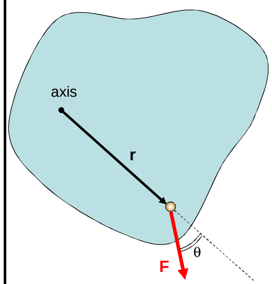
Torque (Latin for “twist”):

$$\tau = Fr\sin\theta$$



To twist an object, one needs to apply force

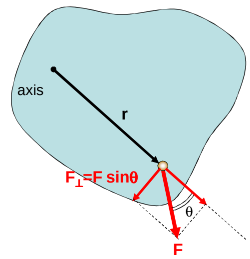
The ease of twisting will depend on

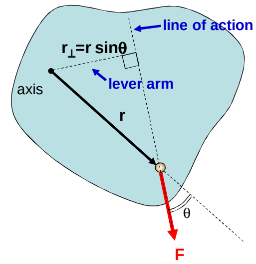
- force
- distance between the force and the axis
- direction of the force

Torque: two perspectives

$$\tau = Fr\sin\theta$$

$$\tau = (F\sin\theta)r = F_{\perp}r \qquad \tau = F(r\sin\theta) = Fr_{\perp}$$





Torque:

Sign convention:

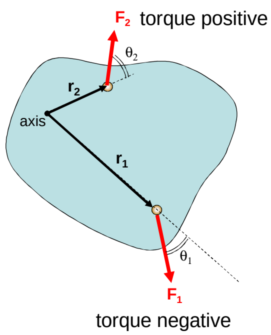
- CW (clockwise): negative
- CCW(counter-clockwise): positive

Multiple forces:

- $\tau_{net} = \tau_1 + \tau_2 + \ldots$

Units:

- N-m

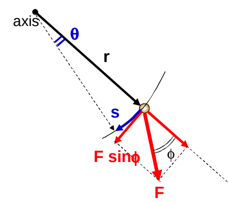


Torque and work

$$W = \tau\theta$$

Note analogy with *W=Fs*

Proof for simple case of one particle



$$W = (F\sin\phi)s$$

$$W = (F\sin\phi)r\frac{s}{r}$$

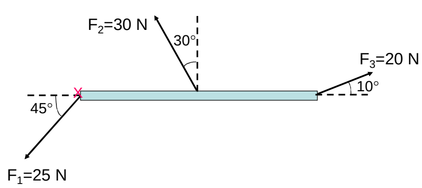
$$W = (F\sin\phi r)\frac{s}{r}$$

$$W = \tau\theta$$

$$P = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

Example

Example: Calculate the torque due to the three forces shown about the left end of the bar (the red X). The length of the bar is 4m and F_2 acts in the middle of the bar.



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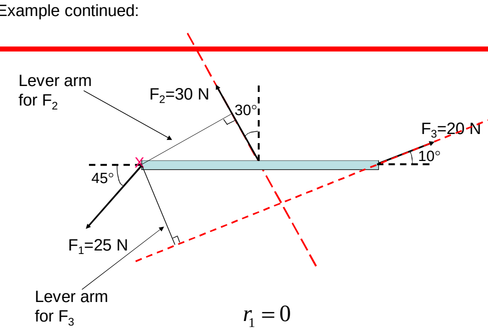
Example continued:



The lever arms are:

$$r_1 = 0$$

$$r_2 = (2\text{m})\sin 60° = 1.73\,\text{m}$$

$$r_3 = (4\text{m})\sin 10° = 0.695\,\text{m}$$

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Example continued:

The torques are:

$$\tau_1 = 0$$
$$\tau_2 = +(1.73\,\text{m})(30\,\text{N}) = +51.9\,\text{Nm}$$
$$\tau_3 = +(0.695\,\text{m})(20\,\text{N}) = +13.9\,\text{Nm}$$

The net torque is +65.8 Nm and is the sum of the above results.

Mechanical Energy of Rigid Body

The mechanical energy of a rigid body with fixed axis:

$$E_{mec} = U + K_{rot}$$

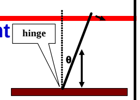
Gravitational Potential Energy

$$U = \sum m_i g y_i = g \sum m_i y_i = g y_{CM} M$$

Conservation of Energy
Work-Energy

Exam 2 Fall 2011: Problem 13

- A thin stick with mass M, length L, and moment of inertia $ML^2/3$ is hinged at its lower end and allowed to fall freely as shown in the figure. If its length L = 2 m and it starts from rest at an angle θ = 20°, what is the speed (in m/s) of the free end of the stick when it hits the table?



$$h = \frac{L}{2}\cos\theta$$

Answer: 7.43 m/s
% Right: 14%

$$E_i = Mgy_{CM} = Mg\frac{L}{2}\cos\theta \quad E_f = \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}I\frac{v_f^2}{L^2} \quad E_i = E_f$$

$$v_f = \sqrt{\frac{mgL^3\cos\theta}{I}} = \sqrt{\frac{mgL^3\cos\theta}{\frac{1}{3}mL^2}} = \sqrt{3gL\cos\theta}$$

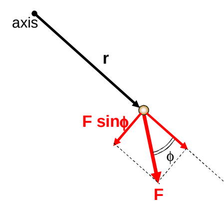
$$= \sqrt{3(9.8m/s^2)(2m)\cos(20^\circ)} \approx 7.43m/s$$

Torque and angular acceleration

$$\tau = I\alpha$$

Note analogy with F=ma

Proof for simple case of one particle



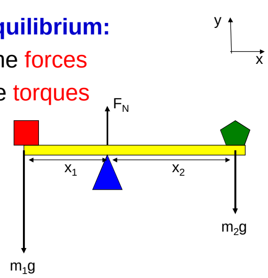
$$F\sin\phi = ma_t$$
$$F\sin\phi = m(\alpha r)$$
$$(F\sin\phi)r = m(\alpha r)r = (mr^2)\alpha$$
$$\tau = I\alpha$$

Static Equilibrium

Conditions for static equilibrium:

- Two equations for the forces
- One equation for the torques

$$F_{net,x} = \sum F_x = 0$$
$$F_{net,y} = \sum F_y = 0$$
$$\tau_{net} = \sum \tau = 0$$

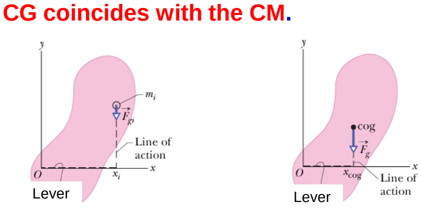


$$F_N - m_1 g - m_2 g = 0$$
$$m_1 g x_1 - m_2 g x_2 = 0$$

Center of Gravity (CG)

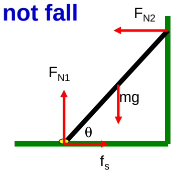
The gravity force on a body effectively acts at a single point, called center of gravity (CG)

If $\vec{g}$ is the same for all parts of the body then the CG coincides with the CM.



Example: a ladder against a wall

What must the friction coefficient with the floor be so that the ladder does not fall down?



$$\sum F_x = 0 \qquad f_s - F_{N2} = 0$$

$$\sum F_y = 0 \qquad F_{N1} - mg = 0$$

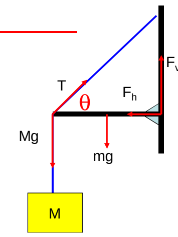
$$\sum \tau_z = 0 \qquad F_{N2} L \sin\theta - mg\frac{L}{2}\cos\theta = 0$$

$$f_s = \mu_s F_{N1} = \mu_s mg$$

$$\mu_s mgL\sin\theta - mg\frac{L}{2}\cos\theta = 0 \qquad \mu_s \sin\theta - \frac{1}{2}\cos\theta = 0 \qquad \boxed{\mu_s = \frac{1}{2\tan\theta}}$$

Equilibrium problem

A crate of mass M is hanging by a rope from horizontal beam with mass m. The beam is supported by a cable attached at an angle θ. Find the tension in the cable and the force with which the wall acts on the beam.



$$\sum F_x = 0 \quad \text{x:} \quad T\cos\theta - F_h = 0$$

$$\sum F_y = 0 \quad \text{y:} \quad T\sin\theta + F_v - Mg - mg = 0$$

$$\sum \tau = 0 \quad \tau\text{:} \quad MgL + mg\frac{L}{2} - TL\sin\theta = 0$$

$$\boxed{T = \frac{(M+0.5m)g}{\sin\theta}} \qquad \boxed{F_h = T\cos\theta = \frac{(M+0.5m)g}{\tan\theta}}$$

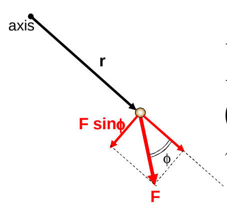
$$\boxed{F_v = (M+m)g - (M+0.5m)g = 0.5mg}$$

Torque and angular acceleration

$$\tau = I\alpha$$

Note analogy with F=ma

Proof for simple case of one particle



$$F\sin\phi = ma_t$$

$$F\sin\phi = m(\alpha r)$$

$$(F\sin\phi)r = m(\alpha r)r = (mr^2)\alpha$$

$$\tau = I\alpha$$

Example: Bicycle wheel

Example (text problem 8.53): A bicycle wheel (a hoop) of radius 0.3 m and mass 2 kg is rotating at 4.00 rev/sec. After 50 sec the wheel comes to a stop because of friction. What is the magnitude of the average torque due to frictional forces?

$$\sum \tau = I\alpha = MR^2\alpha \qquad \omega_i = 4.00\frac{\text{rev}}{\text{sec}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 25.1\text{ rad/sec}$$

$$\omega_f = 0$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = -0.50\text{ rad/s}^2$$

$$|\tau_{av}| = MR^2|\alpha| = 0.09\text{ Nm}$$

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