Chapter 8: Torque and Angular Momentum

rigid body, stationary axis of rotation

rotational variables (and their similarity to translations):

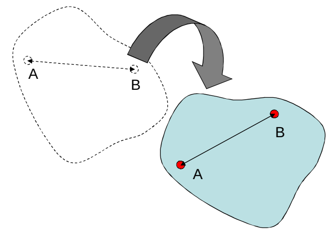
- angle: θ
- angular velocity: $\omega = \Delta\theta/\Delta t$
- angular acceleration: $\alpha = \Delta\omega/\Delta t$

point at distance r from axis: relationship between v and ω

rotations and kinetic energy

rotational inertia

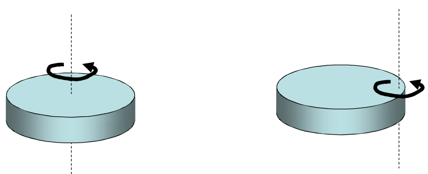
Definition: *rigid body*



Distance between any two points does not change

Fixed axis of rotation

For now, we consider only the case when the axis of rotation, wherever it is, is stationary



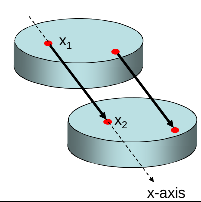
Translation and Rotation

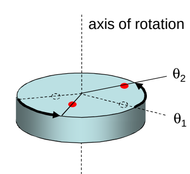
Translation:

all points move and have exactly the same displacements

Rotation:

all points rotate around some axis by exactly the same angle





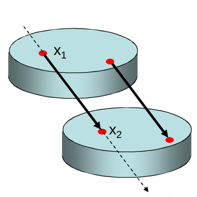
Translation and Rotation

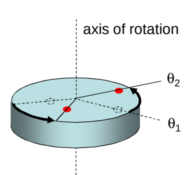
Translation

coordinate: x (meters)

Rotation

angle: θ (radians)

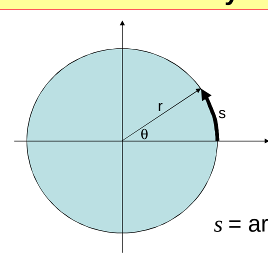




Radians

DO NOT USE DEGREES
(many formulas will work only in radians)

180º = π rad



$$\theta = \frac{s}{r}$$

s = arc length

Kinematics

Translation

position: x

velocity: $v = \Delta x/\Delta t$

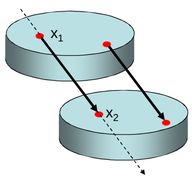
acceleration: $a = \Delta v/\Delta t$

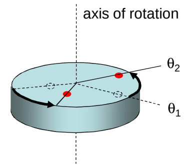
Rotation (Ch. 5)

angle: θ

angular velocity: $\omega = \Delta\theta/\Delta t$

angular acc.: $\alpha = \Delta\omega/\Delta t$





More equations of motion

Translation

$$a = const$$
$$v = v_i + at$$
$$x = x_i + v_i t + \frac{1}{2}at^2$$

Rotation

$$\alpha = const$$
$$\omega = \omega_i + \alpha t$$
$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

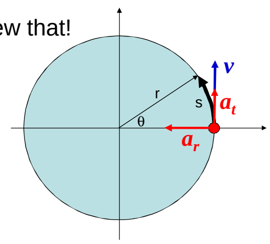
Acceleration and Angular Acceleration

$$a_r = \frac{v^2}{r} = r\omega^2$$

we already knew that!

$$a_t = r\alpha$$

and this !



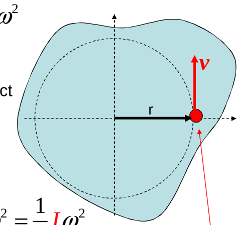
Kinetic Energy and Moment of Inertia

$$K_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}(m_i r_i^2)\omega^2$$

Rotational Inertia I

- characterizes the rotational inertia of the object
- depends on the axis

$$K = \sum_i \frac{1}{2}(m_i r_i^2)\omega^2 = \frac{1}{2}\underbrace{\left(\sum_i m_i r_i^2\right)}_{I}\omega^2 = \frac{1}{2}I\omega^2$$



piece i of rigid body, mass m_i

Kinetic Energy and Rotational Inertia

$$I = \sum m_i r_i^2$$

$$K = \frac{1}{2}I\omega^2$$

(pure rotation)

Compare to linear motion:

$$M = \sum_i m_i$$

$$K = \frac{1}{2}Mv^2$$

Example 1

In a flywheel in the form of a disc, the rotational KE is 2×10^8 J at peak speed of ω=460 rad/s.

1. How much KE does the wheel have at its minimum speed of 300 rad/s?
2. The flywheel has a mass of 20,000 kg. What is its radius?
3. Suppose the flywheel is slowed down from max to minimum speed in 0.2 s. How many kW of avg power would it produce during this interval?

Example 1

In a flywheel in the form of a disc, the rotational KE is 2x10⁸ J at peak speed of ω=460 rad/s.

1. How much KE does the wheel have at its minimum speed of 300 rad/s?

$$K_{\max} = \frac{1}{2} I \omega_{\max}{}^2$$

$$K_{\min} = \frac{1}{2} I \omega_{\min}{}^2$$

$$\frac{K_{\max}}{K_{\min}} = \frac{\omega_{\max}{}^2}{\omega_{\min}{}^2} \quad \Rightarrow \quad K_{\min} = K_{\max} \frac{\omega_{\min}{}^2}{\omega_{\max}{}^2} = (2\times10^8\,\text{J}) \frac{300^2}{460^2} = 8.5\times10^7\,\text{J}$$

Example 1

In a flywheel in the form of a disc, the rotational KE is 2x10⁸ J at peak speed of ω=460 rad/s.

2. The flywheel has a mass of 20,000 kg. What is its radius?

$$K_{\max} = \frac{1}{2} I \omega_{\max}{}^2 = \frac{1}{2}(\frac{1}{2} M R^2) \omega_{\max}{}^2$$

$$\Rightarrow \quad R = \frac{2}{\omega}\sqrt{\frac{K_{\max}}{M}} = \frac{2}{460\,\text{rad/s}}\sqrt{\frac{2\times10^8\,\text{J}}{2\times10^4\,\text{kg}}} = 0.435\text{m}$$

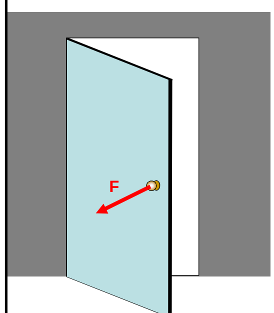
Example 1

In a flywheel in the form of a disc, the rotational KE is 2x10⁸ J at peak speed of ω=460 rad/s.

3. Suppose the flywheel is slowed down from max to minimum speed in 0.2 s. How many kW of avg. power would it produce during this interval?

$$P = \frac{\Delta E}{\Delta t} = \frac{2\times10^8\,\text{J-}8.5\times10^7\,\text{J}}{0.2\text{s}} = 5.75\times10^5\,\text{kW}$$

Torque (Latin for “twist”)



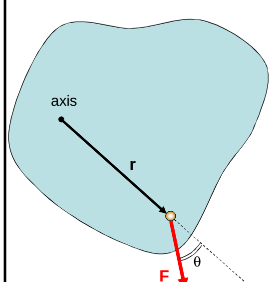
To open door, one needs to apply force

The ease of opening depends on

- distance between the force and the hinge
- the direction of the force

Torque (Latin for “twist”):

$$\tau = F r \sin\theta$$



To twist an object, one needs to apply force

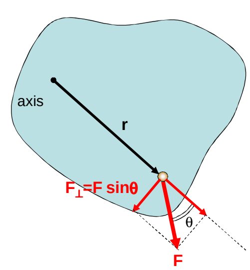
The ease of twisting will depend on

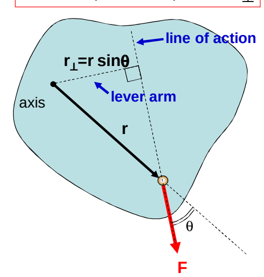
- force
- distance between the force and the axis
- direction of the force

Torque: two perspectives

$$\tau = F r \sin\theta$$

$$\tau = (F\sin\theta) r = F_{\perp} r \qquad \tau = F(r\sin\theta) = F r_{\perp}$$



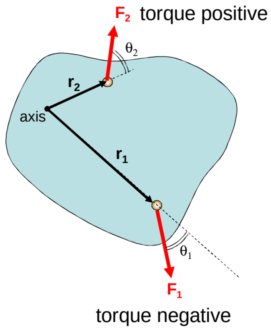


Torque:

Sign convention:
- CW (clockwise): negative
- CCW(counter-clockwise): positive

Multiple forces:
- $\tau_{net} = \tau_1 + \tau_2 + \ldots$

Units:
- N-m

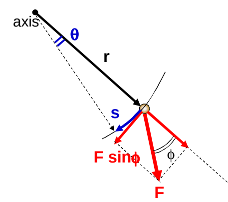


Torque and work

$$W = \tau\theta$$

Note analogy with $W=Fs$

Proof for simple case of one particle



$$W = (F\sin\phi)s$$

$$W = (F\sin\phi)r\frac{s}{r}$$

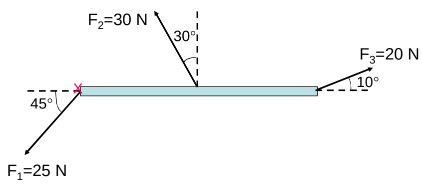
$$W = (F\sin\phi r)\frac{s}{r}$$

$$W = \tau\theta$$

$$P = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

Example

Example: Calculate the torque due to the three forces shown about the left end of the bar (the red X). The length of the bar is 4m and F_2 acts in the middle of the bar.



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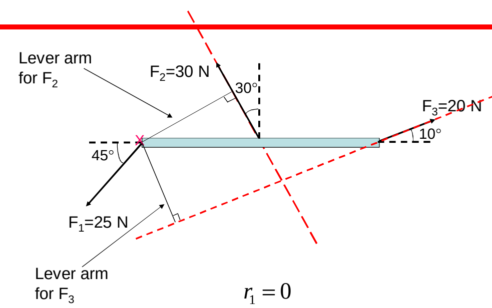
Example continued:



The lever arms are:

$$r_1 = 0$$
$$r_2 = (2\text{m})\sin 60° = 1.73\,\text{m}$$
$$r_3 = (4\text{m})\sin 10° = 0.695\,\text{m}$$

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Example continued:

The torques are:

$$\tau_1 = 0$$
$$\tau_2 = +(1.73\,\text{m})(30\,\text{N}) = +51.9\,\text{Nm}$$
$$\tau_3 = +(0.695\,\text{m})(20\,\text{N}) = +13.9\,\text{Nm}$$

The net torque is +65.8 Nm and is the sum of the above results.

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Mechanical Energy of Rigid Body

The mechanical energy of a rigid body with fixed axis:

$$E_{mec} = U + K_{rot}$$

Gravitational Potential Energy

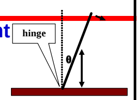
$$U = \sum m_i g y_i = g\sum m_i y_i = g y_{CM} M$$

Conservation of Energy

Work-Energy

Exam 2 Fall 2011: Problem 13

- **A thin stick with mass M, length L, and moment of inertia $ML^2/3$ is hinged at its lower end and allowed to fall freely as shown in the figure. If its length L = 2 m and it starts from rest at an angle θ = 20°, what is the speed (in m/s) of the free end of the stick when it hits the table?**



$$h = \frac{L}{2}\cos\theta$$

Answer: 7.43 m/s
% Right: 14%

$$E_i = Mgy_{CM} = Mg\frac{L}{2}\cos\theta \quad E_f = \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}I\frac{v_f^2}{L^2} \quad E_i = E_f$$

$$v_f = \sqrt{\frac{mgL^3\cos\theta}{I}} = \sqrt{\frac{mgL^3\cos\theta}{\frac{1}{3}mL^2}} = \sqrt{3gL\cos\theta}$$

$$= \sqrt{3(9.8m/s^2)(2m)\cos(20^\circ)} \approx 7.43m/s$$

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