Ch. 11: waves

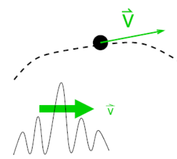
- Waves and Energy Transport
- What determines speed of wave?
- Mathematical form of wave: example
- demos
- Superposition of waves
- HITT quiz

11. Waves

- Mechanical: sound, seismic, water, string.
- Electromagnetic: light, radio, x-rays.
- Gravitational: (not yet detected).
- Matter (quantum mechanics): wave-particle duality.

→ Particle: localized

→ Wave: extended



Waves in nature

When a stone is dropped into a pond, the water is disturbed from its equilibrium positions as the wave passes; it returns to its equilibrium position after the wave has passed.

The water moves up and down as the disturbance moves outward.

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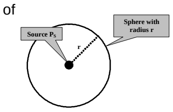
Energy transport

Intensity is a measure of the amount of energy/sec that passes through a square meter of area perpendicular to the wave's direction of travel.

$$I = \frac{\text{Power}}{4\pi r^2} = \frac{P}{4\pi r^2}$$

Intensity has units of watts/m^2 .



This is an inverse square law. The intensity drops as the inverse square of the distance from the source. (Light sources appear dimmer the farther away from them you are.)

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Example

Example: At the location of the Earth's upper atmosphere, the intensity of the Sun's light is 1400 W/m^2. What is the intensity of the Sun's light at the orbit of the planet Mercury?

$$I_e = \frac{P_{sun}}{4\pi r_{es}^2} \qquad I_m = \frac{P_{sun}}{4\pi r_{ms}^2}$$

Divide one equation by the other:

$$\frac{I_m}{I_e} = \frac{\dfrac{P_{sun}}{4\pi r_{ms}^2}}{\dfrac{P_{sun}}{4\pi r_{es}^2}} = \left(\frac{r_{es}}{r_{ms}}\right)^2 = \left(\frac{1.50\times10^{11}\text{ m}}{5.85\times10^{10}\text{ m}}\right)^2 = 6.57$$

$$I_m = 6.57 I_e = 9200\text{ W/m}^2$$

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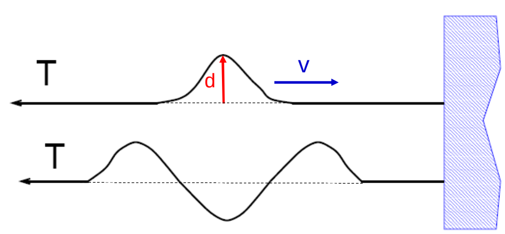
Wave on a string



A *transverse wave*, i.e. displacement, d is perpendicular to direction of wave motion, v!

Wave on a string

A wave traveling on this string will have a speed of $v = \sqrt{\frac{F}{\mu}}$

where F is the force applied to the string (tension) and μ is the mass/unit length of the string (linear mass density).

- **Speed of Propagation: The speed of any mechanical wave depends on both the inertial property of the medium (stores kinetic energy) and the elastic property (stores potential energy).**

$v = \sqrt{\frac{elastic}{inertial}}$ **(wave speed)**

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Example

Example (text problem 11.8): When the tension in a cord is 75.0 N, the wave speed is 140 m/s. What is the linear mass density of the cord?

The speed of a wave on a string is $v = \sqrt{\frac{F}{\mu}}$

Solving for the linear mass density:

$$\mu = \frac{F}{v^2} = \frac{75.0\text{ N}}{(140\text{ m/s})^2} = 3.8\times10^{-3}\text{ kg/m}$$

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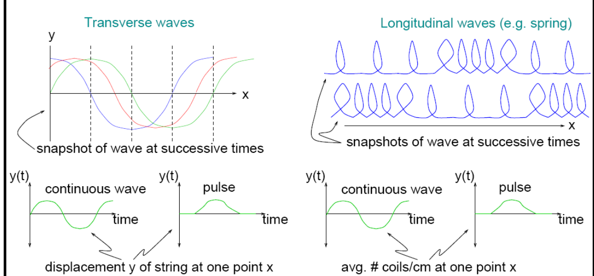
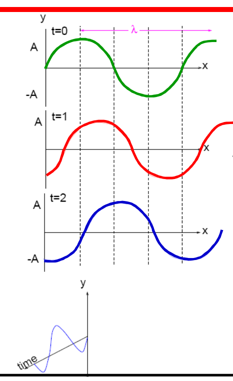
Transverse vs. longitudinal waves



Special case: periodic waves



$y = A\ \sin(kx - \omega t)$

$y(x,0) = A\ \sin\left(2\pi\frac{x}{\lambda}\right)$

$y(0,t) = -A\ \sin(2\pi f t)$

λ: wavelength and f: frequency

Define:

wave number $k = \frac{2\pi}{\lambda}$

angular frequency $\omega = 2\pi f$

$(kx - \omega t)$ is called the phase.

Units

$$y = A\ \sin(kx - \omega t)$$

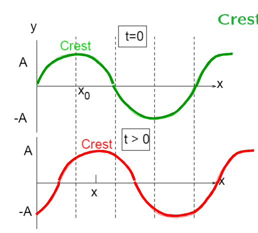
- f: cycles/s or Hertz (audio 20 Hz–20,000 Hz)
- λ: m
- ω: radians/s or s^{-1}
- k: radians/m or m^{-1}
- Also the period: $T = 1/f$: s

$k = \frac{2\pi}{\lambda}$

$\omega = 2\pi f$

Wave speed

$y = A\ \sin(kx - \omega t)$



Crest:

$\sin(\phi) = 1 \Rightarrow \phi = \frac{\pi}{2}$

$kx_0 = \frac{\pi}{2}$

$kx - \omega t = \frac{\pi}{2}$

$kx - \omega t = kx_0$

$\Rightarrow x = x_0 + \frac{\omega}{k}t$

$$v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f$$

General form of a wave

Convince yourself that $\sin(kx+\omega t)$ moves to *left*.
(before we used $\sin(kx-\omega t)$ which moves to *right*)

Generally, any function

$$y(x,t) = f(x \pm vt)$$

is a wave traveling at velocity $\mp v$. Three examples:

$$y_1 = A\sin[k(x-vt)]$$

$$y_2 = B\cos(kx-\omega t)$$

$$y_3 = Ce^{i(kx-\omega t)}$$

Example

Example (text problem 11.21): A wave on a string has an equation:

$$y(x,t) = (4.00\text{ mm})\sin\left((600\text{ rad/sec})t - (6.00\text{ rad/m})x\right)$$

Compare this to

$$y(x,t) = A\sin(\omega t - kx) = -A\sin(kx-\omega t) = A\sin(kx-\omega t+\pi)$$

(a) What is the amplitude of the wave?

A = 4.00 mm

(b) What is the wavelength?

The wave number k is 6.00 rad/m.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.00\text{ rad/m}} = 1.05\text{ m}$$

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Example continued:

(c) What is the period?

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{600\text{ rad/sec}} = 1.05\times10^{-2}\text{ sec}$$

(d) What is the wave speed?

$$v = \lambda f = \left(\frac{\lambda}{2\pi}\right)(2\pi f) = \frac{\omega}{k} = \frac{600\text{ rad/sec}}{6.00\text{ rad/m}} = 100\text{ m/s}$$

(e) What direction is the wave traveling.

Along the +x direction.

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Example continued (SHM):

(f) What is the maximum speed of a point on the string, take x=0

$$y(0,t) = (4.00\text{ mm})\sin\left((600\text{ rad/sec})t\right) = A\sin\omega t \qquad \text{SHM}$$

$$v_{max} = A\omega = 4.00\text{mm}\times600\text{ rad/sec} = 2.4\text{ m/ sec}$$

(g) What is maximum acceleration of a point on the string?

$$a_{max} = A\omega^2 = 1440\text{ m/sec}^2$$

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