Ch. 11: waves

- Last time
- Superposition of waves
- Construction or Destruction
- Reflection and Refraction
- Demos
- Examples
- HITT quiz

Wave Summary

General form of travelling wave: $y(x,t) = f(x \pm vt)$

y = displacement of medium { perpendicular to propagation $\Rightarrow$ transverse; parallel to propagation $\Rightarrow$ longitudinal

Special case: periodic wave $y = A\,\sin(kx - \omega t)$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

Wave speed depends on properties of medium, e.g. $v = \sqrt{\frac{T}{\mu}}$

Intensity I ~ A^2

Superposition of waves

Medium must be "linear" (Hooke's law–type restoring force).

Then

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

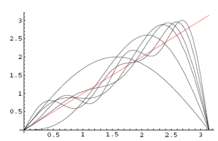
Add displacements

Fourier's theorem (for general culture only!!)

- Complex waveform built of superposition of sine waves.

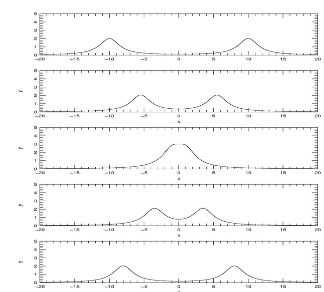
First 6 terms in Fourier Series for $y = x$:

$$x = 2\sin x - \sin 2x + \frac{2\sin 3x}{3} - \frac{\sin 4x}{2} + \frac{2\sin 5x}{5} - \frac{2\sin 6x}{6} + \ldots$$



The Principle of Superposition

Superposition Principle: When two or more waves overlap, the net disturbance at any point is the sum of the individual disturbances due to each wave.



Two traveling wave pulses: left pulse travels right; right pulse travels left.

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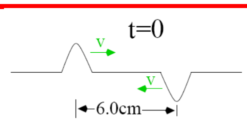
Example



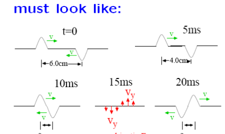
Two pulses travel along a string in opposite directions as shown. Speed of pulses = 2.0 m/s, at t=0 they are 6.0 cm apart.

a) Sketch string at t=5,10,15,20 ms.

b) What has happened to energy of pulse at 15 ms?

Example: soln

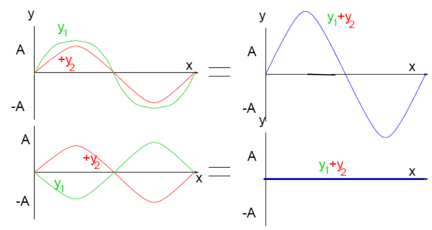
A: a) Pulses travel (2.0m/s)(5 $\times 10^{-3}$s)=1 cm in each 5 ms. Therefore after 5 ms the two pulses are 2 cm closer to each other, and are located at the same point at 15 ms. At this time they cancel exactly and then "pass through" and reappear on the other side at 20 ms. The sequence must look like:



b) Note at 15 ms the string is straight (no string elements are displaced from equilibrium positions $\Rightarrow$ no potential energy), but kinetic energy is maximum. Think of string as sequence of coupled vertical oscillators–at 15 ms all are at equilibrium point with max. velocity.

Superposition of waves cont'd

Superposition of waves of the same frequency (and wavelength)



$$y(x,t) = y_1(x,t) + y_2(x,t)$$

Superposition of waves with same frequency, amplitude and wavelength but different *phase*

$$y(x,t) = A[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)]$$

Use a trig identity:

$$\sin(\alpha)+\sin(\beta) = 2\sin\left(\frac{1}{2}[\alpha+\beta]\right)\cos\left(\frac{1}{2}[\alpha-\beta]\right)$$

so

$$y(x,t) = 2A\left\{\sin\left(\frac{1}{2}[2kx-2\omega t+\phi]\right)\cos\left(\frac{1}{2}[kx-\omega t-kx+\omega t-\phi]\right)\right\}$$

$$= 2A\cos\left(\frac{-\phi}{2}\right)\sin\left(kx-\omega t+\frac{1}{2}\phi\right)$$

$$\boxed{y(x,t) = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{1}{2}\phi\right)}$$

Superposition of waves with same frequency and wavelength but different *phase*

Result depends on value of ϕ. If $\phi = 0$ [in-phase: $\cos(\frac{\phi}{2}) = 1$]

$$y(x,t) = 2A\sin(kx-\omega t)$$

This is constructive interference.

If $\phi = \pi$ [180° out-of-phase: $\cos(\frac{\phi}{2}) = 0$]

$$y(x,t) = 0$$

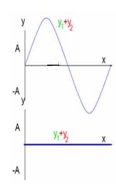
This is destructive interference

If $\phi = \frac{\pi}{2}$ (partially in, partially out)

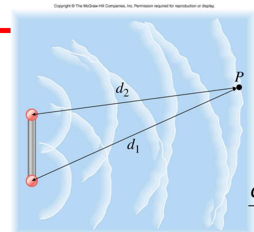
$$\cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$y(x,t) = \frac{2}{\sqrt{2}}A\sin(kx-\omega t+\frac{\pi}{4})$$

$$\frac{2}{\sqrt{2}}A = \sqrt{2}A = 1.414A$$



Interference



When two waves travel different distances to reach the same point, the phase difference is determined by:

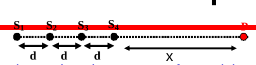
$$\frac{d_1-d_2}{\lambda} = \frac{\text{phase difference}}{2\pi} = \frac{\Delta\phi}{2\pi}$$

$$\text{phase difference } \Delta\phi = \frac{2\pi(d_1-d_2)}{\lambda} = k(d_1-d_2)$$

$$\Delta\phi = k\Delta d = 2\pi n, \text{ Constructive}$$
$$\Delta d = \frac{2\pi n}{k} = n\lambda, \text{ n} = 0, \pm1, \pm2, \cdots$$

$$\Delta\phi = k\Delta d = 2\pi(n+\frac{1}{2}), \text{ Destructive}$$
$$\Delta d = \frac{2\pi}{k}(n+\frac{1}{2}) = (n+\frac{1}{2})\lambda, \text{ n} = 0, \pm1, \pm2, \cdots$$

Example Problem: Superposition



- The figure shows four isotropic point sources of sound that are uniformly spaced on the x-axis. The sources emit sound at the same wavelength λ and the same amplitude A, and they emit in phase. A point P is shown on the x-axis. Assume that as the sound waves travel to the point P, the decrease in their amplitude is negligible. What is the amplitude of the net wave at P if d = λ/4?

Answer: Zero

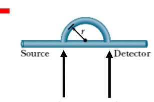
$$d_1 = x+3d$$
$$d_2 = x+2d$$
$$d_3 = x+d$$
$$d_4 = x$$

$$\Delta\phi_{13} = k\Delta d_{13} = \frac{2\pi\Delta d_{13}}{\lambda} = \frac{2\pi 2d}{\lambda} = \frac{2\pi(\lambda/2)}{\lambda} = \pi$$ Destructive

$$\Delta\phi_{24} = k\Delta d_{24} = \frac{2\pi\Delta d_{24}}{\lambda} = \frac{2\pi 2d}{\lambda} = \frac{2\pi(\lambda/2)}{\lambda} = \pi$$ Destructive

Example Problem: Superposition

- Sound with a 40 cm wavelength travels rightward from a source and through a tube that consists of a straight portion and a half-circle as shown in the figure. Part of the sound wave travels through the half-circle and then rejoins the rest of the wave, which goes directly through the straight portion. This rejoining results in interference. What is the smallest radius r that results in an intensity minimum at the detector?



Answer: 17.5 cm

At point A the waves have the same amplitude, wavelength, and frequency and are in phase.

Wave 1 travels a distance $d_1 = 2r$ to reach the point B, while wave 2 travels a distance $d_2 = \pi r$ to reach the point B.

$$\Delta d = d_2 - d_1 = (\pi-2)r = (n+\tfrac{1}{2})\lambda \quad n = 0, \pm1, \pm2, \cdots$$ Destructive

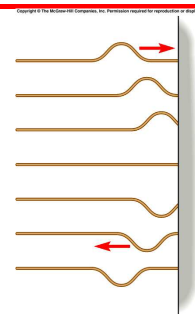
$$r = \frac{(n+\frac{1}{2})\lambda}{(\pi-2)} \xrightarrow{\min} \frac{\lambda}{2(\pi-2)} = \frac{(40cm)}{2(\pi-2)} \approx 17.5cm$$

Reflection and Refraction

At an abrupt boundary between two media, a reflection will occur. A portion of the incident wave will be reflected backward from the boundary.

Reflection

When you have a wave that travels from a "low density" medium to a "high density" medium, the reflected wave pulse will be inverted.



The frequency of the reflected wave remains the same.

Traveling Waves: Refraction

- Refraction: Zero Incident Angle

(speed of propagation is the frequency times the wavelength)

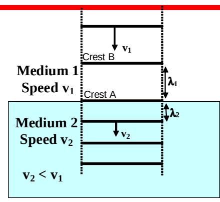
$$v_1 = f_1\lambda_1$$
$$v_2 = f_2\lambda_2$$

$$\Delta t = \frac{\lambda_1}{v_1}$$ (time for crest B to reach medium 2)

$$\lambda_2 = v_2\Delta t = \frac{v_2\lambda_1}{v_1}$$ (distance traveled by crest A in time Δt)

$$\frac{\lambda_2}{v_2} = \frac{\lambda_1}{v_1}$$ (Law of Refraction)

$$f_1 = f_2$$ (frequency is the same)



Traveling Waves: Refraction

- Refraction: Incident Angle θ_1

(speed of propagation is the frequency times the wavelength)

$$v_1 = f\lambda_1$$
$$v_2 = f\lambda_2$$
$$f_1 = f_2$$

$$\sin\theta_1 = \frac{\lambda_1}{L}$$ (incident angle)

$$\sin\theta_2 = \frac{\lambda_2}{L}$$ (refracted angle)

$$\frac{\sin\theta_1}{\lambda_1} = \frac{\sin\theta_2}{\lambda_2}$$ (Law of Refraction)

$$\frac{\lambda_2}{v_2} = \frac{\lambda_1}{v_1}$$ (Law of Refraction)

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$ (Law of Refraction)