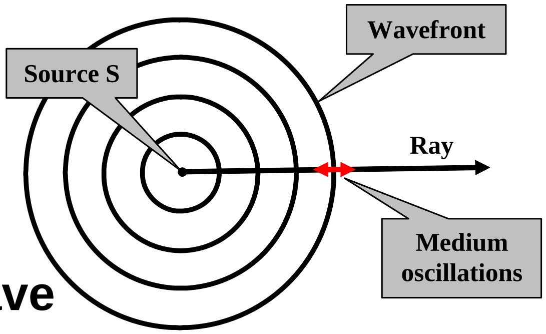
# Sound: Propagation




- **Sound Waves:** Sound waves are longitudinal waves (*i.e.* involve oscillations parallel to the direction of the wave travel) that propagate through a medium (*e.g.* air, water, iron).
- **Speed of Sound:** The speed of any mechanical wave depends on both the inertial property of the medium (stores kinetic energy) and the elastic property (stores potential energy).

$$v = \sqrt{\frac{elastic}{inertial}}$$ (wave speed)

- **Stretched String (Chapter 11):** The speed of the "transverse" wave along a stretched string is

$$v = \sqrt{\frac{\tau}{\mu}}$$ (string, $\tau$ = tension, $\mu$ = linear mass density)

- **Sound:** The speed of the "longitudinal" sound wave is

$$v = \sqrt{\frac{B}{\rho}}$$ (sound, B = bulk modulus = $-\Delta P/(\Delta V/V)$, $\rho$ = volume mass density)

| Medium | Sound Speed (m/s) | Sound Speed (mi/hr) |
|---|---|---|
| Air ($20^oC$) | 343 | 768 |
| Water ($20^oC$) | 1,482 | 3,320 |
| Steel | 5,941 | 13,308 |

# Sound: Speed

- **Speed of Sound:** In Fluids

$$v = \sqrt{\frac{B}{\rho}}$$

(sound, B = bulk modulus = $-\Delta P/(\Delta V/V)$, $\rho$ = volume mass density)

- **Speed of Sound:** In Air (B proportional to absolute T)

$$v(T) = v_0\sqrt{\frac{T}{T_0}}$$

where T is measured in degrees Kelvin with $T_0$ = 273.15 °K and $v_0$ = 331 m/s.

T(in °K) = T(in °C) + 273.15

$$T(^\circ F) = (1.8^\circ F/^\circ C)T(^\circ C) + 32^\circ F$$

(degrees Fahrenheit)

| Medium | Sound Speed (m/s) | Sound Speed (mi/hr) |
|---|---|---|
| Air (20°C) | 343 | 768 |
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- **Speed of Sound:** In Solids

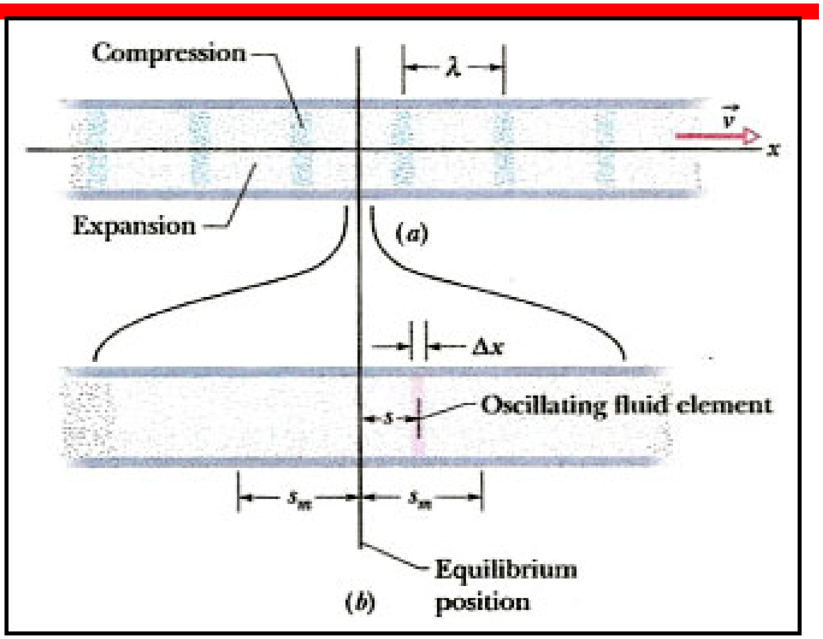
$$v = \sqrt{\frac{Y}{\rho}}$$

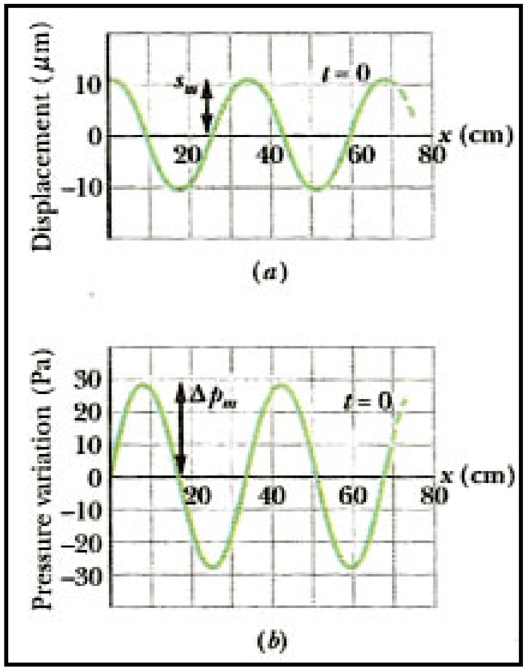
(sound, Y = young's modulus, $\rho$ = volume mass density)

# Sound: Traveling Waves

- **Traveling Sound Waves in Air: A traveling sound waves consist of a moving periodic pattern of expansions and compressions of the air. As the wave passes the air elements oscillate longitudinally in simple harmonic motion.**

**(1000 Hz sound wave with $\Delta P_{max}$ at the threshold of pain!)**




**Longitudinal displacement**

$$s(x,t) = s_{max}\cos(kx - \omega t)$$

**Pressure variation**

$$\Delta P(x,t) = \Delta P_{max}\sin(kx - \omega t)$$

$$\Delta P_{max} = (v\rho\omega)s_{max}$$

**(The pressure amplitude is related to the displacement amplitude!)**

# Sound: Intensity and Level

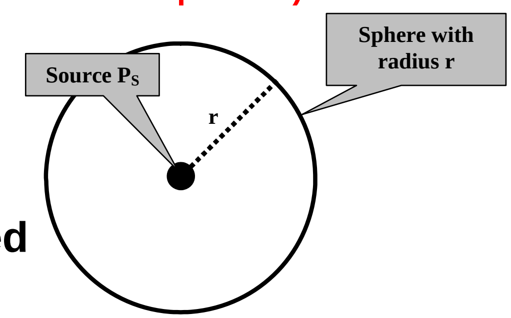
- **Intensity:** Traveling sound waves transport energy (kinetic and potential) from one point to another. The intensity, I, of a sound wave at a surface is the average energy per unit time per unit area transmitted by the wave to the surface (*i.e.* average power per unit area). It is also equal to the average energy per unit volume in the wave times the speed of propagation of the wave.

$$I = \frac{1}{A}\frac{d\bar{E}}{dt} = \frac{\bar{P}_{power}}{A}$$

Watts/m$^2$

$$I = \frac{d\bar{E}}{dV}v = \tfrac{1}{2}\rho v\omega^2 s_{max}^2 = \tfrac{1}{2}\frac{\Delta P_{max}^2}{\rho v}$$

(the intensity is proportional to the square of the amplitude!)




- **Variation with Distance:** If sound is emitted isotropically (*i.e.* equal intensity in all directions) from a point source with power $P_s$ and if the mechanical energy of the wave is conserved then

$$I = \frac{P_s}{4\pi r^2}$$

(intensity from isotropic point source)

| Sound | level (dB) | Intensity (W/m$^2$) |
|---|---|---|
| Hearing threshold | 0 | $10^{-12}$ |
| Conversation | 60 | $10^{-6}$ |
| Pain threshold | 120 | 1 |

- **The Decibel Scale:** Instead of speaking about the intensity I of sound, it is more convenient to speak of the sound level $\beta$, where

$$\beta = (10dB)\log_{10}(I/I_0)$$

$$I = I_0 \times 10^{\beta/10dB}$$

(sound level, dB = "decibel", $I_0 = 10^{-12}$ W/m$^2$)

# Sound Waves: Example Problem

- **At a baseball game, a spectator is 60.0 m away from the batter. How long does it take the sound of the bat connecting with the ball to travel to the spectator's ears? The air temperature is 27.0 $^oC$.** **Answer: 172.9 ms**

**The speed of sound in the air depends on the temperature as follows:**

$$v(T) = v_0\sqrt{\frac{T}{T_0}}$$

**where T is measured in degrees Kelvin with $T_0 = 273.15$ $^oK$ and $v_0 = 331$ m/s. Also, T($^o$K) = T($^o$C) + 273.15. Hence,**

$$v(27^\circ C) = (331 m/s)\sqrt{\frac{300.15}{273.15}} \approx 347.0 m/s$$

**Note that 27$^o$C is about 81$^o$F.**

$$t = \frac{d}{v(27^\circ C)} = \frac{60m}{347.0 m/s} \approx 0.173 s = 172.9 ms$$

# Sound Waves: Example Problem

- **Stan and Ollie are standing next to a train track. Stan puts his ear to the steel track to hear the train coming. He hears the sound of the train whistle through the track 2.1 s before Ollie hears it through the air. How far away is the train? Take the speed of sound in air and steel to be 343 m/s and 5790 m/s, respectively.** **Answer: 765.7 m**

$$\Delta t = t_{air} - t_{steel} = \frac{d}{v_{air}} - \frac{d}{v_{steel}} = d\left(\frac{1}{v_{air}} - \frac{1}{v_{steel}}\right) = d\,\frac{v_{steel} - v_{air}}{v_{air}v_{steel}}$$

$$d = \Delta t\,\frac{v_{air}v_{steel}}{v_{steel} - v_{air}} = (2.1s)\frac{(343m/s)(5790m/s)}{(5790m/s) - (343m/s)} \approx 765.7m$$

# Sound Waves: Example Problem

- You drop a stone into a deep well and hear it hit the bottom 3.2 s later. How deep is the well? Take the speed of sound to be 343 m/s.

**Answer: 46.05 m**

$$t_{tot} = t_{drop} + t_{sound} = \sqrt{\frac{2d}{g}} + \frac{d}{v_{sound}}$$

$$\frac{2d}{g} = \left(t_{tot} - \frac{d}{v_{sound}}\right)^2 = t_{tot}^2 - 2\frac{dt_{tot}}{v_{sound}} + \frac{d^2}{v_{sound}^2} \longrightarrow d^2 - \left(2t_{tot}v_{sound} + \frac{2v_{sound}^2}{g}\right)d + v_{sound}^2 t_{tot}^2 = 0$$

$$d = v_{sound}\left[\left(t_{tot} + \frac{v_{sound}}{g}\right) \pm \sqrt{\frac{2t_{tot}v_{sound}}{g} + \frac{v_{sound}^2}{g^2}}\right]$$

$$= (343m/s)\left[\left(3.2s + \frac{343m/s}{9.8m/s^2}\right) \pm \sqrt{\frac{2(3.2s)(343m/s)}{9.8m/s^2} + \frac{(343m/s)^2}{(9.8m/s^2)^2}}\right]$$

$$= (343m/s)[(38.2s) \pm (38.06573s)] = \begin{matrix} 26{,}159m \\ 46.05m \end{matrix}$$

# **Sound Waves:** Example Problem

- **The sound level 25 m from a loudspeaker is 71 dB. What is the rate at which sound energy is produced by the loudspeaker, assuming it to be an isotropic source?** **Answer: 98.9 mW**

$$\beta = (10dB)\log_{10}(I/I_0)$$

**where $I_0 = 10^{-12}$ W/m$^2$. Hence,**

$$I = I_0 \times 10^{\beta/10dB}$$

$$P = 4\pi r^2 I = 4\pi r^2 I_0 10^{\beta/10dB} = 4\pi(25m)^2(10^{-12}W/m^2)10^{7.1} \approx 98.9mW$$