Exam 2: Tonight 8:20-10:10pm

- **Room Assignments:**

Last Name	Room
A-G	CLB C130
H-O	WEIM 1064
P-Z	MCCC 100

- **Breakdown of the 20 Problems**

Material	# of Problems
Chapter 7	6
Chapter 8	5
Chapter 9	5
Chapter 10	4

- **Crib Sheet:** You may bring a single hand written formula sheet on 8½ x 11 inch paper (both sides).

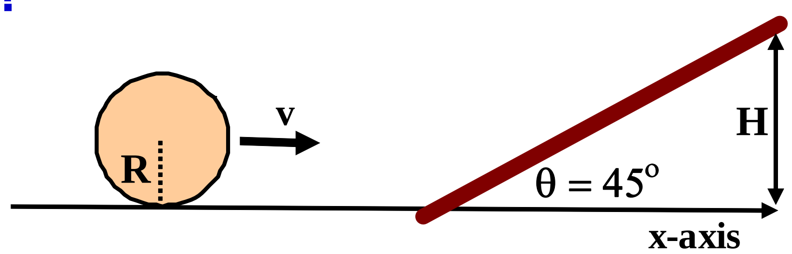
- **Calculator:** You should bring a calculator (any type).

- **Scratch Paper:** We will provide scratch paper.

Final Exam Spring 2011: Problem 12

- A uniform disk with mass M, radius R = 0.50 m, and moment of inertia I = $MR^2/2$ rolls without slipping along the floor at 2 revolutions per second when it encounters a long ramp angled upwards at 45° with respect to the horizontal. How high above its original level will the center of the disk get (in meters)?

Answer: 3.0
% Right: 39%



$$E_i = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 + MgR = E_f = Mg(H+R)$$

$$v = R\omega$$

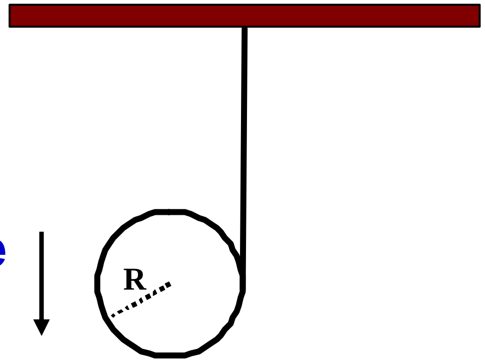
$$\omega = 2\pi f$$

$$H = \frac{1}{2g}\left(v^2 + \frac{I}{M}\omega^2\right) = \frac{1}{2g}\left(R^2\omega^2 + \frac{I}{M}\omega^2\right)$$

$$= \frac{R^2}{2g}\left(1+\frac{I}{MR^2}\right)\omega^2 = \frac{2\pi^2R^2}{g}\left(1+\frac{I}{MR^2}\right)f^2 = \frac{2\pi^2(0.5m)^2}{9.8m/s^2}\left(\frac{3}{2}\right)(2/s)^2 \approx 3.0m$$

Exam 2 Fall 2011: Problem 17

- A cloth tape is wound around the outside of a non-uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the figure. The cylinder is held with the tape vertical and then release from rest. If the acceleration of the center-of-mass of the cylinder is 3g/5, what is its moment of inertia about its symmetry axis?



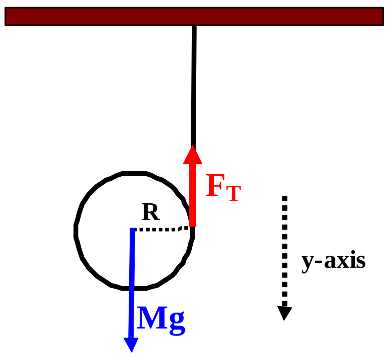
Answer: $\frac{2}{3}MR^2$

% Right: 48%

$$Mg - F_T = Ma_y$$

$$F_T = M(g - a_y)$$

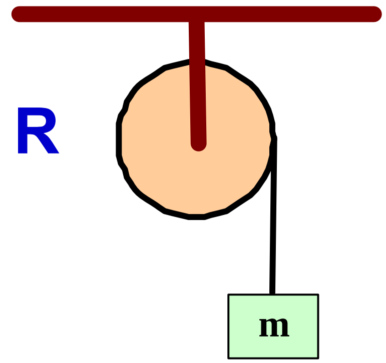
$$\tau = RF_T = I\alpha = \frac{Ia_y}{R}$$



$$I = \frac{R^2 F_T}{a_y} = \left(\frac{g - a_y}{a_y}\right)MR^2 = \left(\frac{g - \frac{3}{5}g}{\frac{3}{5}g}\right)MR^2 = \frac{2}{3}MR^2$$

Final Exam Fall 2010: Problem 12

- **A block of mass m is attached to a cord that is wrapped around the rim of a flywheel of radius R and hangs vertically, as shown. The rotational inertia of the flywheel is I = MR²/2. If when the block is released and the cord unwinds the acceleration of the block is equal to g/2, what is the mass m of the block?**



Answer: M/2
% Right: 51%

$$mg - F_T = ma_y$$

$$\tau = I\alpha = RF_T \qquad F_T = \frac{I}{R}\alpha = \frac{I}{R^2}a_y$$

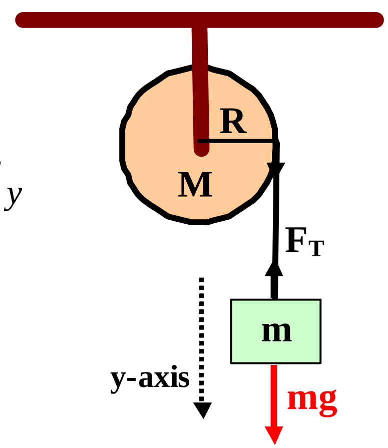
$$s = R\theta$$

$$v = R\omega$$

$$a = R\alpha$$

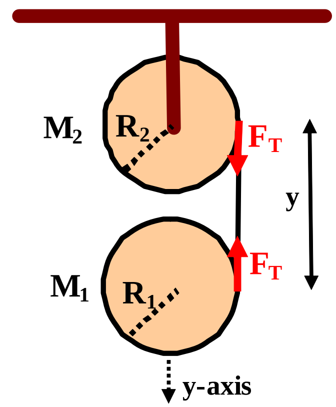
$$mg - \frac{I}{R^2}a_y = ma_y$$

$$m = \frac{I}{R^2}\left(\frac{a_y}{g - a_y}\right) = \frac{\frac{1}{2}MR^2}{R^2}\left(\frac{\frac{1}{2}g}{g - \frac{1}{2}g}\right) = \frac{1}{2}M$$



Exam 2 Fall 2013: Problem 39

- **Near the surface of the Earth, a cloth tape is wound around the outside of the two uniform solid cylinders shown in the figure. Solid cylinder 1 has mass M_1, radius R_1, and moment of inertia $I_1 = ½M_1R_1^2$. Solid cylinder 2 has mass M_2, radius R_2, and moment of inertia $I_2 = ½M_2R_2^2$. Cylinder 2 is attached to the ceiling and rotates without friction and cylinder 1 hangs vertically. If $M_1 = 2M_2$, when the cylinders are released from rest and the tape unwinds off both cylinders, what is the acceleration of the center-of-mass of cylinder 1 (in m/s_2)?**



Answer: 8.4
% Right: 8%

$$y = R_1\theta_1 + R_2\theta_2$$
$$v_y = R_1\omega_1 + R_2\omega_2$$
$$a_y = R_1\alpha_1 + R_2\alpha_2$$

$$M_1 g - F_T = M_1 a_y \longrightarrow F_T = M_1(g - a_y)$$
$$\tau_1 = R_1 F_T = I_1\alpha_1 \longrightarrow \alpha_1 = R_1 F_T / I_1$$
$$\tau_2 = R_2 F_T = I_2\alpha_2 \longrightarrow \alpha_2 = R_2 F_T / I_2$$

$$a_y = R_1\alpha_1 + R_2\alpha_2 = \frac{R_1^2 F_T}{I_1} + \frac{R_2^2 F_T}{I_2} = \left(\frac{M_1R_1^2}{I_1} + \frac{M_1R_2^2}{I_2}\right)(g - a_y) = X(g - a_y)$$

$$X = \left(\frac{M_1R_1^2}{I_1} + \frac{M_1R_2^2}{I_2}\right) = 2\left(1 + \frac{M_1}{M_2}\right) \xrightarrow[M_1 = 2M_2]{} 6$$

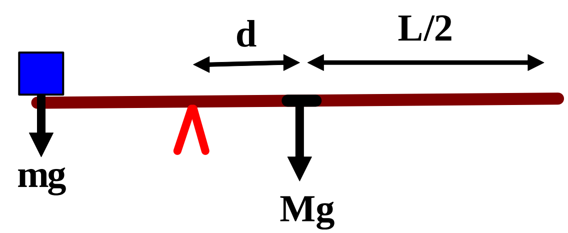
$$a_y = \frac{Xg}{1+X} = \tfrac{6}{7} g = \tfrac{6}{7}(9.8 m/s^2) = 8.4 m/s^2$$

Exam 2 Fall 2010: Problem 13

- **A uniform plank is 6 m long and weighs 80 N. It is balanced on a sawhorse at its center. An additional 160 N weight is now placed on the left end of the plank. To keep the plank balanced, the sawhorse must be moved what distance to the left?**

Answer: 2 m
% Right: 48%

$$mg\left(\frac{L}{2} - d\right) - Mgd = 0$$



$$Mg = 80N$$

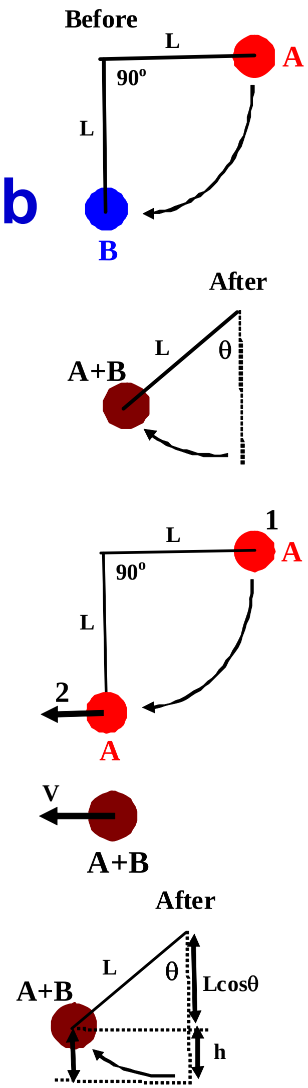
$$mg = 160N$$

$$L = 6m$$

$$d = \left(\frac{mg}{mg + Mg}\right)\frac{L}{2} = \left(\frac{160N}{80N + 160N}\right)\frac{L}{2} = \frac{1}{3}L = 2m$$

Exam 2 Fall 2011: Problem 10

- **Two pendulum bobs have masses M_A = 3 kg and M_B = 2 kg and equal lengths L as shown in the figure. Bob A is initially held horizontally while bob B hangs vertically at rest. Bob A is released and collides with bob B. The two masses then stick together and swing upward to the right to a maximum angle θ. What is the maximum swing angle θ?**



Answer: 50.2°

% Right: 31%

$$E_1 = M_A gL = E_2 = \tfrac{1}{2} M_A v_2^2 \qquad v_2 = \sqrt{2gL}$$

$$p_i = M_A v_2 = M_A \sqrt{2gL} = p_f = (M_A + M_B)V$$

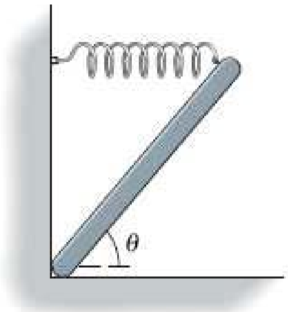
$$E_i = \tfrac{1}{2}(M_A + M_B)V^2 = E_f = (M_A + M_B)gh \qquad h = L(1 - \cos\theta)$$

$$h = \frac{V^2}{2g} = \left(\frac{M_A}{M_A + M_B}\right)^2 L \qquad \cos\theta = 1 - \frac{h}{L} = 1 - \left(\frac{M_A}{M_A + M_B}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\theta \approx 50.2^\circ$$

Exam 2 Spring 2012: Problem 39

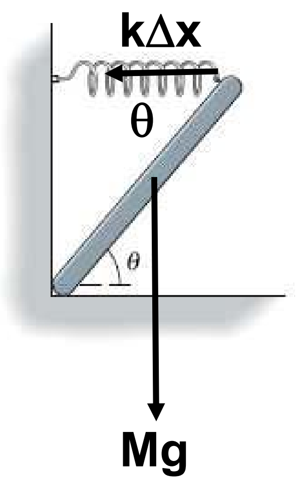
- A modern sculpture has a large horizontal spring, that is attached to a 60-kg piece of uniform metal at its end and holds the metal at rest at an angle of θ = 60° above the horizontal direction as shown in the figure. The other end of the metal is wedged into a corner and is free to rotate. If the spring is stretched 0.2 m what is the spring constant k (in N/m)?



Answer: 849
% Right: 31%

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

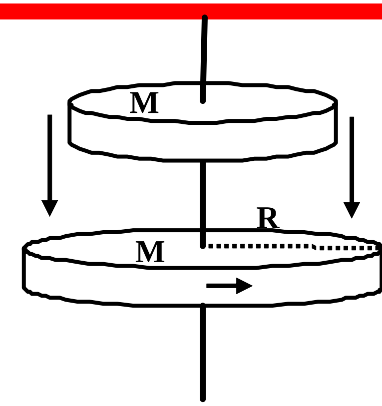
$$Lk\Delta x \sin\theta - \frac{L}{2}Mg\cos\theta = 0$$



$$k = \frac{Mg}{2\Delta x}\cot\theta = \frac{(60kg)(9.8m/s^2)}{2(0.2m)}\cot(60^\circ) \approx 849N/m$$

Exam 2 Fall 2011: Problem 31

- **A uniform solid disk with mass M and radius R is mounted on a vertical shaft with negligible rotational inertia and is initially rotating with angular speed ω. A non-rotating uniform solid disk with mass M and radius R/2 is suddenly dropped onto the same shaft as shown in the figure. The two disks stick together and rotate at the same angluar speed. What is the new angular speed of the two disk system?**



Answer: 4ω/5
% Right: 47%

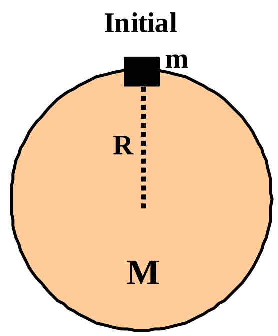
$$L_i = I_i\omega_i = \tfrac{1}{2}MR_1^2\omega = L_f = I_f\omega_f = (\tfrac{1}{2}MR_1^2 + \tfrac{1}{2}MR_2^2)\omega_f$$

$$\omega_f = \frac{R_1^2}{R_1^2 + R_2^2}\omega = \frac{R^2}{R^2 + (R/2)^2}\omega = \tfrac{4}{5}\omega$$

Exam 2 Fall 2010: Problem 16

- A mouse of mass M/6 lies on the rim of a solid uniform disk of mass M that can rotate freely about its center like a merry-go-round. Initially the mouse and disk rotate together with an angular velocity of ω. If the mouse walks to a new position that is at the center of the disk what is the new angular velocity of the mouse-disk system?

Answer: 4ω/3
% Right: 58%

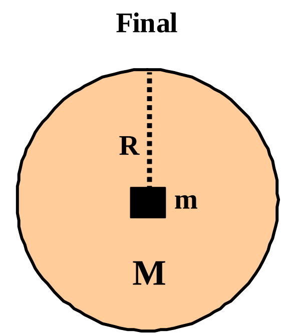


$$L_i = I_i\omega_i = (I_{disk} + mR^2)\omega$$

$$I_{disk}\omega_{new} = (I_{disk} + mR^2)\omega$$

$$L_f = I_f\omega_f = I_{disk}\omega_f$$

$$L_i = L_f$$



$$\omega_{new} = \left(1 + \frac{mR^2}{I_{disk}}\right)\omega = \left(1 + \frac{mR^2}{\frac{1}{2}MR^2}\right)\omega = \left(1 + \frac{2m}{M}\right)\omega = \tfrac{4}{3}\omega$$

Note that energy is not conserved in this problem!

$$\Delta E = E_f - E_i$$
$$= \frac{L_f^2}{2I_f} - \frac{L_i^2}{2I_i} = \tfrac{1}{9}MR^2\omega^2$$

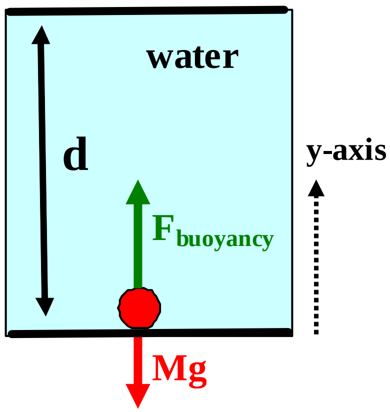
Exam 2 Fall 2010: Problem 20

- **Suppose that you release a small ball from rest at a depth of 39.2 m below the surface in a pool of water (with density ρ_{water}) near the surface of the Earth. If the density of the ball is 1/3 the density of water, how long does it take the ball to reach the surface?**

Answer: 2 s
% Right: 53%

$$d = \tfrac{1}{2} a_y t^2$$

$$t = \sqrt{\frac{2d}{a_y}}$$



$$M_{water} g - M_{ball} g = M_{ball} a_y$$

$$\rho_{water} V_{ball} g - \rho_{ball} V_{ball} g = \rho_{ball} V_{ball} a_y$$

$$a_y = \frac{\rho_{water} - \rho_{ball}}{\rho_{ball}} g = \left(\frac{\rho_{water}}{\rho_{ball}} - 1 \right) g$$

$$t = \sqrt{\frac{2d}{\left(\frac{\rho_{water}}{\rho_{ball}} - 1 \right) g}} = \sqrt{\frac{2(39.2m)}{(3-1)(9.8m/s^2)}} = \sqrt{4s^2} = 2s$$

Final Exam Fall 2010: Problem 13

- **A block of wood has a mass of 4 kg and density of 600 kg/m^3. It is loaded on top with lead (density = 11,400 kg/m^3) so that the block of wood will float in water with 90% of its volume submerged. What is the mass of the lead if the water density is 1,000 kg/m^3?**

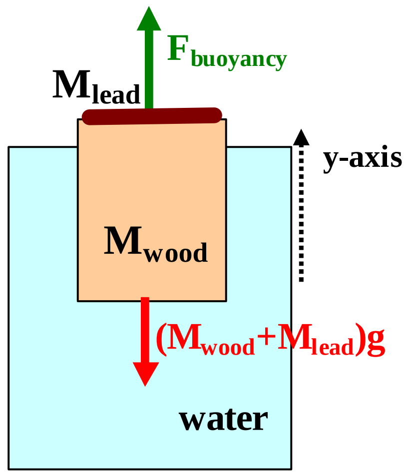
Answer: 2 kg
% Right: 46%

$$M_{water}g - (M_{wood} + M_{lead})g = (M_{wood} + M_{lead})a_y = 0$$

$$M_{lead} = M_{water} - M_{wood} = V_{sub}\rho_{water} - M_{wood} =$$

$$V_{sub}\rho_{water}\left(\frac{M_{wood}}{V_{wood}\rho_{wood}}\right) - M_{wood} = \left(\frac{V_{sub}\rho_{water}}{V_{wood}\rho_{wood}} - 1\right)M_{wood}$$

$$= \left(\frac{0.9(1{,}000kg/m^3)}{600kg/m^3} - 1\right)M_{wood} = \tfrac{1}{2}M_{wood} = 2kg$$

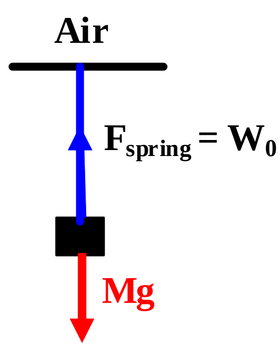


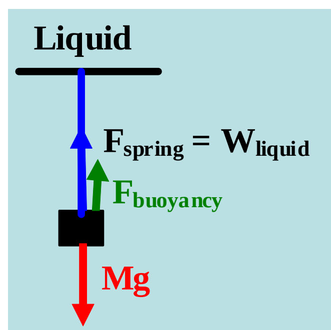
Exam 2 Fall 2010: Problem 19

- **An object hangs from a spring balance. When submerged in water the object weighs one-half what it weighs in air. If when submerged in an unknown liquid with density ρ_X the object weighs three-fourths what it weighs in air, what is the density of the unknown liquid?**

Answer: $\frac{1}{2}\rho_{water}$
% Right: 41%

$$W_0 = Mg$$





$$W_{liquid} - Mg + M_{liquid}g = 0$$

$$W_0 - W_{liquid} = M_{liquid}g = \rho_{liquid}Vg$$

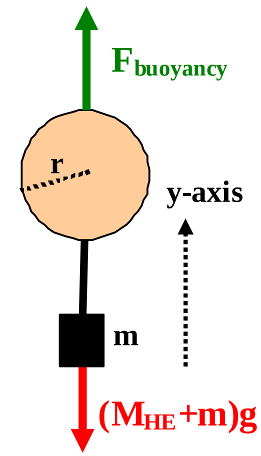
$$W_0 - W_{water} = \rho_{water}Vg$$

$$W_0 - W_X = \rho_X Vg$$

$$\frac{\rho_X}{\rho_{water}} = \frac{W_0 - W_X}{W_0 - W_{water}} = \frac{W_0 - \frac{3}{4}W_0}{W_0 - \frac{1}{2}W_0} = \frac{1}{2}$$

Exam 2 Spring 12: Problem 46

- **What is the minimum radius (in m) that a spherical helium balloon must have in order to lift a total mass of m = 10-kg (including the mass of the empty balloon) off the ground? The density of helium and the air are, ρ_{HE} = 0.18 kg/m^3 and ρ_{air} = 1.2 kg/m^3, respectively.**



Answer: 1.33
% Right: 37%

$$F_{buoyancy} - (M_{HE} + m)g = 0$$

$$\rho_{air} V_{ballon} g - (\rho_{HE} V_{ballon} + m)g = 0$$

$$V_{ballon} = \tfrac{4}{3}\pi r^3 = \frac{m}{\rho_{air} - \rho_{HE}}$$

$$r = \left(\frac{3m}{4\pi(\rho_{air} - \rho_{HE})}\right)^{\frac{1}{3}} = \left(\frac{3(10kg)}{4\pi(1.02kg/m^3)}\right)^{\frac{1}{3}} = \left(2.34m^3\right)^{\frac{1}{3}} \approx 1.33m$$

Exam 3 Fall 2012: Problem 43

- An ideal spring-and-mass system is undergoing simple harmonic motion (SHM). If the speed of the block is 1.0 m/s when the displacement from equilibrium is 2.0 m, and the speed of the block is 3.0 m/s when the displacement from equilibrium is 1.0 m, what is the angular frequency of the oscillations (in rad/s)?

Answer: 1.63
% Right: 33%

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2$$

$$\frac{k}{m}A^2 = v_x^2(t) + \frac{k}{m}x^2(t)$$

$$\omega^2 A^2 = v_x^2(t_2) + \omega^2 x^2(t_2)$$

$$\omega^2 A^2 = v_x^2(t_1) + \omega^2 x^2(t_1)$$

$$0 = v_x^2(t_2) - v_x^2(t_1) + \omega^2(x^2(t_2) - x^2(t_1))$$

$$\omega^2 = \frac{v_x^2(t_2) - v_x^2(t_1)}{x^2(t_1) - x^2(t_2)}$$

$$\omega = \sqrt{\frac{v_x^2(t_2) - v_x^2(t_1)}{x^2(t_1) - x^2(t_2)}} = \sqrt{\frac{(3m/s)^2 - (1m/s)^2}{(2m)^2 - (1m)^2}} \approx 1.63/s$$