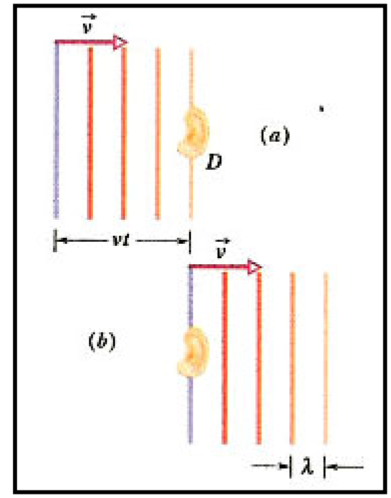
Sound Waves: Doppler Effect

- **Doppler Shift: If either the detector or the source of sound is moving, or if both are moving, then the emitted frequency, f_s, of the source and the detected frequency, f_{obs}, are different. If both the source and the detector are at rest with respect to the air and $v_{sound} = \lambda_s f_s$ is the speed of sound in air then**

$$f_{obs} = \frac{N_{pulse}}{t} = \frac{v_{sound} t / \lambda_s}{t} = \frac{v_{sound}}{\lambda_s} = f_s$$

(source stationary, detector stationary)

(Source and detector at rest)

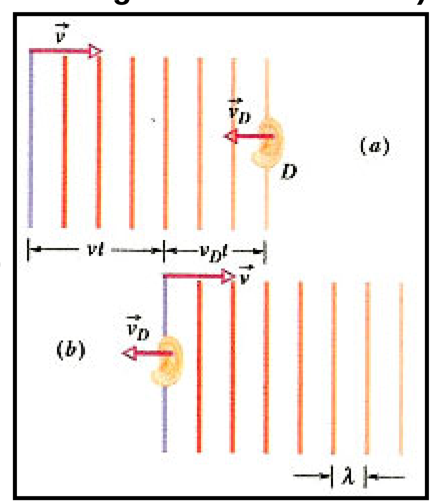


- **Detector Moving, Source Stationary: If source is at rest and the detector is moving toward the source with speed v_D with respect to the air and $v = \lambda_s f_s$ is the speed of sound in air then**

$$f_{obs} = \frac{N_{pulse}}{t} = \frac{(v_{sound} t + v_D t)/\lambda_s}{t} = \frac{v_{sound} + v_D}{\lambda_s} = \frac{v_{sound} + v_D}{v_{sound} / f_s} = \frac{v_{sound} + v_D}{v_{sound}} f_s$$

(source stationary, detector moving toward the source)

(Source at rest, Detector moving toward the source)



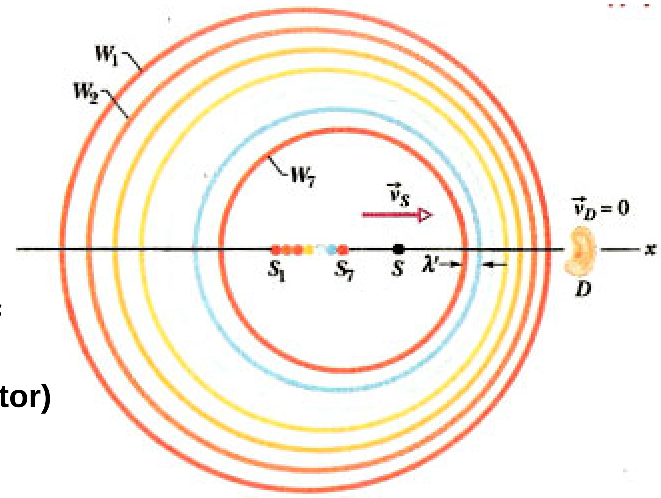
$$f_{obs} = \frac{v_{sound} \pm v_D}{v_{sound}} f_s$$

(source stationary, detector moving toward (+) or away (-) from the source)

Sound Waves: Doppler Effect

- **Detector Stationary, Source Moving:** If detector is at rest and the source is moving toward the detector with speed v_s with respect to the air and $v_{sound} = \lambda_s f_s$ is the speed of sound in air and $T_s = 1/f_s$ is the time between emitted pulses then

(detector stationary, source moving toward the detector)



$$f_{obs} = \frac{v_{sound}}{\lambda_{obs}} = \frac{v_{sound}}{v_{sound}T_s - v_s T_s} = \frac{v_{sound}}{v_{sound}/f_s - v_s/f_s} = \frac{v_{sound}}{v_{sound} - v_s} f_s$$

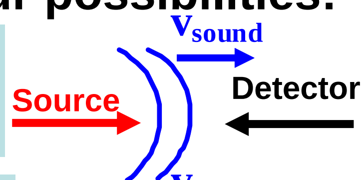
(detector stationary, source moving toward the detector)

$$f_{obs} = \frac{v_{sound}}{v_{sound} \mp v_s} f_s$$

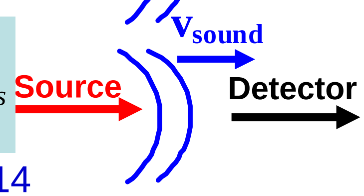
(detector stationary, source moving toward (-) or away (+) from the source)

- **Detector and Source Moving:** If detector is moving with speed v_D and the source is moving with speed v_s with respect to the air then there are the following four possibilities:

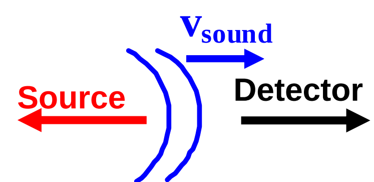
$$f_{obs} = \frac{v_{sound} + v_D}{v_{sound} - v_s} f_s$$



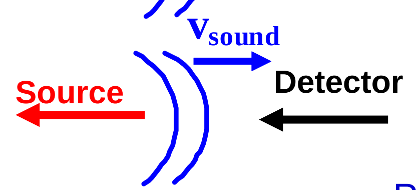
$$f_{obs} = \frac{v_{sound} - v_D}{v_{sound} - v_s} f_s$$



$$f_{obs} = \frac{v_{sound} - v_D}{v_{sound} + v_s} f_s$$



$$f_{obs} = \frac{v_{sound} + v_D}{v_{sound} + v_s} f_s$$



Summary: Doppler Effect

- **Detector and Source Moving:** If detector is moving with speed v_D and the source is moving with speed v_s, and $v_{sound} = \lambda_s f_s$ is the speed of sound in air then the observed frequency at the detector is

$$f_{obs} = \frac{v_{sound} - v_D}{v_{sound} - v_s} f_s$$

Take v_D positive if the detector is moving in the direction of the propagation of the sound wave.

Take v_D negative if the detector is moving opposite the direction of propagation of the sound wave.

Take v_s positive if the source is moving in the direction of propagation of the sound wave.

Take v_S negative if the source is moving opposite the direction of propagation of the sound wave.

Doppler Effect: Examples

- **A low flying airplane skims the ground at a speed of 200 m/s as it approaches a stationary observer. A loud horn whose fundamental frequency is 400 Hz is carried on the plane. What frequency does the ground observer hear? (Assume that the speed of sound in the air is 343 m/s.)** **Answer: 959 Hz**

$$f_{obs} = \frac{v_{sound}}{v_{sound} - v_s} f_s = \frac{1}{1 - v_s / v_{sound}} f_s = \frac{400Hz}{1 - (200/343)} \approx 959Hz$$

- **If instead the horn were on the ground, what frequency would the airplane pilot hear as she approached?** **Answer: 633 Hz**

$$f_{obs} = \frac{v_{sound} + v_D}{v_{sound}} f_s = (1 + v_D / v_{sound}) f_s$$

$$= (1 + 200/343)(400Hz) \approx 633Hz$$

Doppler Effect: Example

- **The pitch of the sound from a race car engine drops the musical interval of a fourth when it passes the spectators. This means the frequency of the sound after passing is 0.75 times what it was before. How fast is the race can moving? (Assume that the speed of sound in the air is 343 m/s.)**

Answer: 49.0 m/s

$$f_{toward} = \frac{v_{sound}}{v_{sound} - v_s} f_s \qquad f_{away} = \frac{v_{sound}}{v_{sound} + v_s} f_s$$

$$\xi = \frac{f_{away}}{f_{toward}} = \frac{v_{sound} - v_s}{v_{sound} + v_s}$$

$$v_s = \left(\frac{1-\xi}{1+\xi}\right) v_{sound} = \left(\frac{1-0.75}{1+0.75}\right)(343 m/s) \approx 49.0 m/s$$

Doppler Effect: Example

- **A stationary motion detector sends sound waves of frequency of 600 Hz toward a truck that is speeding away. The waves sent out by the detector are reflected off the truck and then are received back at the detector. If the frequency of the waves received back at the detector is 400 Hz, what is the speed of the receding truck (in m/s)? (Take the speed of sound to be 343 m/s.)** **Answer: 68.6 m/s**

Part 1: The truck is the detector and f_{truck} is the frequency observed by the truck and f_0 is the original frequency emitted by the motion detector which is the source.

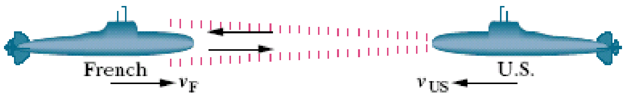
$$f_{truck} = \frac{v_{sound} - v_{truck}}{v_{sound}} f_0$$

Part 2: The truck is now the source emitting frequency f_{truck} and the motion detector is the detector which observes frequency f_{obs}.

$$f_{obs} = \frac{v_{sound}}{v_{sound} + v_{truck}} f_{truck} = \frac{v_{sound} - v_{truck}}{v_{sound} + v_{truck}} f_0 \qquad \frac{f_{obs}}{f_0} = \frac{v_{sound} - v_{truck}}{v_{sound} + v_{truck}}$$

$$v_{truck} = \left(\frac{f_0 - f_{obs}}{f_0 + f_{obs}}\right) v_{sound} = \left(\frac{600Hz - 400Hz}{600Hz + 400Hz}\right)(343m/s) = 68.6m/s$$

Doppler Effect: Example



- **In the figure, a French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic. The French sub moves at speed v_F = 100 km/h, and the U.S. sub at v_{US} = 200 km/h. The French sub sends out a sonar signal (sound wave in water) at 1,000 Hz. Sonar waves travel at 5000 km/h. What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?** **Answer: 1,127.6 Hz**

Part 1: The US sub is the detector and f_{US} is the frequency observed by the US sub and f_0 is the original frequency emitted by the French sub which is the source.

$$f_{US} = \frac{v_{sound} + v_{US}}{v_{sound} - v_F} f_0$$

Part 2: The US sub is now the source emitting frequency f_{US} and the French sub is the detector which observes frequency f_{obs}.

$$f_{obs} = \frac{v_{sound} + v_F}{v_{sound} - v_{US}} f_{US} = \frac{(v_{sound} + v_F)(v_{sound} + v_{US})}{(v_{sound} - v_{US})(v_{sound} - v_F)} f_0$$

$$= \frac{(5000km/h + 100km/h)(5000km/h + 200km/h)}{(5000km/h - 200km/h)(5000km/h - 100km/h)}(1000Hz) = 1127.6Hz$$