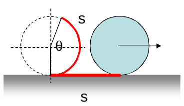
Smooth Rolling

Smooth rolling,
if a wheel does not slip at the point of contact

$$s = \theta R$$

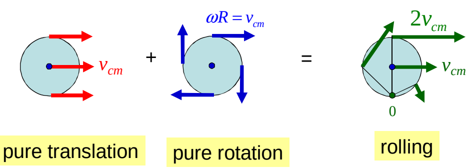


so, because arc length s is the same as linear distance moved forward,

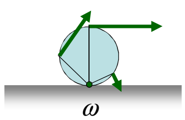
$$v_{cm} = \omega R$$
$$a_{cm} = \alpha R$$

rolling without slipping: translation + rotation

As seen by an observer at rest:



Rolling and Kinetic Energy

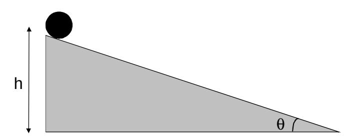


If the object rolls without slipping then $v_{cm} = R\omega$.

$$K_{\text{tot}} = K_{\text{T}} + K_{\text{rot}}$$
$$= \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$$
$$\Rightarrow \frac{1}{2}(mR^2 + I)\omega^2$$
$$= \frac{1}{2}\left(m + \frac{I}{R^2}\right)v_{\text{cm}}^2$$

Rolling Example

Example: Two objects (a solid disk and a solid sphere) are rolling down a ramp. Both objects start from rest and from the same height. Which object reaches the bottom of the ramp first?



The object with the largest linear velocity (v) at the bottom of the ramp will win the race.

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Example continued:

Apply conservation of mechanical energy:

$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$
$$mgh = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2$$

Solving for v: $$v = \sqrt{\frac{2mgh}{\left(m + \frac{I}{R^2}\right)}}$$

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Example continued:

The moments of inertia are: $$I_{\text{disk}} = \frac{1}{2}mR^2$$
$$I_{\text{sphere}} = \frac{2}{5}mR^2$$

For the disk: $$v_{\text{disk}} = \sqrt{\frac{4}{3}gh}$$

For the sphere: $$v_{\text{sphere}} = \sqrt{\frac{10}{7}gh}$$

Since $V_{\text{sphere}} > V_{\text{disk}}$ the sphere wins the race.

Compare these to a box sliding down the frictionless ramp. $$v_{\text{box}} = \sqrt{2gh}$$

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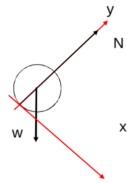
Acceleration

How do objects in the previous example roll?

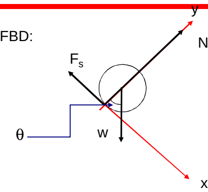
FBD:



Both the normal force and the weight act through the center of mass so Στ = 0. This means that the object cannot rotate when only these two forces are applied.

Add friction:

FBD:



$$\sum \tau = F_s r = I\alpha$$
$$\sum F_x = w\sin\theta - F_s = ma_{cm}$$
$$\sum F_y = N - w\cos\theta = 0$$

Also need $a_{cm} = \alpha R$ and

$$v^2 = v_0^2 + 2a\Delta x$$

The above system of equations can be solved for v at the bottom of the ramp. The result is the same as when using energy methods. (See text example 8.13.)

It is static friction that makes an object roll.

angular momentum

REMINDERS

	Linear Motion		Rotations	
coordinate	x		θ	angle
velocity	v		ω	angular velocity
acceleration	a		α	angular acceleration
mass	m		I	rotational inertia
force	F		τ	torque
1st Newton's Law	$F=0$: v=const		$\tau=0$: ω=const	
2nd Newton's Law	$F = ma$		$\tau = I\alpha$	
Kinetic Energy	$K = ½mv^2$		$K = ½I\omega^2$	Kinetic Energy
Momentum	$p = mv$		???	Angular Momentum

Angular Momentum and AMC

$$\mathbf{F}_{net} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{p}}{\Delta t}$$
$$\mathbf{p} = m\mathbf{v}$$

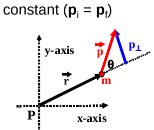
Units of **p** are kg m/s

$$\tau_{net} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{L}}{\Delta t}$$
$$\mathbf{L} = I\boldsymbol{\omega}$$

Units of **L** are kg m²/s

When no net external forces act, the momentum of a system remains constant ($\mathbf{p}_i = \mathbf{p}_f$)

When no net external torques act, the angular momentum of a system remains constant ($\mathbf{L}_i = \mathbf{L}_f$).



Angular momentum of a moving particle:

$$L = rp\sin\theta = rp_\perp = r_\perp p$$

Example

Example (text problem 8.69): A turntable of mass 5.00 kg has a radius of 0.100 m and spins with a frequency of 0.500 rev/sec. What is the angular momentum? Assume a uniform disk.

$$\omega = 0.500\frac{\text{rev}}{\text{sec}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 3.14\text{ rad/sec}$$

$$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = 0.079\text{ kg m}^2/\text{s}$$

AMC Example

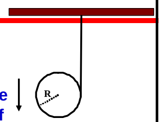
Example (text problem 8.75): A skater is initially spinning at a rate of 10.0 rad/sec with I=2.50 kg m² when her arms are extended. What is her angular velocity after she pulls her arms in and reduces I to 1.60 kg m²?

The skater is on ice, so we can ignore external torques.

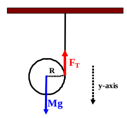
$$L_i = L_f$$
$$I_i\omega_i = I_f\omega_f$$
$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{2.50\text{ kg m}^2}{1.60\text{ kg m}^2}\right)(10.0\text{ rad/sec}) = 15.6\text{ rad/sec}$$

Exam 2 Fall 2011: Problem 17

- A cloth tape is wound around the outside of a non-uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the figure. The cylinder is held with the tape vertical and then release from rest. If the acceleration of the center-of-mass of the cylinder is 3g/5, what is its moment of inertia about its symmetry axis?



Answer: $\frac{2}{3}MR^2$
% Right: 48%



$$Mg - F_T = Ma_y$$

$$F_T = M(g - a_y)$$

$$\tau = RF_T = I\alpha = \frac{Ia_y}{R}$$

$$I = \frac{R^2 F_T}{a_y} = \left(\frac{g - a_y}{a_y}\right)MR^2 = \left(\frac{g - \frac{3}{5}g}{\frac{3}{5}g}\right)MR^2 = \frac{2}{3}MR^2$$

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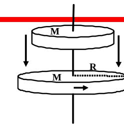
Exam 2 Fall 2011: Problem 31

- A uniform solid disk with mass M and radius R is mounted on a vertical shaft with negligible rotational inertia and is initially rotating with angular speed ω. A non-rotating uniform solid disk with mass M and radius R/2 is suddenly dropped onto the same shaft as shown in the figure. The two disks stick together and rotate at the same angluar speed. What is the new angular speed of the two disk system?



Answer: 4ω/5
% Right: 47%

$$L_i = I_i\omega_i = \frac{1}{2}MR_1^2\omega = L_f = I_f\omega_f = (\frac{1}{2}MR_1^2 + \frac{1}{2}MR_2^2)\omega_f$$

$$\omega_f = \frac{R_1^2}{R_1^2 + R_2^2}\omega = \frac{R^2}{R^2 + (R/2)^2}\omega = \frac{4}{5}\omega$$

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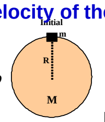
Exam 2 Fall 2010: Problem 16

- A mouse of mass M/6 lies on the rim of a solid uniform disk of mass M that can rotate freely about its center like a merry-go-round. Initially the mouse and disk rotate together with an angular velocity of ω. If the mouse walks to a new position that is at the center of the disk what is the new angular velocity of the mouse-disk system?

Answer: 4ω/3
% Right: 58%

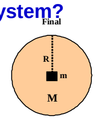


$$L_i = I_i\omega_i = (I_{disk} + mR^2)\omega$$

$$I_{disk}\omega_{new} = (I_{disk} + mR^2)\omega$$

$$L_f = I_f\omega_f = I_{disk}\omega_f$$

$$L_i = L_f$$



Note that energy is not conserved in this problem!

$$\Delta E = E_f - E_i = \frac{L_f^2}{2I_f} - \frac{L_i^2}{2I_i} = \frac{1}{9}MR^2\omega^2$$

$$\omega_{new} = \left(1 + \frac{mR^2}{I_{disk}}\right)\omega = \left(1 + \frac{mR^2}{\frac{1}{2}MR^2}\right)\omega = \left(1 + \frac{2m}{M}\right)\omega = \frac{4}{3}\omega$$

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