

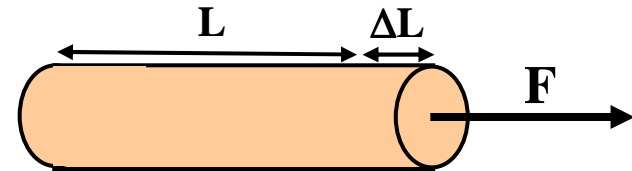
# Definition of Strain

Define variables the cause deformation of a material and the response of the material.

- **System Response – Linear Deformation:**

Suppose that a wire of length  $L$  is stretched by a force  $F$  then:

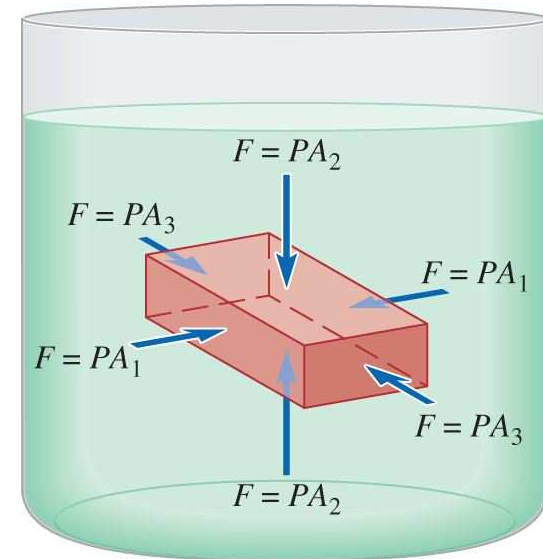
$$\text{Linear Strain} = \frac{\Delta L}{L}$$



- **System Response – Volume Deformation:**

Consider an object immersed in a fluid. The fluid will exert a pressure  $P$  from all sides causing a volume deformation.

$$\text{Volume Strain} = \frac{\Delta V}{V}$$



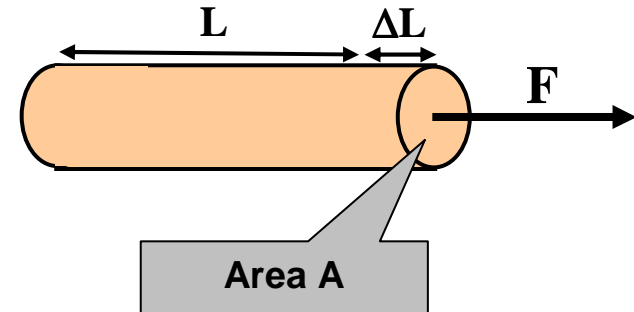
# Definition of Stress

Define variables the cause deformation of a material and the response of the material.

- **Linear Deformation:**

Suppose that a wire of length  $L$  is stretched by a force  $F$  then:

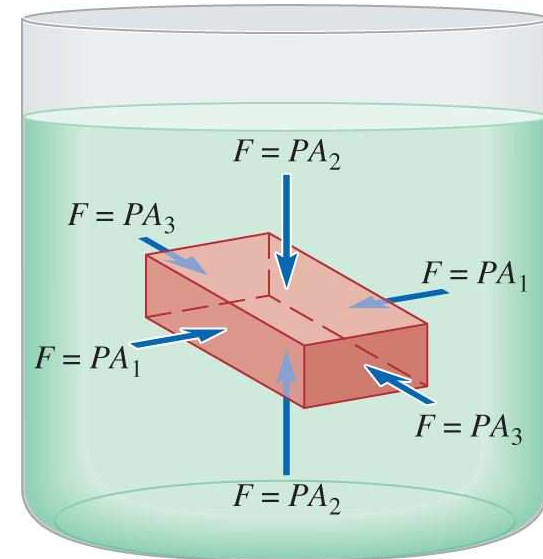
$$\text{Linear Stress} = \frac{F}{A}$$



- **Volume Deformation:**

Consider an object immersed in a fluid. The fluid will exert a pressure  $P$  from all sides causing a volume deformation.

$$\text{Volume Stress} = \frac{F}{A} = P$$



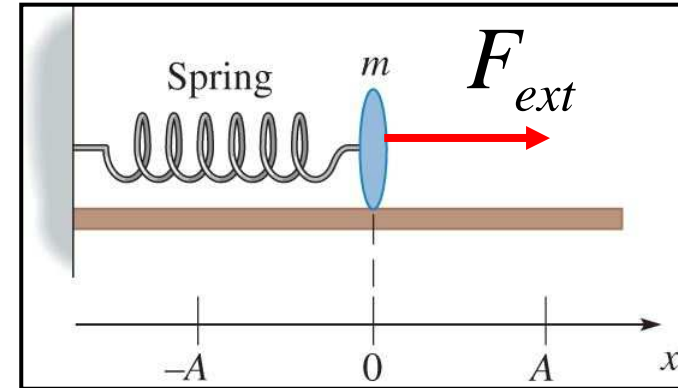
# Hooke's Law Spring

- Linear Restoring Force: Ideal Spring

$$F_{spring} = -k\Delta x$$

Spring Force

Spring Constant  $k$



$$F_{ext} = k\Delta L \quad \frac{F_{ext}}{A} = \left( \frac{kL}{A} \right) \frac{\Delta L}{L} = C \frac{\Delta L}{L}$$

$$\text{Stress} = C \times \text{Strain}$$

Generalized Hooke's Law!

Constant

# Stress Proportional to Strain

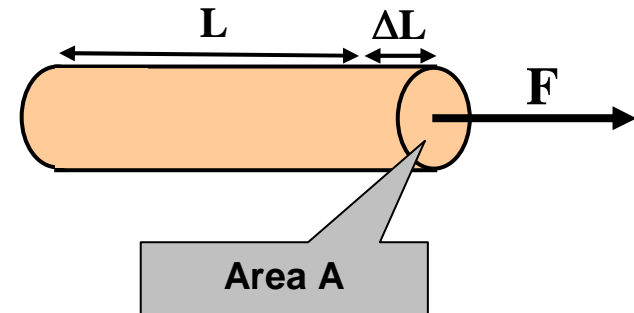
- Linear Deformation:

Suppose that a wire of length  $L$  is stretched by a force  $F$  then:

$$\text{Stress} = Y \times \text{Strain}$$

Young's Modulus  
Units = Pa

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$



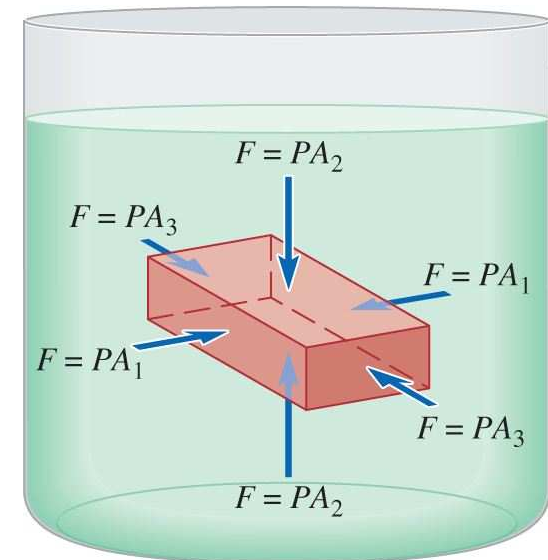
- Volume Deformation:

Consider an object immersed in a fluid. The fluid will exert a pressure  $P$  from all sides causing a volume deformation.

$$\text{Stress} = -B \times \text{Strain}$$

Bulk Modulus  
Units = Pa

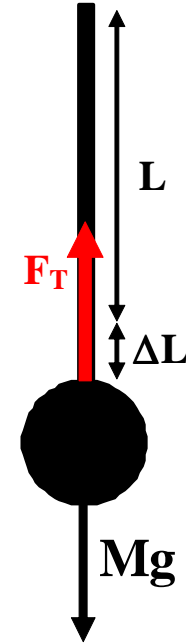
$$\frac{F}{A} = P = -B \frac{\Delta V}{V}$$



# Example Problem: Young's Modulus

- Linear Deformation:

A wrecking ball with mass  $M$  is to be lifted by a crane with a steel cable that has a diameter of 1.5 cm and an unstretched length of 30 m. The Young's modulus of steel is  $2.0 \times 10^{11}$  Pa. Ignoring the weight of the cable itself, when the ball is lifted and held at rest the cable stretches by 1.66 cm, what is the mass  $M$  of the wrecking ball?



$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

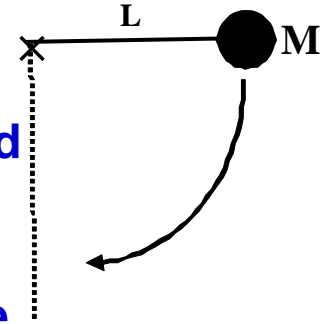
$$F = F_T = Mg$$
$$A = \pi r^2 = \pi (D/2)^2$$
$$F = Mg = AY \frac{\Delta L}{L}$$

$$M = AY \frac{\Delta L}{gL} = \pi (D/2)^2 Y \frac{\Delta L}{gL}$$

$$= \pi (0.0075m)^2 (2 \times 10^{11} N/m^2) \frac{0.0166m}{(9.8m/s^2)(30m)} \approx 2,000kg$$

# Exam 2 Fall 2013: Problem 59

- Near the surface of the Earth, a 2,000 kg wrecking ball is connected to a steel cable that has a diameter of 2.5 cm and an unstretched length of  $L = 40$  m. The other end of the cable is fixed in position and the ball is initially held at rest horizontally, as shown in the figure. When the ball is released from rest it swings down. The Young's modulus of steel is  $2.0 \times 10^{11}$  Pa. Ignoring the weight of the cable itself, when the ball-cable system swing through vertical (i.e. ball at its lowest point) how much does the cable stretch (in cm)?



Answer: 2.40

% Right: 6%

$$F_T - Mg = Ma_{radial} = M \frac{v^2}{R} = \frac{Mv^2}{L + \Delta L}$$

$$F_T = Mg + \frac{Mv^2}{L + \Delta L} = 3Mg$$

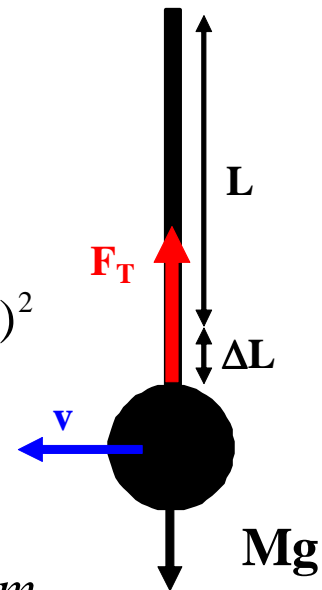
$$\frac{1}{2} Mv^2 = Mg(L + \Delta L)$$

$$Mv^2 = 2Mg(L + \Delta L)$$

$$\frac{\Delta L}{L} = \frac{F_T}{AY} = \frac{3Mg}{\pi(D/2)^2 Y}$$

$$\Delta L = \frac{3MgL}{\pi(D/2)^2 Y} = \frac{3(2000\text{kg})(9.8\text{m/s}^2)(40\text{m})}{\pi(0.0125\text{m})^2 (2 \times 10^{11}\text{N/m}^2)} \approx 2.4\text{cm}$$

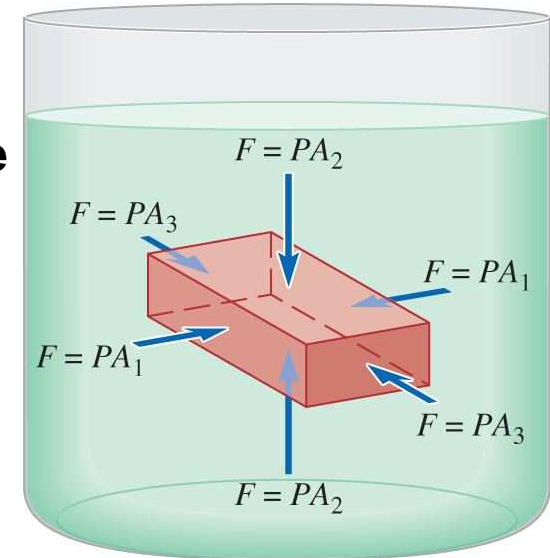
$$A = \pi r^2 = \pi(D/2)^2$$



# Example Problem: Bulk Modulus

- Volume Deformation:**

The atmospheric pressure at the surface of a clear lake is 101 kPa. By what percentage does the density of lake water increase at a depth of 1.0 km below the surface of the lake? The bulk modulus for water is  $2.2 \times 10^9$  Pa?



$$\frac{F}{A} = P = -B \frac{\Delta V}{V}$$

$$P_d = P_0 + \rho g d$$

$$\Delta V_d = \frac{-VP_d}{B}$$

$$\rho g d = (1,000 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)(1,000 \text{ m}) = 9.8 \times 10^6 \text{ Pa}$$

$$\rho_0 = \frac{M}{V_0} = \frac{M}{V + \Delta V_0} = \frac{M}{V - (VP_0 / B)} = \frac{M / V}{1 - P_0 / B}$$

$$\rho_d = \frac{M}{V_d} = \frac{M}{V + \Delta V_d} = \frac{M}{V - (VP_d / B)} = \frac{M / V}{1 - P_d / B}$$

$$\frac{\rho_d - \rho_0}{\rho_0} = \frac{1 - P_0 / B}{1 - P_d / B} - 1 = \frac{P_d - P_0}{B - P_d} = \frac{\rho g d}{B - (P_0 + \rho g d)} = \frac{9.8 \times 10^6}{2.2 \times 10^9 - 9.901 \times 10^6} \approx 0.00447$$

0.447%