

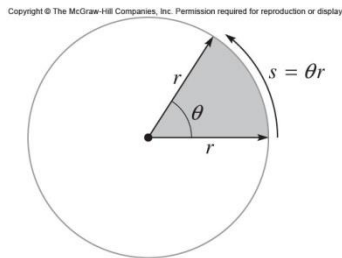
## Chapter 5 Circular Motion

The language used to describe rotational motion is very similar to the language used to describe linear motion. The symbols are different.

Description	Linear	Angular
position	$x$	$\theta$
displacement	$\Delta x$	$\Delta \theta$
rate of change of position	$v_x$	$\omega$
average rate of change of position	$v_{x,av} = \frac{\Delta x}{\Delta t}$	$\omega_{av} = \frac{\Delta \theta}{\Delta t}$
instantaneous rate of change of position	$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$	$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$

While we are familiar with angles measured in degrees, we measure rotations in radians. In the figure, the angle measure in radians is defined as the ratio of the arc length  $s$  to the radius  $r$ :

$$\theta = \frac{s}{r}$$



For a complete rotation,  $s = 2\pi r$  and the angle for a complete rotation is  $2\pi$  radians. This gives the conversion of radians to degrees

$$360^\circ = 2\pi \text{ rad}$$

There is a relation between the speed of a point on the rim and the angular velocity of the wheel.

$$\begin{aligned}
 s &= r\theta \\
 \Delta s &= r\Delta \theta \\
 \frac{\Delta s}{\Delta t} &= \frac{r\Delta \theta}{\Delta t} \\
 v &= r\omega
 \end{aligned}$$

Just like displacements, rotations have directions. We take counterclockwise rotations as positive and clockwise rotations as negative.

### Other useful parameters

The period ( $T$ ) – The time it takes for a complete revolution.

The frequency ( $f$ ) – The number of revolutions per unit time.

These parameters are related. By definition

$$f = \frac{1}{T}$$

A complete revolution is the circumference  $2\pi r$ . The speed is the distance divided by the time. For a complete revolution, the time is the period  $T$

$$v = \frac{2\pi r}{T} = 2\pi r f$$

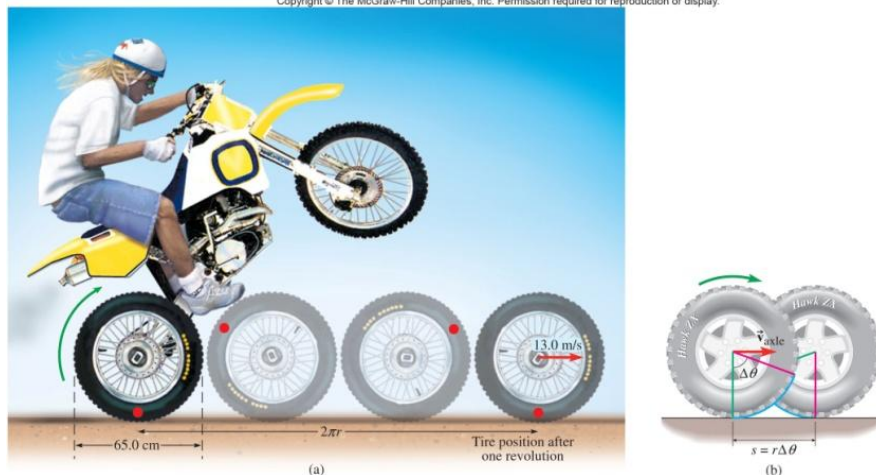
Using the equation near the top of the page, we have

$$\omega r = 2\pi f$$

$$\omega = 2\pi f$$

### Rolling motion

If the wheel rolls across the ground, its speed depends on how fast it spins.



If the wheel rolls without slipping, the axle of the wheel moves with a speed of  $v$  given by

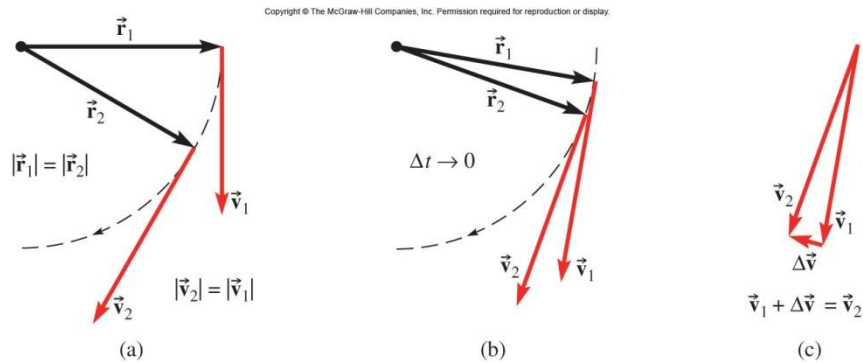
$$v = \omega r$$

This is a relative velocity problem! A – Axle of wheel, G – Ground, R – Rim of wheel

$$\begin{aligned}
 \vec{v}_{AG} &= \vec{v}_{AR} + \vec{v}_{RG} \\
 |v_{AG}| &= |v_{AR}| + 0 \\
 &= |v_{RA}| \\
 &= \omega r
 \end{aligned}$$

## Radial Acceleration

A particle undergoing uniform circular motion ( $\omega$  is constant in time), still accelerates even though its speed is constant. The velocity continuously changes direction.

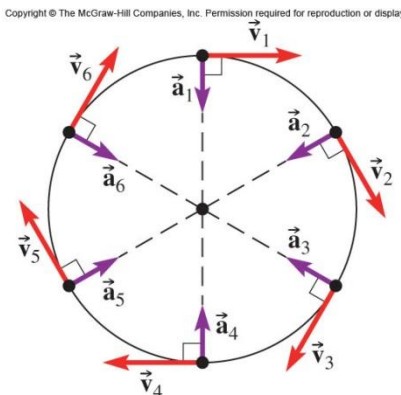


The change in velocity points towards the center of the circle. The magnitude of the radial acceleration is

$$a_r = \frac{v^2}{r}$$

A derivation is given on pages 153-154.

The acceleration is perpendicular to the velocity as long as the magnitude of the velocity is constant (the speed is constant)



### Problem-Solving Strategy for an Object in Uniform Circular Motion (page 155)

1. Begin as for any Newton's second law problem: identify all the forces acting on the object and draw an FBD.
2. Choose perpendicular axes at the point of interest so that one is radial and the other is tangential to the circular path.
3. Find the radial component of each force.
4. Apply Newton's second law as follows:

$$\sum F_r = ma_r$$

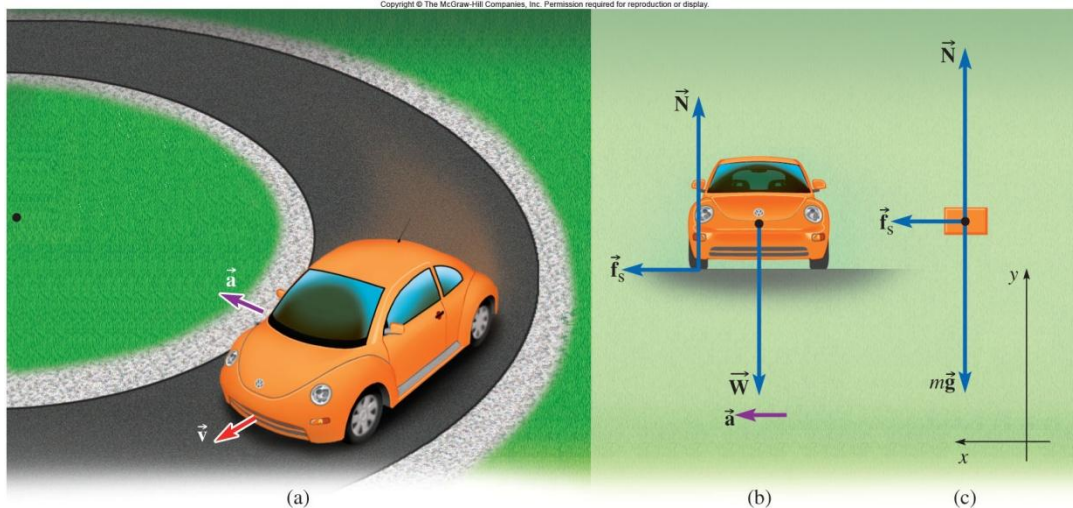
where  $\sum F_r$  = is the radial component of the net force and the radial acceleration is

$$a_r = \frac{v^2}{r} = \omega^2 r$$

(For uniform circular motion, neither the net force nor the acceleration has a tangential component.)

### Banked and Unbanked curves

A car turning through a curve at constant speed is accelerated towards the center of the curve. If the road is flat, the radial force is supplied by friction.



If the car is rolling, the radial force is static friction. If it is sliding, it is kinetic friction. From the diagram above (note the direction of  $x$ -axis is contrary to our convention)

$$\sum F_x = ma_x$$
$$f_s = m \frac{v^2}{r}$$

$$\sum F_y = ma_y$$
$$N - mg = 0$$

What is the fastest safe speed for a curve? That is when the car needs the largest possible static frictional force in the curve.

$$f_{s,\max} = \mu_s N$$

$$m \frac{v^2}{r} = \mu_s mg$$

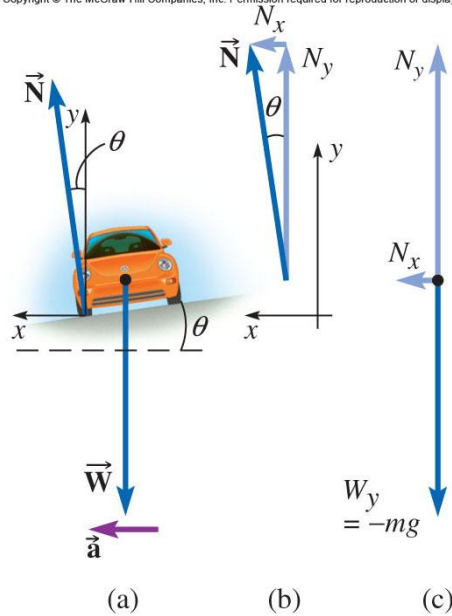
$$v_{\max} = \sqrt{\mu_s r g}$$

The maximum safe speed depends on

- the coefficient of friction. The lower  $\mu_s$ , the slower the car must go. (Slow down on wet streets!)
- the radius of the curve. Go slow around sharp curves (where  $r$  is small)!
- the acceleration due to gravity. (Be careful driving on the moon!!!)

The situation can be improved by banking the curve.

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The needed radial force is supplied by a component of the normal force. It is important to notice that the  $x$ -axis does not point down the incline. It points in the direction of the radial acceleration to the left. The car does not slide down the incline. It is accelerated towards the center of the curve.

$$\sum F_x = ma_x$$

$$N_x = m \frac{v^2}{r}$$

$$N \sin \theta = m \frac{v^2}{r}$$

$$\sum F_y = ma_y$$

$$N_y - W = 0$$

$$N \cos \theta = mg$$

Dividing the two equations to eliminate the normal force

$$\frac{N \sin \theta}{N \cos \theta} = \frac{m \frac{v^2}{r}}{mg}$$

$$\tan \theta = \frac{v^2}{rg}$$

A car traveling this speed around a curve banked at angle  $\theta$  with radius  $r$ , will not require any friction to safely travel around the curve. What happens if the car goes too fast? What happens if the car goes too slow?

### Circular orbits

The Earth remains in orbit around the sun because of the gravitational pull of the sun. Gravitation supplies the radial force needed to keep the Earth in its orbit. The same physics occurs with satellites in orbit around the Earth.

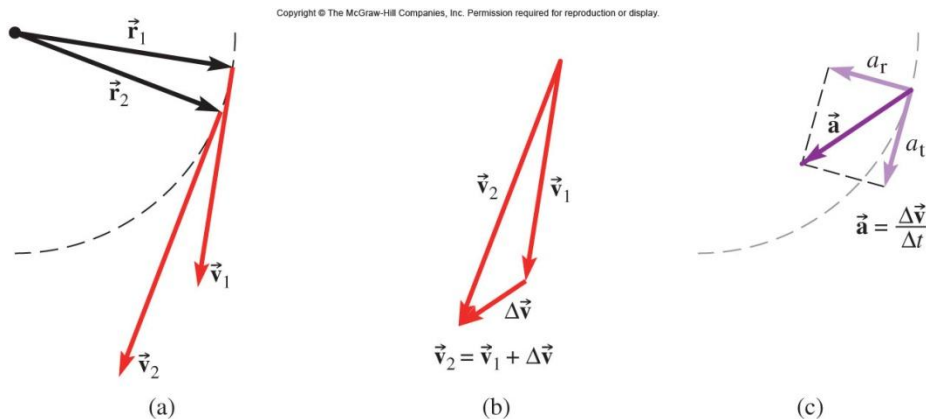
After working for 20 years with the observations of Tycho Brahe, Johannes Kepler stated his **three laws of planetary motion**:

- The planets travel in elliptical orbits with the Sun at one focus of the ellipse.
- A line drawn from a planet to the Sun sweeps out equal areas in equal time intervals.
- The square of the orbital period is proportional to the cube of the average distance from the planet to the Sun.

In a remarkable verification of his law of gravitation, Newton was able to derive Kepler's laws.

### Nonuniform Circular Motion

Suppose the rotating wheel changes its angular speed.



This time  $\Delta \mathbf{v}$  does not point towards the center of the wheel!

There is now a tangential acceleration as well as the radial acceleration we have studied. Since the radial and tangential directions are perpendicular to each other, the overall acceleration is

$$a = \sqrt{a_r^2 + a_t^2}$$

**Problem-Solving Strategy for an Object in Nonuniform Circular Motion (page 165)**

1. Begin as for any Newton's second law problem: Identify all the forces acting on the object and draw an FBD.
2. Choose perpendicular axes at the point of interest so that one axis is radial and the other is tangent to the circular path.
3. Find the radial component of each force.
4. Apply Newton's second law along the radial direction:

$$\sum F_r = ma_r$$

where

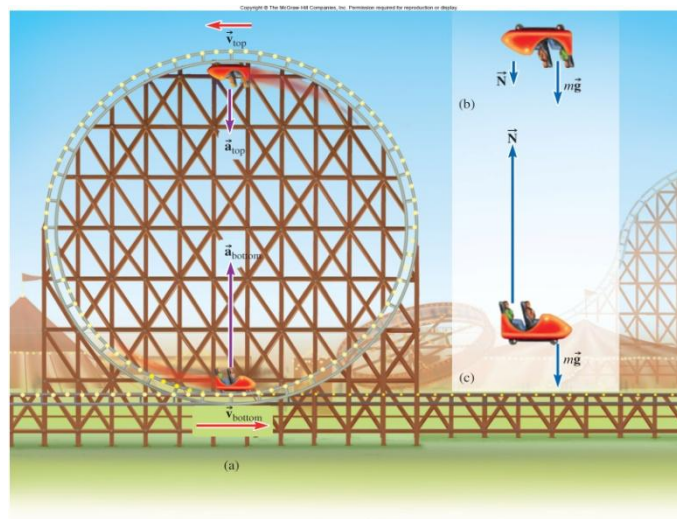
$$a_r = \frac{v^2}{r} = \omega^2 r$$

5. If necessary, apply Newton's second law to the tangential force components:

$$\sum F_t = ma_t$$

The tangential acceleration component at determines how the speed of the object changes.

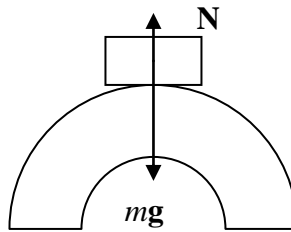
**Apparent Weight**



At what point of the ride would you feel heaviest? Lightest?

To travel around a circle, there must be an acceleration towards the center of the circle. At the top, some of the radial acceleration is supplied by the weight. The rest is supplied by the normal force, the force of the seat on the passenger. At the bottom, the normal force must overcome the weight. Weight does not change, but the normal force does. The reaction to the normal force (the seat pushing on the passenger) is the passenger pushing on the seat. That force is largest at the bottom, where you feel the heaviest.

**Problem 5.41** A car approaches the top of a hill that is shaped like a vertical circle with a radius of 55.0 m. What is the fastest speed that the car can go over the hill without losing contact with the ground?



The radial direction is in the  $-y$ -direction. Using Newton's second law,

$$\sum F_r = ma_r$$

$$mg - N = m \frac{v^2}{r}$$

When the car is just about to leave the ground  $N = 0$ .

$$mg - 0 = m \frac{v^2}{r}$$

$$v = \sqrt{rg} = \sqrt{(55.0\text{ m})(9.8\text{ m/s}^2)} = 23.2\text{ m/s}$$

### Tangential and Angular Acceleration

We can add rows to our table

Description	Linear	Angular
position	$x$	$\theta$
displacement	$\Delta x$	$\Delta \theta$
Rate of change of position	$v_x$	$\omega$
Average rate of change of position	$v_{x,av} = \frac{\Delta x}{\Delta t}$	$\omega_{av} = \frac{\Delta \theta}{\Delta t}$
Instantaneous rate of change of position	$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$	$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$



Average rate of change of speed	$a_{x,av} = \frac{\Delta v_x}{\Delta t}$	$\alpha_{av} = \frac{\Delta \omega}{\Delta t}$
Instantaneous rate of change of speed	$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t}$	$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$

The components of acceleration are

$$a_r = \frac{v^2}{r} = \omega^2 r \quad a_t = \alpha r$$

Using similar reasoning to what we used for uniform linear motion, we can create equations for uniform angular motion.

Uniform Linear Motion	Uniform Angular Motion
$a_x = \text{constant}$	$\alpha = \text{constant}$
$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$	$\Delta \omega = \omega_f - \omega_i = \alpha \Delta t$
$\Delta x = \frac{1}{2}(v_{fx} + v_{ix})\Delta t$	$\Delta \theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t$
$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$	$\Delta \theta = \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$
$v_{fx}^2 - v_{ix}^2 = 2a_x\Delta x$	$\omega_f^2 - \omega_i^2 = 2\alpha\Delta \theta$

**Problem 5.52** A disk rotates with constant angular acceleration. The initial speed of the disk is  $2\pi$  rad/s. After the disk rotates through  $10\pi$  radians, the angular speed is  $7\pi$  rad/s. (a) What is the magnitude of the angular acceleration? (b) How much time did it take for the disk to rotate through  $10\pi$  radians? (c) What is the tangential acceleration of a point located at a distance of 5.0 cm from the center of the disk?

(a)

$$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{(7\pi \text{ rad/s})^2 - (2\pi \text{ rad/s})^2}{2(10\pi \text{ rad})} = 2.25\pi \text{ rad/s}^2$$

(b)

$$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t$$

$$\Delta t = \frac{2\Delta\theta}{\omega_f + \omega_i} = \frac{2(10\pi \text{ rad})}{7\pi \text{ rad/s} + 2\pi \text{ rad/s}} = 2.22 \text{ s}$$

(c)

$$a_t = \alpha r = (2.25\pi \text{ rad/s}^2)(0.05 \text{ m}) = 0.353 \text{ m/s}^2$$

## Weightlessness



Are these people weightless? Recall,

$$g = \frac{GM_E}{r^2}$$

Even at 200 miles up,  $g$  is not zero. Why do things float in the space station? Both the acceleration of the space station and the astronauts are the same. It is similar to the weightlessness experienced by a person falling in a severed elevator.

To make artificial gravity, spin the space station. This is the principle of a centrifuge.

