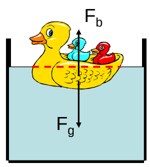
Floating

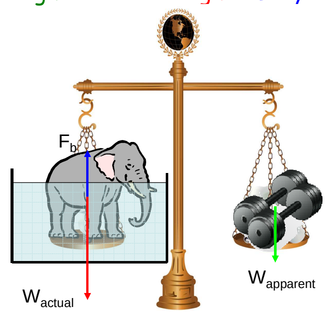
- When a body floats:
  - The gravity force F_g balances the buoyancy force F_b!
  - The gravity force is equal to the weight of the displaced fluid
  - The mass M of the floating body is equal to the mass of the displaced fluid m_f

$$F_g = F_b$$
$$F_g = m_f g$$
$$M = m_f$$



Apparent weight in a fluid

Apparent weight = Actual weight - Buoyancy force



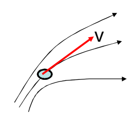
Example problem: Iceberg

- What percentage of the volume of an iceberg can be seen above the surface of the sea?

$$F_b = F_g$$
$$m_f g = Mg$$
$$\rho_f V_f g = \rho V g$$
$$\frac{V_f}{V} = \frac{\rho}{\rho_f}$$
$$\frac{V_{above}}{V} = 1 - \frac{V_f}{V} = \frac{\rho_f - \rho}{\rho_f}$$
$$\frac{V_{above}}{V} = \frac{1.024 - 0.917}{1.024} = 0.10 = 10\%$$

Ideal Fluids in Motion

- Steady flow:
  - the velocity does not change with time.
- Incompressible flow:
  - the density is constant.
- Nonviscous flow:
  - no friction
- Irrotational flow:
  - There is no rotation
- Streamline: the path of a fluid element.
  - The velocity of a fluid is tangent to the streamline.
  - Two streamlines do not intersect.
  - A surface fallowing streamlines acts like a tube.



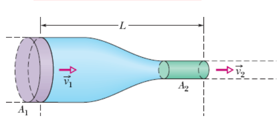
The equation of continuity

- For an ideal fluid the volume flow rate R_v is constant along a tube of flow.

$$R_v = \frac{\Delta V}{\Delta t} = \frac{A\upsilon\Delta t}{\Delta t} = A\upsilon = \text{constant}$$

- For a tube with variable cross section A:

$$R_v = A_1\upsilon_1 = A_2\upsilon_2$$



Example (text problem 9.41): A garden hose of inner radius 1.0 cm carries water at 2.0 m/s. The nozzle at the end has radius 0.20 cm. How fast does the water move through the constriction?

$$A_1 v_1 = A_2 v_2$$
$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right) v_1$$
$$= \left(\frac{1.0\ \text{cm}}{0.20\ \text{cm}}\right)^2 (2.0\ \text{m/s}) = 50\ \text{m/s}$$

6

Bernoulli's Equation

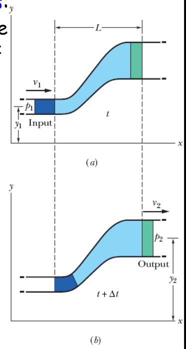
- Conservation of energy for fluids:
  - the work done by the pressure force is equal to the change of the kinetic and the potential energy of a fluid element.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = \text{constant}$$

- If the motion is horizontal:

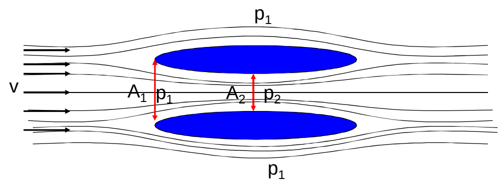
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 = \text{constant}$$

- If the velocity is increasing, the pressure is decreasing!



Example: parallel ships

- Two ships move parallel with a velocity v.



$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_2 = v_1 \frac{A_1}{A_2} \quad \Rightarrow \quad v_2 > v_1$$

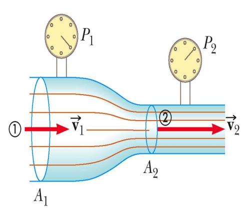
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 = p_2 + \frac{1}{2}\rho (v_2^2 - v_1^2) \quad \Rightarrow \quad p_1 > p_2$$

A net force pushes the ships towards each other

Applications of Bernoulli's Principle: Venturi Tube

- Shows fluid flowing through a horizontal constricted pipe
- Speed changes as diameter changes
- Can be used to measure the speed of the fluid flow
- Swiftly moving fluids exert less pressure than do slowly moving fluids



$$\frac{1}{2}\rho v^2 + P = \text{constant}$$

Example (text problem 9.49): A nozzle is connected to a horizontal hose. The nozzle shoots out water moving at 25 m/s. What is the gauge pressure of the water in the hose? Neglect viscosity and assume that the diameter of the nozzle is much smaller than the inner diameter of the hose.

Let point 1 be inside the hose and point 2 be outside the nozzle.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

The hose is horizontal so $y_1 = y_2$. Also $P_2 = P_{atm}$.

10

Example continued:

Substituting: $$P_1 + \frac{1}{2}\rho v_1^2 = P_{atm} + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_{atm} = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

v_2 = 25 m/s and v_1 is unknown. Use the continuity equation.

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \left(\frac{\pi\left(\frac{d_2}{2}\right)^2}{\pi\left(\frac{d_1}{2}\right)^2}\right) v_2 = \left(\frac{d_2}{d_1}\right)^2 v_2$$

Since $d_2 << d_1$ it is true that $v_1 << v_2$.

11

Example continued:

$$\begin{aligned} P_1 - P_{atm} &= \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \\ &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \approx \frac{1}{2}\rho v_2^2 \\ &= \frac{1}{2}\left(1000\ \text{kg/m}^3\right)\left(25\ \text{m/s}\right)^2 \\ &= 3.1\times10^5\ \text{Pa} \end{aligned}$$

12

Bernoulli's Equation: Applications

- **Example (velocity of efflux):**

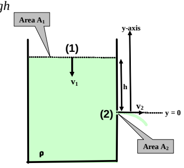
We can use Bernoulli's equation to calculate the speed of efflux, v_2, from a horizontal orifice (and area A_2) located a depth h below the water level of a large talk (with area A_1).

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \textbf{(1}\leftrightarrow\textbf{2)}$$

$P_1 = P_2 = P_{atm}$ **$v_1 = v_2 A_2/A_1$** $v_2^2 = v_1^2 + 2gh = (A_2/A_1)^2 v_2^2 + 2gh$

$$v_2 = \sqrt{\frac{2gh}{(1-(A_2/A_1)^2)}} \xrightarrow[A_2 << A_1]{} \sqrt{2gh}$$

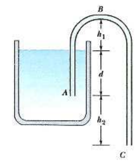
(Torricelli's Law)



Bernoulli's Equation: Application

- **Siphon:**

The figure shows a siphon, which is a device for removing liquid from a container. Tube ABC must initially be filled, but once this is done, liquid will flow until the liquid surface of the container is level with the tube opening A. With what speed does the liquid emerge from the tube at C? What is the greatest possible height h_1 that a siphon can lift water?



$VA = va$

$V = va/A$

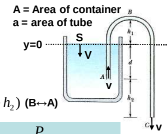
A = Area of container
a = area of tube

$$P_{atm} + \tfrac{1}{2}\rho V^2 = P_A + \tfrac{1}{2}\rho v^2 - \rho g d \quad \textbf{(S}\leftrightarrow\textbf{A)}$$

$$P_{atm} = P_A + \tfrac{1}{2}\rho v^2\left(1 - \frac{a^2}{A^2}\right) - \rho g d \xrightarrow[a << A]{} P_A + \tfrac{1}{2}\rho v^2 - \rho g d$$

$$P_A + \tfrac{1}{2}\rho v^2 - \rho g d = P_{atm} + \tfrac{1}{2}\rho v^2 - \rho g(d + h_2) \quad \textbf{(A}\leftrightarrow\textbf{C)}$$

$$\tfrac{1}{2}\rho v^2 = \rho g(d + h_2) \qquad P_B + \tfrac{1}{2}\rho v^2 + \rho g h_1 = P_{atm} + \tfrac{1}{2}\rho v^2 - \rho g(d + h_2) \quad \textbf{(B}\leftrightarrow\textbf{A)}$$



$$v = \sqrt{2g(d + h_2)} \qquad h_1 = \frac{(P_{atm} - P_B)}{\rho g} - (d + h_2) \longrightarrow (h_1)_{max} = \frac{P_{atm}}{\rho g} - (d + h_2)$$

An Object Moving Through a Fluid

- Many common phenomena can be explained by Bernoulli's equation
  - At least partially
- In general, an object moving through a fluid is acted upon by a net upward force as the result of any effect that causes the fluid to change its direction as it flows past the object

Application – Golf Ball

- The dimples in the golf ball help move air along its surface
- The ball pushes the air down
- Newton's Third Law tells us the air must push up on the ball
- The spinning ball travels farther than if it were not spinning