

Final Exam: 4/26/14 8:00pm-10:00pm

- **Room Assignments:**

Last Name	Room
A-G	CLB C130
H-R	NRN 137
S-U	FLG 280
V-Z	MAT 18

- **Breakdown of the 20 Problems**

Material	# of Problems
Your Exam 1	4
Your Exam 2	4
Chapter 11.1-12.8	4
Chapter 1.1-10.8	8

- **Crib Sheet:** You may bring **two** hand written formula sheet on 8½ x 11 inch paper (both sides).
- **Calculator:** You should bring a calculator (any type).
- **Scratch Paper:** We will provide scratch paper.

Make-Up Exam: 4/23/14 5:10-7:00pm

For anyone who had to miss an exam during the semester.

- **Room:** MAEA 327.
- **Breakdown of the 20 Problems:** Similar to the Final Exam.
- **Crib Sheet:** You may bring **two** hand written formula sheet on 8½ x 11 inch paper (both sides).
- **Calculator:** You should bring a calculator (any type).
- **Scratch Paper:** We will provide scratch paper.

Final Exam Fall 2010: Problem 9

- Near the surface of the Earth a startled armadillo leaps vertically upward at time $t = 0$, at time $t = 0.5$ s it is a height of 0.98 m above the ground. At what time does it land back on the ground?

Answer: 0.9 s

% Right: 47%

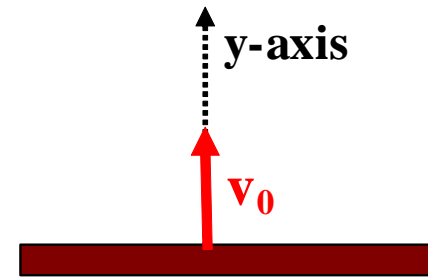
$$y(t) = v_o t - \frac{1}{2} g t^2 = t(v_o - \frac{1}{2} g t)$$

$$y(t_f) = 0 = t_f (v_o - \frac{1}{2} g t_f)$$

$$t_f = \frac{2v_o}{g}$$

$$y(t_h) = h = v_o t_h - \frac{1}{2} g t_h^2 \quad \rightarrow \quad v_o = \frac{h + \frac{1}{2} g t_h^2}{t_h}$$

$$t_f = \frac{2v_o}{g} = \frac{2(h + \frac{1}{2} g t_h^2)}{g t_h} = \frac{2h}{g t_h} + t_h = \frac{2(0.98\text{m})}{(9.8\text{m/s}^2)(0.5\text{s})} + (0.5\text{s}) = (0.4\text{s}) + (0.5\text{s}) = 0.9\text{s}$$



Final Exam Fall 2011: Problem 7

- Near the surface of the Earth a suspension bridge is a height H above the level base of a gorge. Two identical stones are simultaneously thrown from the bridge. One stone is thrown straight down with speed v_0 and the other is thrown straight up at the same speed v_0 . Ignore air resistance. If one of the stones lands at the bottom of the gorge 2 seconds before the other, what is the speed v_0 (in m/s)?

Answer: 9.8

$$y_{up}(t) = H + v_0 t - \frac{1}{2} g t^2$$

$$y_{down}(t) = H - v_0 t - \frac{1}{2} g t^2$$

% Right: 36%

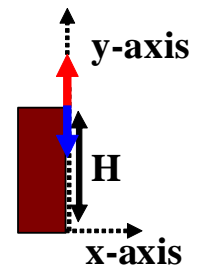
$$y_{up}(t_{up}) = 0 = H + v_0 t_{up} - \frac{1}{2} g t_{up}^2 \quad y_{down}(t_{down}) = 0 = H - v_0 t_{down} - \frac{1}{2} g t_{down}^2$$

$$v_0 t_{up} = \frac{1}{2} g t_{up}^2 - H$$

$$v_0 t_{down} = -\frac{1}{2} g t_{down}^2 + H$$

$$v_0 (t_{up} + t_{down}) = \frac{1}{2} g (t_{up}^2 - t_{down}^2)$$

$$v_0 = \frac{1}{2} g \frac{(t_{up}^2 - t_{down}^2)}{(t_{up} + t_{down})} = \frac{1}{2} g (t_{up} - t_{down}) = \frac{1}{2} (9.8 \text{ m/s}^2)(2 \text{ s}) = 9.8 \text{ m/s}$$



Exam 1 Fall 2010: Problem 12

- Near the surface of the Earth, a block of mass M is at rest on a plane inclined at angle θ to the horizontal. If the coefficient of static friction between the block and the surface of the plane is 0.7, what is the largest angle θ without the block sliding?

Answer: 35°
% Right: 83%

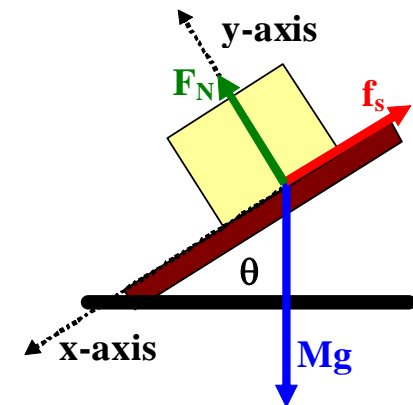
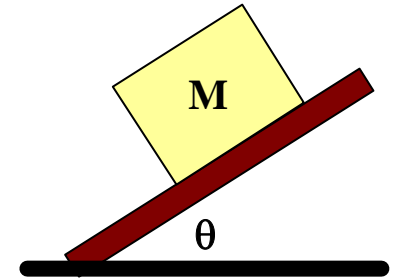
$$Mg \sin \theta - f_s = Ma_x = 0$$

$$F_N - Mg \cos \theta = Ma_y = 0$$

$$Mg \sin \theta = f_s = \mu_s F_N = \mu_s Mg \cos \theta$$

$$\tan \theta = \mu_s = 0.7$$

$$\theta \approx 35^\circ$$



Exam 1 Spring 2014: Problem 22

- In the figure, blocks A and C have a mass of 30 kg and 10 kg, respectively. If the surface of the table is frictionless and the static coefficient of friction, μ_s , between block A and block C is 0.60, what is the maximum mass of block B (in kg) such that, when the system is released from rest, block C will not slip off block A?

Answer: 60

% Right: 19%

$$f_s = M_C a$$

$$M_C a_{\max} = (f_s)_{\max} = \mu_s F_N = \mu_s M_C g$$

$$a_{\max} = \mu_s g$$

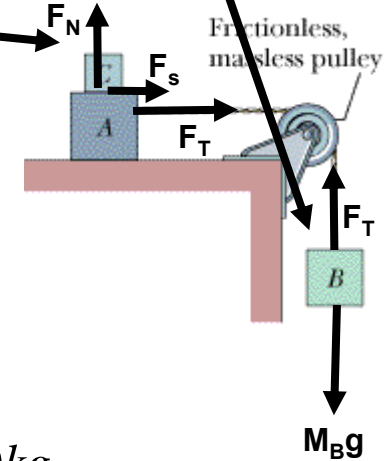
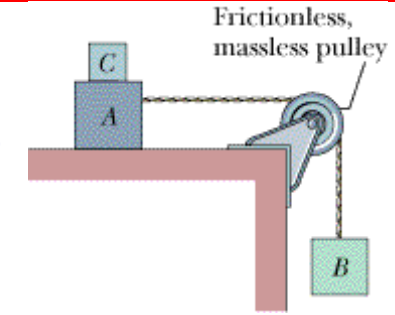
$$\mu_s = \frac{M_B}{M_A + M_B + M_C}$$

$$M_B g = (M_A + M_B + M_C) a$$

$$a = \frac{M_B}{M_A + M_B + M_C} g$$

$$\mu_s (M_A + M_B + M_C) = M_B$$

$$M_B = \frac{\mu_s}{1 - \mu_s} (M_A + M_C) = \frac{0.6}{1 - 0.6} (30\text{kg} + 10\text{kg}) = \frac{3}{2} (40\text{kg}) = 60\text{kg}$$



Exam 1 Spring 2014: Problem 55

- A race car starts from rest at $t = 0$ and travels around a circular track of radius R with a constant angular acceleration. If the race car completes its first revolution around the track in 3 minutes, at what time t (in s) is the magnitude of the tangential acceleration of the car equal to the magnitude of the radial acceleration (i.e. centripetal acceleration) of the car?

Answer: 50.8
% Right: 12%

$$\alpha(t) = \alpha$$

$$\omega(t) = \alpha t$$

$$\theta(t) = \frac{1}{2} \alpha t^2$$

$$a_{\text{radial}}(t) = R\omega^2(t) = R\alpha^2 t^2$$

$$a_t(t) = R\alpha$$

$$a_{\text{radial}}(t) = R\alpha^2 t^2 = a_t = R\alpha$$

$$t = \frac{1}{\sqrt{\alpha}} = \frac{t_1}{\sqrt{4\pi N(t_1)}} = \frac{(3 \text{ min})(60 \text{ s/min})}{\sqrt{4\pi(1)}} \approx 50.8 \text{ s}$$

$$N(t_1) = \frac{1}{2} \left(\frac{\alpha}{2\pi} \right) t_1^2$$

Let t_1 be the time for the first revolution, $N(t_1) = 1$.

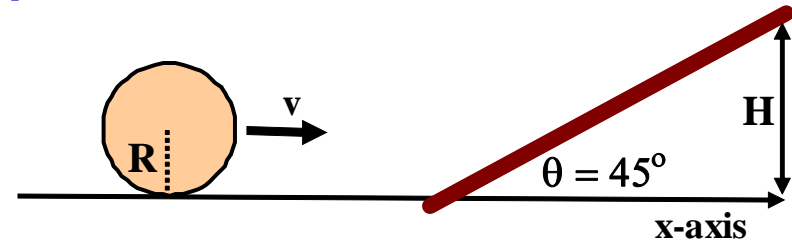
$$\alpha = \frac{4\pi N(t_1)}{t_1^2}$$

Final Exam Spring 2011: Problem 12

- A uniform disk with mass M , radius $R = 0.50$ m, and moment of inertia $I = MR^2/2$ rolls without slipping along the floor at 2 revolutions per second when it encounters a long ramp angled upwards at 45° with respect to the horizontal. How high above its original level will the center of the disk get (in meters)?

Answer: 3.0

% Right: 39%



$$E_i = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + MgR = E_f = Mg(H + R)$$

$$v = R\omega$$

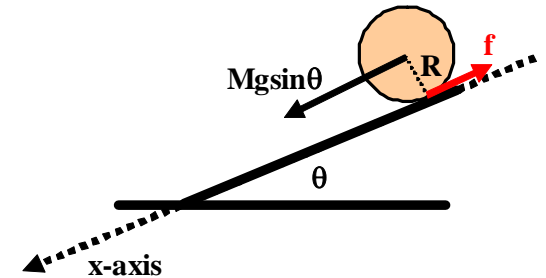
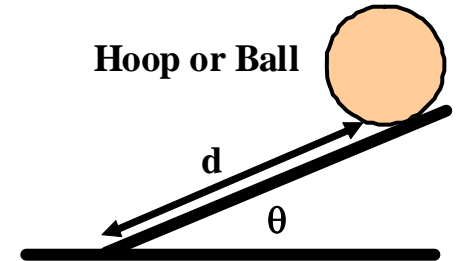
$$\omega = 2\pi f$$

$$H = \frac{1}{2g} \left(v^2 + \frac{I}{M} \omega^2 \right) = \frac{1}{2g} \left(R^2 \omega^2 + \frac{I}{M} \omega^2 \right)$$

$$= \frac{R^2}{2g} \left(1 + \frac{I}{MR^2} \right) \omega^2 = \frac{2\pi^2 R^2}{g} \left(1 + \frac{I}{MR^2} \right) f^2 = \frac{2\pi^2 (0.5m)^2}{9.8m/s^2} \left(\frac{3}{2} \right) (2/s)^2 \approx 3.0m$$

Final Exam Spring 2012: Problem 40

- A thin hoop ($I = MR^2$) and a solid spherical ball ($I = \frac{2}{5}MR^2$) start from rest and roll without slipping a distance d down an incline ramp as shown in the figure. If it takes 2 s for the ball to roll down the incline, how long does it take for the hoop (in s)?



Answer: 2.39
% Right: 53%

$$Mg \sin \theta - f = Ma_x$$

$$v = R\omega$$

$$\tau = I\alpha = \frac{I}{R}a_x = Rf$$

$$\alpha = Ra_x$$

$$Mg \sin \theta - \frac{I}{R^2}a_x = Ma_x \rightarrow a_x = \frac{g \sin \theta}{\left(1 + \frac{I}{MR^2}\right)}$$

$$d = \frac{1}{2}a_x t^2 \rightarrow t = \sqrt{\frac{2d}{a_x}}$$

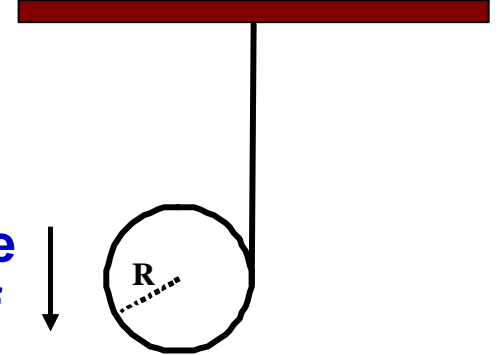
$$\frac{a_x(\text{hoop})}{a_x(\text{ball})} = \frac{\left(1 + \frac{I_{\text{ball}}}{MR^2}\right)}{\left(1 + \frac{I_{\text{hoop}}}{MR^2}\right)} = \frac{\left(1 + \frac{2}{5}\right)}{(1+1)} = \frac{7}{10} = 0.7$$

$$\frac{t(\text{hoop})}{t(\text{ball})} = \sqrt{\frac{a_x(\text{ball})}{a_x(\text{hoop})}} = \frac{1}{\sqrt{0.7}} = 1.1952$$

$$t(\text{hoop}) = 1.1952 \times t(\text{ball}) = 1.1952 \times (2\text{s}) \approx 2.39\text{s}$$

Exam 2 Fall 2011: Problem 17

- A cloth tape is wound around the outside of a non-uniform solid cylinder (mass M , radius R) and fastened to the ceiling as shown in the figure. The cylinder is held with the tape vertical and then released from rest. If the acceleration of the center-of-mass of the cylinder is $3g/5$, what is its moment of inertia about its symmetry axis?



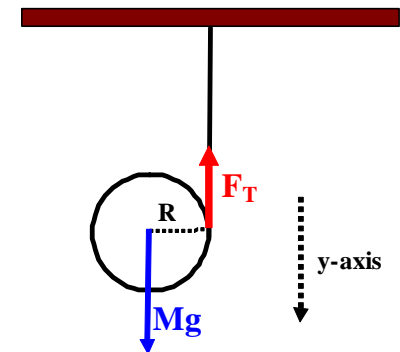
Answer: $\frac{2}{3}MR^2$
 % Right: 48%

$$Mg - F_T = Ma_y$$

$$F_T = M(g - a_y)$$

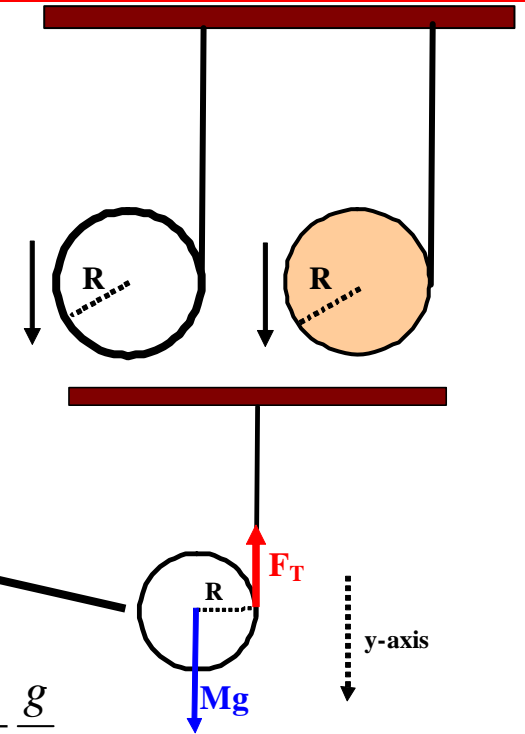
$$\tau = RF_T = I\alpha = \frac{Ia_y}{R}$$

$$I = \frac{R^2 F_T}{a_y} = \left(\frac{g - a_y}{a_y} \right) MR^2 = \left(\frac{g - \frac{3}{5}g}{\frac{3}{5}g} \right) MR^2 = \frac{2}{3}MR^2$$



Exam 2 Spring 2014: Problem 30

- Near the surface of the Earth a cloth tape is wound around the outside of a thin uniform hoop with mass M , radius R , and moment of inertia $I_{\text{hoop}} = MR^2$. A cloth tape is also wound around the outside of a non-uniform cylinder with the same mass M and the same radius R , but with moment of inertia I_{cylinder} . Both the hoop and the cylinder are fastened to the ceiling as shown in the figure. They are both released from rest at $t = 0$ at the same distance from the ceiling. If at $t = 1.81$ s the two objects are a vertical distance of 2 meters apart, what is the moment of inertia of the cylinder I_{cylinder} ?



Answer: $0.60MR^2$

% Right: 20%

$$y_{\text{hoop}} = \frac{1}{2} a_{\text{hoop}} t^2$$

$$y_{\text{cylinder}} = \frac{1}{2} a_{\text{cylinder}} t^2$$

$$\Delta y = y_{\text{cylinder}} - y_{\text{hoop}} = \frac{1}{2} \Delta a t^2$$

$$\frac{I_{\text{cylinder}}}{MR^2} = \frac{1 - \frac{4\Delta y}{gt^2}}{1 + \frac{4\Delta y}{gt^2}} \approx \frac{1 - 0.25}{1 + 0.25} \approx 0.6$$

$$a_{\text{hoop}} = \frac{g}{(1 + I_{\text{hoop}} / MR^2)} = \frac{1}{2} g$$

$$a_{\text{cylinder}} = \frac{g}{(1 + I_{\text{cylinder}} / MR^2)}$$

$$\Delta a = a_{\text{cylinder}} - a_{\text{hoop}} = \frac{g}{(1 + I_{\text{cylinder}} / MR^2)} - \frac{g}{2}$$

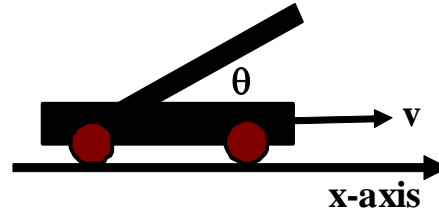
$$\frac{4\Delta y}{gt^2} = \frac{4(2\text{m})}{(9.8\text{m/s}^2)(1.81\text{s})^2} \approx 0.25$$

$$Mg - F_T = Ma_y$$

$$\tau = RF_T = I\alpha = \frac{Ia_y}{R}$$

$$a_y = \frac{g}{(1 + I / MR^2)}$$

Exam 2 Spring 2012: Problem 10



- A cannon on a railroad car is facing in a direction parallel to the tracks as shown in the figure. The cannon can fire a 100-kg cannon ball at a muzzle speed of 150 m/s at an angle of θ above the horizontal as shown in the figure. The cannon plus railway car have a mass of 5,000 kg. If the cannon and one cannon ball are travelling to the right on the railway car a speed of $v = 2$ m/s, at what angle θ must the cannon be fired in order to bring the railway car to rest? Assume that the track is horizontal and there is no friction.

Answer: 48.2°
% Right: 57%

$$(p_x)_i = (M_{car} + M_{Ball})V_i \quad (p_x)_f = M_{Ball}V_{ball} = M_{Ball}(V_i + V_{muzzle} \cos \theta)$$

$$(p_x)_i = (p_x)_f \quad (M_{car} + M_{Ball})V_i = M_{Ball}(V_i + V_{muzzle} \cos \theta)$$

$$\cos \theta = \frac{M_{car}V_i}{M_{Ball}V_{muzzle}} = \frac{(5000\text{kg})(2\text{m/s})}{(100\text{kg})(150\text{m/s})} = 0.6667 \quad \theta \approx 48.2^\circ$$

Exam 2 Spring 2014: Problem 16

- A firecracker is tossed straight upward into the air. It explodes into three pieces (A, B, and C) of equal mass just as it reaches the highest point. If piece A moves off at a speed of 4 m/s and piece B and C move off with equal speeds of 5 m/s, what is the angle between piece B and C?

Answer: 132.8°

% Right: 16%

$$\vec{p}_A + \vec{p}_B + \vec{p}_C = M\vec{v}_A + M\vec{v}_B + M\vec{v}_C = 0$$

$$\vec{v}_C = v_0 \hat{x} \quad \vec{v}_B = v_0 \cos \theta \hat{x} + v_0 \sin \theta \hat{y}$$

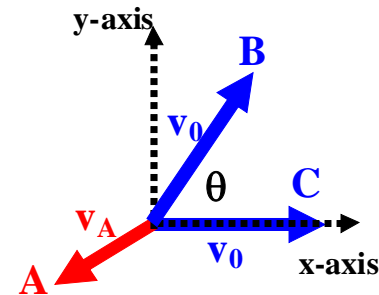
$$\vec{v}_A = -(\vec{v}_B + \vec{v}_C)$$

$$\vec{v}_A = -v_0(1 + \cos \theta) \hat{x} - v_0 \sin \theta \hat{y}$$

$$v_A^2 = |\vec{v}_A|^2 = v_0^2 [(1 + \cos \theta)^2 + \sin^2 \theta] = v_0^2 [2 + 2 \cos \theta]$$

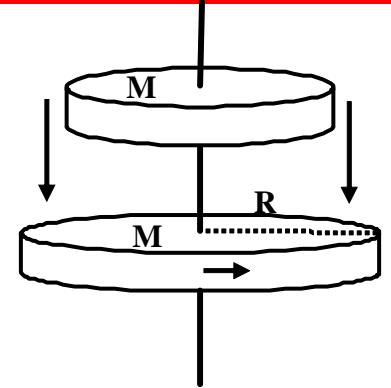
$$\cos \theta = \frac{v_A^2}{2v_0^2} - 1 = \frac{(4 \text{ m/s})^2}{2(5 \text{ m/s})^2} - 1 = -0.68$$

$$\theta \approx 132.8^\circ$$



Exam 2 Fall 2011: Problem 31

- A uniform solid disk with mass M and radius R is mounted on a vertical shaft with negligible rotational inertia and is initially rotating with angular speed ω . A non-rotating uniform solid disk with mass M and radius $R/2$ is suddenly dropped onto the same shaft as shown in the figure. The two disks stick together and rotate at the same angular speed. What is the new angular speed of the two disk system?



Answer: $4\omega/5$
% Right: 47%

$$L_i = I_i \omega_i = \frac{1}{2} MR_1^2 \omega = L_f = I_f \omega_f = \left(\frac{1}{2} MR_1^2 + \frac{1}{2} MR_2^2 \right) \omega_f$$

$$\omega_f = \frac{R_1^2}{R_1^2 + R_2^2} \omega = \frac{R^2}{R^2 + (R/2)^2} \omega = \frac{4}{5} \omega$$

Exam 1 Spring 2014: Problem 36

- A stone of mass m is released from rest a height $h = 1.5R_E$ above the surface of the Earth, where R_E is the radius of the Earth. What is the speed of the stone (in km/s) when it hits the Earth? Ignore air resistance and take $R_E = 6.371 \times 10^6$ m, $M_E = 5.974 \times 10^{24}$ kg, and $G = 6.674 \times 10^{-11}$ N·m²/kg², where M_E is the mass of the Earth and G is Newton's constant.

Answer: 8.67
% Right: 18%

$$g = \frac{GM_E}{R_E^2} = \frac{(6.674 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2)(5.974 \times 10^{24} \text{ kg})}{(6.371 \times 10^6 \text{ m})^2} \approx 9.823 \text{ m/s}^2$$

Energy conservation: $E_f = E_i$

$$E_i = -\frac{GmM_E}{h + R_E}$$

$$E_f = \frac{1}{2}mv_f^2 - \frac{GmM_E}{R_E}$$

$$v_f^2 = 2 \left[\frac{GM_E}{R_E} - \frac{GM_E}{h + R_E} \right] = \frac{2GM_E}{R_E} \left[1 - \frac{R_E}{h + R_E} \right] = \frac{2gR_E h}{h + R_E}$$

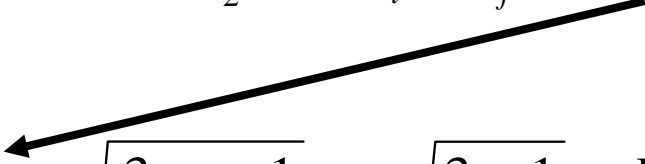
$$v_f = \sqrt{\frac{2gR_E h}{h + R_E}} \xrightarrow{h=1.5R_E} \sqrt{\frac{2g(1.5R_E)}{2.5}} = \sqrt{\frac{2(9.823 \text{ m/s}^2)(1.5)(6.371 \times 10^6 \text{ m})}{2.5}} \approx 8.67 \text{ km/s}$$

Final Exam Spring 2011: Problem 5

- A toy merry-go-round consists of a uniform disk of mass M and radius R that rotates freely in a horizontal plane about its center. A mouse of the same mass, M , starts at the rim of the disk. Initially the mouse and disk rotate together with an angular velocity of ω . If the mouse walks to a new position a distance r from the center of the disk the new angular velocity of the mouse-disk system is 2ω . What is the new distance r ?

Answer: $R/2$
% Right: 65%

$$L_i = I_i \omega_i = (MR^2 + \frac{1}{2}MR)\omega_i = L_f = (Mr^2 + \frac{1}{2}MR)\omega_f$$


$$r = R \sqrt{\frac{3\omega_i}{2\omega_f} - \frac{1}{2}} = R \sqrt{\frac{3}{4} - \frac{1}{2}} = \frac{R}{2}$$

Exam 2 Fall 2011: Problem 36

- Two small balls are simultaneously released from rest in a deep pool of water (with density ρ_{water}). The first ball is released from rest at the surface of the pool and has a density three times the density of water (i.e. $\rho_1 = 3\rho_{\text{water}}$). A second ball of unknown density is released from rest at the bottom of the pool. If it takes the second ball the same amount of time to reach the surface of the pool as it takes for the first ball to reach the bottom of the pool, what is the density of the second ball? Neglect hydrodynamic drag forces.

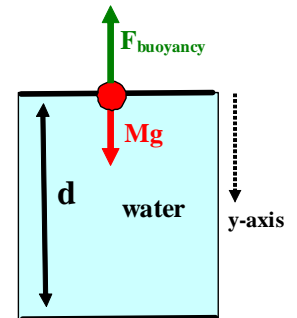
Answer: $3\rho_{\text{water}}/5$
 % Right: 22%

$$\rho_{\text{down}} V_{\text{ball}} g - \rho_{\text{water}} V_{\text{ball}} g = \rho_{\text{down}} V_{\text{ball}} a_{\text{down}}$$

$$a_{\text{down}} = \frac{\rho_{\text{down}} - \rho_{\text{water}}}{\rho_{\text{down}}} g = \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{down}}}\right) g$$

$$d = \frac{1}{2} a_y t^2$$

$$t = \sqrt{\frac{2d}{a_y}}$$

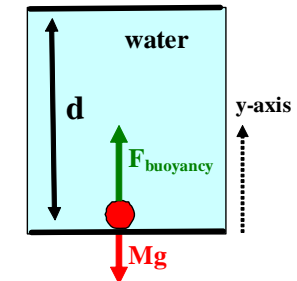


$$a_{\text{up}} = \left(\frac{\rho_{\text{water}}}{\rho_{\text{up}}} - 1\right) g = a_{\text{down}} = \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{down}}}\right) g$$

$$\rho_{\text{up}} = \frac{\rho_{\text{water}}}{2 - \rho_{\text{water}} / \rho_{\text{down}}} = \frac{\rho_{\text{water}}}{2 - \frac{1}{3}} = \frac{3}{5} \rho_{\text{water}}$$

$$\rho_{\text{water}} V_{\text{ball}} g - \rho_{\text{up}} V_{\text{ball}} g = \rho_{\text{up}} V_{\text{ball}} a_{\text{up}}$$

$$a_{\text{up}} = \frac{\rho_{\text{water}} - \rho_{\text{up}}}{\rho_{\text{up}}} g = \left(\frac{\rho_{\text{water}}}{\rho_{\text{up}}} - 1\right) g$$



Exam 2 Spring 12: Problem 46

- What is the minimum radius (in m) that a spherical helium balloon must have in order to lift a total mass of $m = 10\text{-kg}$ (including the mass of the empty balloon) off the ground? The density of helium and the air are, $\rho_{HE} = 0.18 \text{ kg/m}^3$ and $\rho_{air} = 1.2 \text{ kg/m}^3$, respectively.

Answer: 1.33

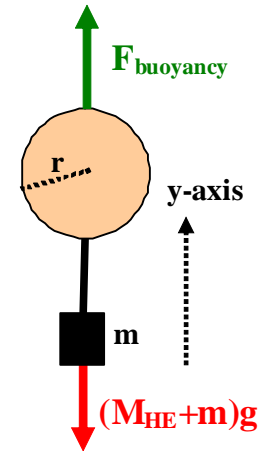
% Right: 37%

$$F_{buoyancy} - (M_{HE} + m)g = 0$$

$$\rho_{air} V_{ballon} g - (\rho_{HE} V_{ballon} + m)g = 0$$

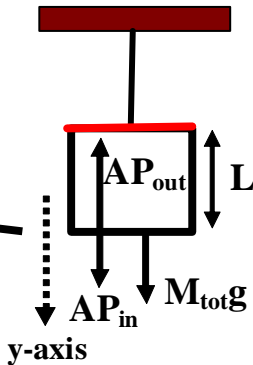
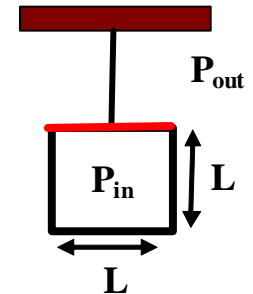
$$V_{ballon} = \frac{4}{3} \pi r^3 = \frac{m}{\rho_{air} - \rho_{HE}}$$

$$r = \left(\frac{3m}{4\pi(\rho_{air} - \rho_{HE})} \right)^{\frac{1}{3}} = \left(\frac{3(10\text{kg})}{4\pi(1.02\text{kg} / \text{m}^3)} \right)^{\frac{1}{3}} = (2.34\text{m}^3)^{\frac{1}{3}} \approx 1.33\text{m}$$



Exam 2 Spring 2014: Problem 43

- A cubical metal box with sides of mass M and length L has a square lid also with mass M and length L . The lid is not attached to the box, however, the lid and the box form an airtight seal. Near the surface of the Earth, the lid is held at rest by a steel cable, as shown in the figure. The pressure outside the box is the atmospheric pressure, $P_{out} = P_{atm} = 101 \text{ kPa}$. The box is partially evacuated to an inside pressure P_{in} so that the box remains at rest. If $M = 25 \text{ kg}$ and $L = 0.4 \text{ m}$, what is the maximum inside pressure P_{in} (in kPa) such that the box will not fall?



$$P_{out} A - P_{in} A - M_{tot} g = 0$$

$$P_{in} = P_{out} - \frac{M_{tot} g}{A} = P_{atm} - \frac{5Mg}{L^2} = P_{atm} - \frac{5(25\text{kg})(9.8\text{m/s}^2)}{(0.4\text{m})^2} = 101\text{kPa} - 7.656\text{kPa} \approx 93.3\text{kPa}$$

Answer: 93.3
% Right: 18%

Exam 2 Fall 2011: Problem 50

- An ideal spring-and-mass system is undergoing simple harmonic motion (SHM) with period $T = 3.14$ s. If the mass $m = 0.5$ kg and the total energy of the spring-and-mass system is 5 J, what is the speed (in m/s) of the mass when the displacement is 1.0 m?

Answer: 4.0

% Right: 34%

$$E = \frac{1}{2} kA^2 = \frac{1}{2} mv_x^2 + \frac{1}{2} kx^2$$

$$\omega = \sqrt{\frac{k}{m}}$$
$$\omega = \frac{2\pi}{T}$$

$$v_x^2 = \frac{2E}{m} - \frac{k}{m} x^2 = \frac{2E}{m} - \omega^2 x^2 = \frac{2E}{m} - \frac{4\pi^2}{T^2} x^2$$

$$v_x = \sqrt{\frac{2E}{m} - \frac{4\pi^2}{T^2} x^2} = \sqrt{\frac{2(5J)}{0.5kg} - \frac{4\pi^2}{(3.14s)^2} (1)^2} = \sqrt{(20m^2/s^2) - (m^2/s^2)} = 4m/s$$

Final Exam Spring 2011: Problem 20

- An ambulance moving at a constant speed of 20 m/s with its siren on overtakes a car moving at a constant speed of 10 m/s in the same direction as the ambulance. As the ambulance approaches the car the driver of the car perceives the siren to have a frequency of 1200 Hz. What frequency does the driver of the car perceive after the ambulance has passed? This takes place on a cold night in Alaska where the speed of sound is 320 m/s.

Answer: 1127 Hz

% Right: 53%

$$f_{\text{toward}} = \frac{v_{\text{sound}} - v_{\text{car}}}{v_{\text{sound}} - v_A} f_0$$

$$f_{\text{away}} = \frac{v_{\text{sound}} + v_{\text{car}}}{v_{\text{sound}} + v_A} f_0$$

$$\frac{f_{\text{away}}}{f_{\text{toward}}} = \frac{(v_{\text{sound}} + v_{\text{car}})(v_{\text{sound}} - v_A)}{(v_{\text{sound}} + v_A)(v_{\text{sound}} - v_{\text{car}})}$$

$$f_{\text{away}} = \frac{(v_{\text{sound}} + v_{\text{car}})(v_{\text{sound}} - v_A)}{(v_{\text{sound}} + v_A)(v_{\text{sound}} - v_{\text{car}})} f_{\text{toward}} = \frac{(320 + 10)(320 - 20)}{(320 + 20)(320 - 10)} (1200 \text{ Hz}) \approx 1127 \text{ Hz}$$

