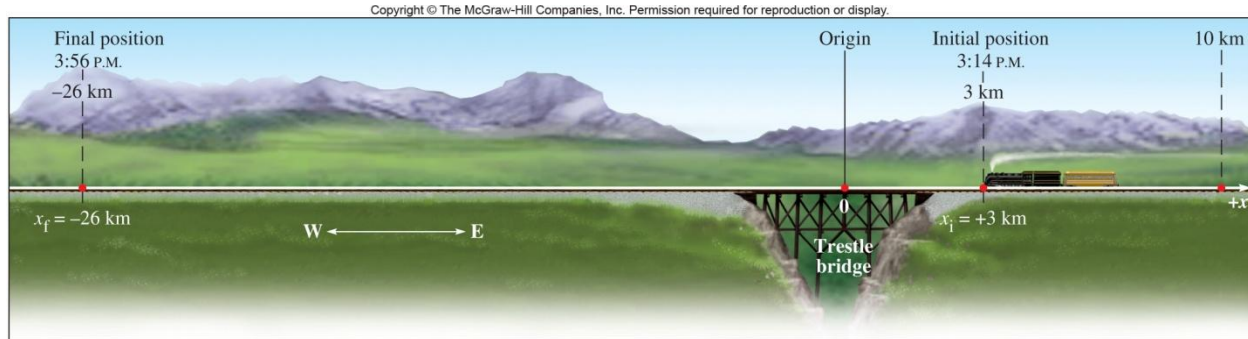


## Chapter 2 Motion Along a Line

In order to specify a position, it is necessary to choose an origin. We talk about the football field is 100 yards from goal line to goal line, it is 35 miles until we arrive at grandma's, and so on.



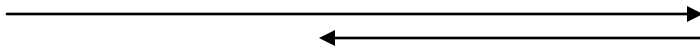
When motion occurs along a line, we customarily choose the  $x$ -axis along the motion. An exception occurs when things are moving up and down. In those cases the motion occurs in the  $y$  direction.

We define the displacement as the change in position:

$$\Delta x = x_f - x_i$$

We will use the  $\Delta$  (delta) notation often in class. It denotes the change in whatever follows. The change is the difference in the final and initial values.

Displacement is different from distance.



The upper displacement is 18 m and the lower displacement is 9 m. What is the total displacement? What is the distance?

The displacement depends only on the beginning and ending points, not the path taken.

The total displacement for multiple displacements is the sum of the individual displacements. How does that work for the problem above?

Even though positions depend on the location of the origin, displacements do not.

When a displacement  $\Delta x$  occurs in a time  $\Delta t$ , we define the average velocity as

$$v_{av,x} = \frac{\Delta x}{\Delta t}$$

Since the time interval is always positive (why?), the average velocity is in the same direction as the displacement.

The  $\Delta$ s do not cancel in the definition. The  $\Delta$  operates on the variable, it does not multiply it.

The average speed is different from the average velocity. In the problem above, suppose the motion occurs in 20 s. Average velocity is  $9 \text{ m}/20 \text{ s} = 0.45 \text{ m/s}$ . Average speed is  $27 \text{ m}/20 \text{ s} = 1.35 \text{ m/s}$ . If you run around a track, your average velocity will be zero if the starting and ending points coincide.

What are the units of velocity?

Can the average speed be greater than the average velocity?

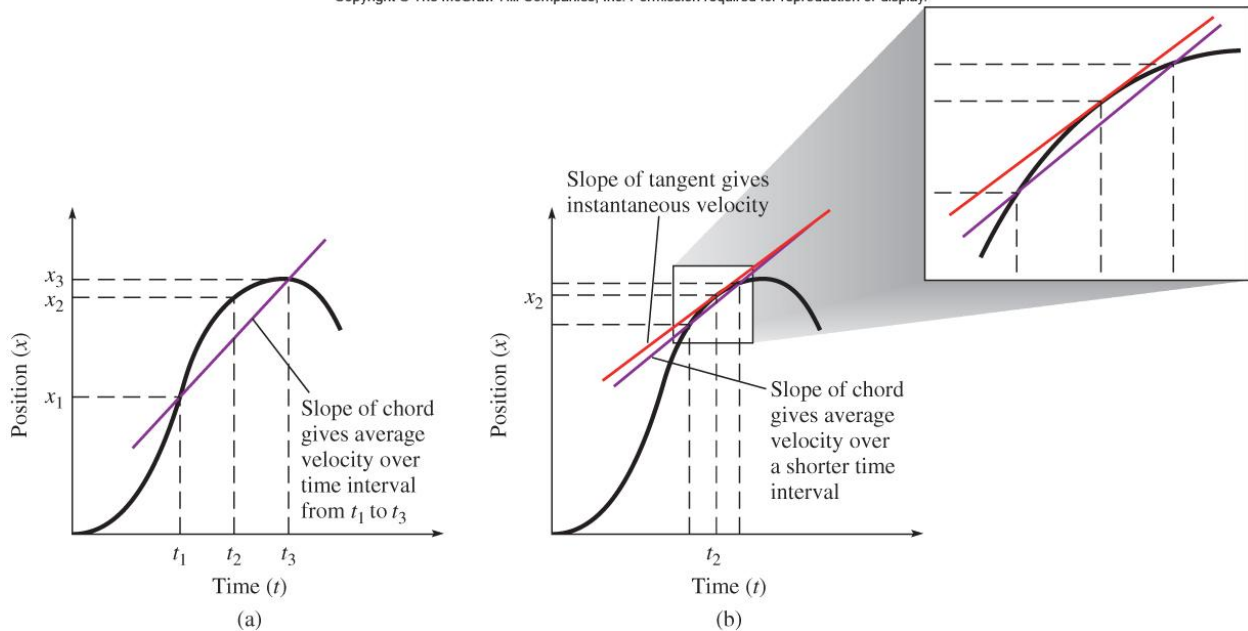
The instantaneous velocity is the velocity of the object at some moment in time. There can be a big difference between average velocity and instantaneous velocity. Just consider any trip around Gainesville.

The instantaneous velocity is the limit of taking small displacements over smaller time intervals:

$$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

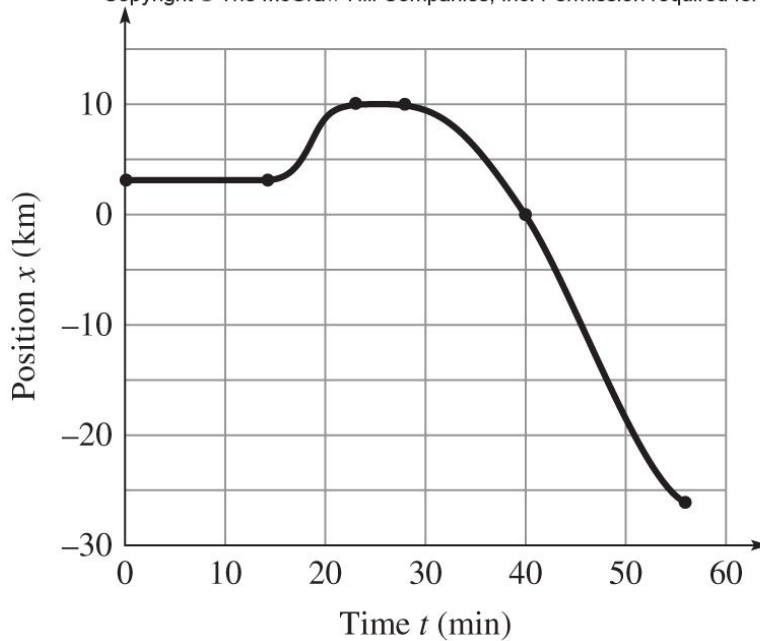
Okay, calculus people. What do we call this?

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As the time interval shrinks, the slope of the chord approaches a constant value and becomes a tangent line. The instantaneous velocity is the slope of a position vs. time graph.

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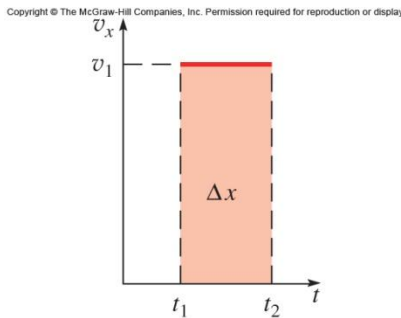


$x$ (km)	$t$ (min)
+3	0
+3	14
+10	23
+10	28
0	40
-26	56

Let's talk about the graph above.

From now on, we drop the instantaneous velocity and just call it velocity.

If we look at a velocity vs. time graph, we can find the displacement. For constant velocity, the graph is



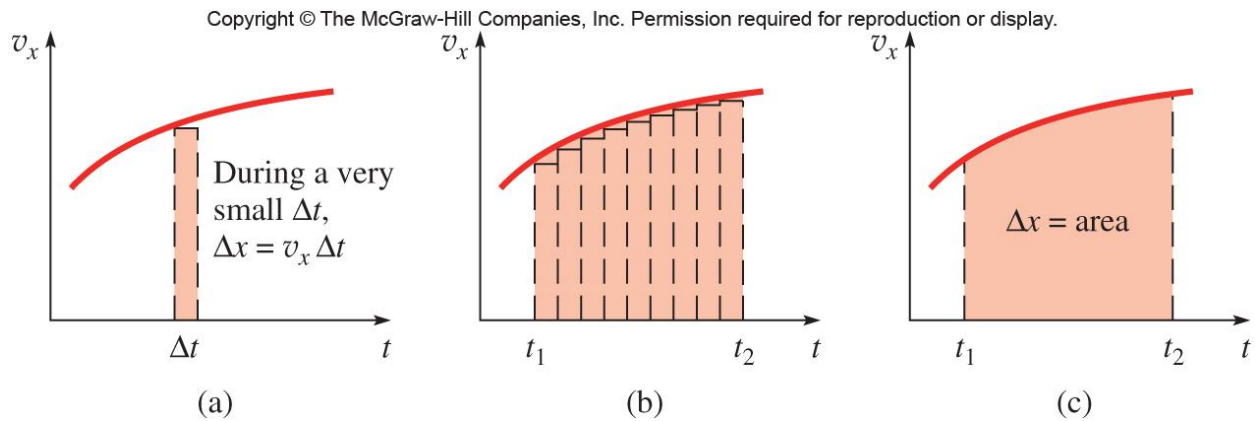
How do we know the velocity is constant? If the velocity is constant, the average and instantaneous velocities are the same,

$$v_x = v_{av,x} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x \Delta t$$

The displacement is the area under the curve in a velocity vs. time graph. Read the warning in page 32.

If the velocity changes, it can be harder to find  $\Delta x$ :



Okay calculus people, what do we call this?

In a similar way, we can talk about the rate of change of velocity. We call that the acceleration. The average acceleration is

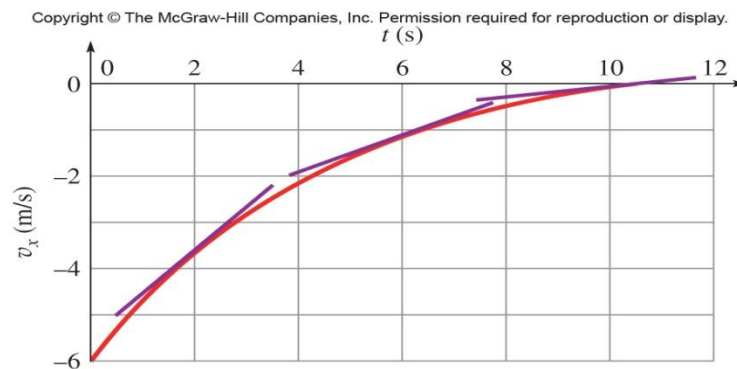
$$a_{av,x} = \frac{\Delta v_x}{\Delta t}$$

What are the units of acceleration?

The instantaneous acceleration (or just the acceleration) is

$$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t}$$

The acceleration is the slope of a velocity vs. time graph. The area under an acceleration vs. time graph is the velocity.



Which way is the object heading in the graph above?

There can be confusion when we talk about acceleration. For example, what to you push on the make your car travel with constant speed? We will not use the word deceleration in this class. Rather we will concern ourselves with the direction of the acceleration and velocity. Here are four possibilities:

Situation	Result
$v_x$ in positive direction, $a_x$ in positive direction	Magnitude of velocity increases
$v_x$ in positive direction, $a_x$ in negative direction	Magnitude of velocity decreases
$v_x$ in negative direction, $a_x$ in positive direction	Magnitude of velocity decreases
$v_x$ in negative direction, $a_x$ in negative direction	Magnitude of velocity increases

To summarize: Same direction, go faster. Opposite direction, go slower.

“In straight line motion, the acceleration is always in the same direction as the velocity, in the direction opposite to the velocity, or zero.” p. 34

Our first set of formulas: The equations of uniformly accelerated motion.  
I will follow the derivation in the book and then do it a different way.

If the acceleration is constant, the average acceleration is equal to the instantaneous acceleration.

$$a_x = a_{av,x} = \frac{\Delta v_x}{\Delta t}$$

$$\Delta v_x = a_x \Delta t$$

Our first equation is

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$$

It is just a slight rearrangement of the definition of acceleration. The second equation is found from the diagram. The average velocity is the average of the initial and final velocities:

$$v_{av,x} = \frac{1}{2}(v_{fx} + v_{ix})$$

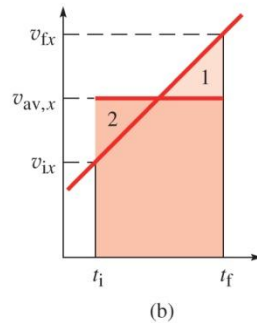
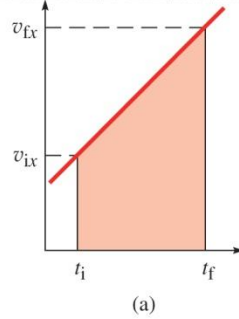
This handy fact is true only because the acceleration is constant. That corresponds to a constant slope in the velocity vs. time graph.

Using this with the definition of average velocity,

$$v_{av,x} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_{av,x} \Delta t = \frac{1}{2}(v_{fx} + v_{ix}) \Delta t$$

$$\Delta x = \frac{1}{2}(v_{fx} + v_{ix}) \Delta t$$



Now let us substitute for  $v_{fx}$  in the above equation.

$$\Delta x = \frac{1}{2}(v_{fx} + v_{ix})\Delta t = \frac{1}{2}((v_{ix} + a_x \Delta t) + v_{ix})\Delta t$$

$$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

We could have divided the two equations, and found

$$\frac{\Delta x}{v_{fx} - v_{ix}} = \frac{\frac{1}{2}(v_{fx} + v_{ix})\Delta t}{a_x \Delta t}$$

$$\frac{1}{2}(v_{fx} + v_{ix})(v_{fx} - v_{ix}) = a_x \Delta x$$

$$v_{fx}^2 - v_{ix}^2 = 2a_x \Delta x$$

These are our four formulas of uniformly accelerated motion

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$$

The difference in velocity is related to the acceleration and how long the acceleration acts.

$$\Delta x = \frac{1}{2}(v_{fx} + v_{ix})\Delta t$$

The displacement is the average velocity times the time.

$$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

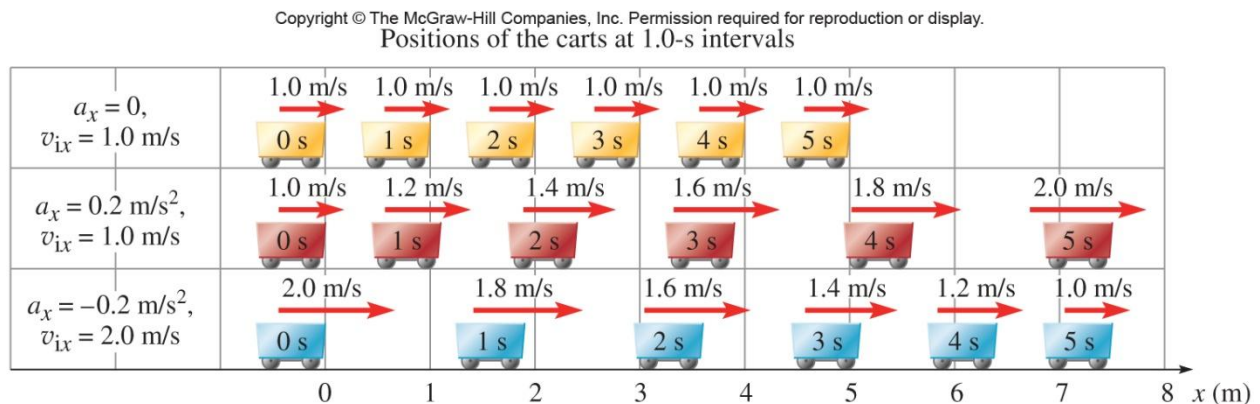
The displacement is the sum of the displacement due to the initial speed and the displacement due to the acceleration

$$v_{fx}^2 - v_{ix}^2 = 2a_x \Delta x$$

The difference in the squares of the velocities is related to the acceleration and the displacement. While this one is hard to see, it will have HUGE consequences later.

Now we do it with calculus.

Motion diagrams can be helpful. The pictures are taken at equal time intervals.



Which is moving with constant speed? Which has an increasing velocity? A decreasing velocity? (I know the answer is in the picture!)

A very common situation with constant acceleration is free fall. If we neglect air resistance, all objects fall with the same acceleration, regardless of the objects mass or state of motion. Surprised? Many people were surprised when Galileo discovered this fact.

Consider the following. What if heavier objects accelerated faster?

The acceleration due to gravity is denoted by  $g$  and has the value

$$g = 9.8 \text{ m/s}^2$$

The direction is downwards, towards the center of the earth. In fact, it is how we define down!