
Conservation of E of a body if only conservative forces do work.

$$\Delta E = 0$$

$$E_2 = E_1$$

$$K_2 + U_2 = K_1 + U_1$$

Example (text problem 6.84): A roller coaster car is about to roll down a track. Ignore friction and air resistance.



(a) At what speed does the car reach the top of the loop?

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2g(y_i - y_f)} = 19.8 \text{ m/s}$$

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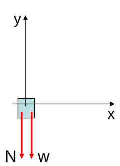
Example continued:

(b) What is the force exerted on the car by the track at the top of the loop?

FBD for the car:



Apply Newton's Second Law:

$$\sum F_y = -N - w = -ma_r = -m\frac{v^2}{r}$$

$$N + w = m\frac{v^2}{r}$$

$$N = m\frac{v^2}{r} - mg = 2.9\times10^4 \text{ N}$$

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Example continued:

(c) From what minimum height above the bottom of the track can the car be released so that it does not lose contact with the track at the top of the loop?

Using conservation of mechanical energy:

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_{\min}^2$$

Solve for the starting height $y_i = y_f + \frac{v_{\min}^2}{2g}$

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Example continued:

What is v_{min}? $v = v_{min}$ when N = 0. This means that

$$N + w = m\frac{v^2}{r}$$

$$w = mg = m\frac{v_{\min}^2}{r}$$

$$v_{\min} = \sqrt{gr}$$

The initial height must be

$$y_i = y_f + \frac{v_{\min}^2}{2g} = 2r + \frac{gr}{2g} = 2.5r = 25.0\text{ m}$$

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Work by Nonconservative force

What do you do when there are nonconservative forces? For example, if friction is present

$$\Delta E = E_f - E_i = W_{\text{fric}}$$

The work done by friction.

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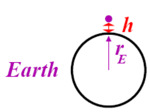
Why can we take g=const. at earth's surface?

Newton says force on mass m a distance r from center of earth is $F = \left(\frac{GmM_E}{r^2}\right)$

But earlier we said $F=mg$

So it must be that $g = \frac{GM_E}{r^2} \neq const.$

A: We assumed $h \ll R_E$



$r = R_E + h$

$$g = \frac{GM_E}{(R_E + h)^2} \approx \frac{GM_E}{R_E^{\ 2}}$$

Gravitational Potential Energy Part 2

Newton says force on mass m a distance r from center of earth is $F = \left(\frac{GmM_E}{r^2}\right)$

When $h \ll R_E$ $F=mg$

The general expression for gravitational potential energy is:

$$U(r) = -\frac{GM_1M_2}{r}$$

$$\text{where } U(r=\infty) = 0$$

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Examples

Example: What is the gravitational potential energy of a body of mass m on the surface of the Earth?

$$U(r = R_e) = -\frac{GM_1M_2}{r} = -\frac{GM_e m}{R_e}$$

Example: Drop a stone from height of h=3 R_e . What is the speed when it hits the Earth?

$$v = \sqrt{\frac{3GM_e}{2R_e}}$$

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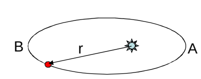
Example

A planet of mass m has an elliptical orbit around the Sun. The elliptical nature of the orbit means that the distance between the planet and Sun varies as the planet follows its orbital path. How does the speed of a planet vary as it orbits the Sun once. Take the planet to move counterclockwise from its initial location.

The mechanical energy of the planet-sun system is:



$$E = \frac{1}{2}mv^2 - \frac{GmM_{\text{sun}}}{r} = \text{constant}$$

At the planet moves r decreases and the planet moves faster.

As the planet moves past point A r begins to increase and the planet moves slower.

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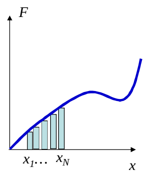
Work by variable force

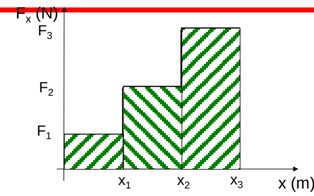
In each interval $\Delta x = x_{i+1} - x_i$, F is roughly constant. So *approximate*

$W = F\Delta x_1 + F\Delta x_2 + \ldots F\Delta x_N$



$$W = \text{area under the curve}$$

Example: What is the work done by the variable force shown below?



The work done by F_1 is $W_1 = F_1(x_1 - 0)$

The work done by F_2 is $W_2 = F_2(x_2 - x_1)$

The work done by F_3 is $W_3 = F_3(x_3 - x_2)$

The net work is then $W_1+W_2+W_3$.

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Spring force

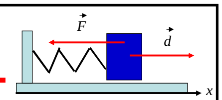
Robert Hooke 1600s

$$F = -kx$$

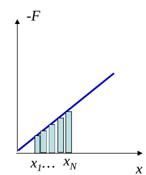
- force direction is opposite to direction of *displacement from the equilibrium point*
- force is linearly proportional to *displacement*
- k – spring constant:
  - large k: hard spring
  - small k: weak spring

As a theorist, Robert Hooke (1635-1703) is mostly remembered for arguing with Isaac Newton over the nature of light and gravity, a long-running debate that is said to have left both men forever bitter toward each other.

Work done *by* spring



Spring Force: $F = -kx$



Area under the curve, minus sign!!

$$W = -\left(\frac{kx_f^2}{2} - \frac{kx_i^2}{2}\right)$$

Q: what work do *you* do?

Elastic potential energy

The work done in stretching/compressing a spring transfers energy to the spring.

$$U_s = \frac{1}{2}kx^2$$

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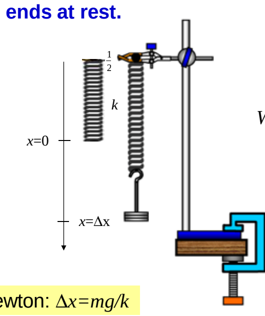
Vertical spring analysis

Lower mass such that it starts and ends at rest.



Newton: $\Delta x = mg/k$

Work done by gravity is

$$W_g = mg\Delta x = m^2g^2/k$$

Work done by spring is

$$W_{spr} = -\frac{1}{2}k\Delta x^2 = -m^2g^2/(2k)$$

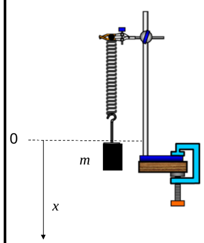
W-KE theorem says

$$W_{tot} = W_g + W_{spr} + W_{hand} = \Delta K = 0$$

so the work done by your hand is

$$W_{hand} = -m^2g^2/(2k)$$

Example 2



A mass m=2kg is attached to a hanging spring with force constant k=10N/m and then released. The spring tension is zero at time of release. a) In resulting oscillations, what is max extension L of spring? b) what is the speed of m when it is at ½ the max extension?

Example - spring and gravity

A block of mass m is dropped from height h onto a spring. The spring is compressed by a length d before the body stops. Find the spring constant k.

For the block-Earth-spring system:

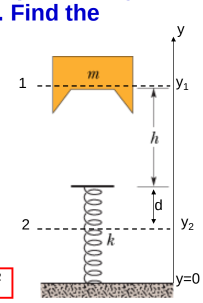
$$E_{mec1} = K_1 + U_{g1} + U_{s1} = 0 + mgy_1 + 0$$

$$E_{mec2} = K_2 + U_{g2} + U_{s2} = 0 + mgy_2 + \frac{1}{2}kd^2$$

$$E_{mec1} = E_{mec2}$$

$$mgy_1 = mgy_2 + \frac{1}{2}kd^2$$

$$k = 2mg(y_1 - y_2)/d^2 \qquad \boxed{k = 2mg(h+d)/d^2}$$



Power

Power is the rate of energy transfer.

Average Power $$P_{av} = \frac{\Delta E}{\Delta t}$$

Instantaneous Power $$P = \frac{W}{\Delta t} = \frac{F\Delta r\cos\theta}{\Delta t} = Fv\cos\theta$$

The unit of power is the watt. 1 watt = 1 J/s = 1 W.

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Example

Example (text problem 6.75): A race car with a mass of 500.0 kg completes a quarter-mile (402 m) race in a time of 4.2 s starting from rest. The car's final speed is 125 m/s. What is the engine's average power output? Neglect friction and air resistance.

$$P_{av} = \frac{\Delta E}{\Delta t} = \frac{\Delta U + \Delta K}{\Delta t}$$

$$= \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}mv_f^2}{\Delta t} = 9.3\times10^5 \text{ watts}$$

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