## REVIEW AND SYNTHESIS: CHAPTERS 1-5

## Review Exercises

1. Strategy Replace the quantities with their units.

Solution Find the units of the spring constant $k$.
$F=k x$, so $k=\frac{F}{x}$, and the units of $k$ are $\mathrm{N} / \mathrm{m}=\frac{\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}}{\mathrm{~m}}=\mathrm{kg} / \mathrm{s}^{2}$.
2. (a) Strategy Find Harrison's total displacement. The return trip has the same magnitude but the opposite direction. Draw a diagram.

Solution Compute the displacement using the component method.
$\Delta x=x_{1}+x_{2}+x_{3}=-2.00 \mathrm{~km}+(5.00 \mathrm{~km}) \cos 233^{\circ}+(1.00 \mathrm{~km}) \cos 120^{\circ}$
$\Delta y=y_{1}+y_{2}+y_{3}=0+(5.00 \mathrm{~km}) \sin 233^{\circ}+(1.00 \mathrm{~km}) \sin 120^{\circ}$
Compute the magnitude.

$\Delta r=\sqrt{\left[-2.00 \mathrm{~km}+(5.00 \mathrm{~km}) \cos 233^{\circ}+(1.00 \mathrm{~km}) \cos 120^{\circ}\right]^{2}+\left[(5.00 \mathrm{~km}) \sin 233^{\circ}+(1.00 \mathrm{~km}) \sin 120^{\circ}\right]^{2}}$

$$
=6.33 \mathrm{~km}
$$

Compute the direction.
$\theta=\tan ^{-1} \frac{\Delta y}{\Delta x}=\tan ^{-1} \frac{(5.00 \mathrm{~km}) \sin 233^{\circ}+(1.00 \mathrm{~km}) \sin 120^{\circ}}{-2.00 \mathrm{~km}+(5.00 \mathrm{~km}) \cos 233^{\circ}+(1.00 \mathrm{~km}) \cos 120^{\circ}}=29.6^{\circ}$ below the $-x$-axis (S of W)
Since Harrison must travel in the opposite direction, his return displacement should be
6.33 km at $29.6^{\circ}$ north of east.
(b) Strategy and Solution Since Harrison will travel 6.33 km at a speed of $5.00 \mathrm{~m} / \mathrm{s}$, his return trip will take
$\Delta t=\frac{\Delta r}{v}=\frac{6.33 \times 10^{3} \mathrm{~m}}{5.00 \mathrm{~m} / \mathrm{s}}\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=21.1 \mathrm{~min}$.
3. (a) Strategy Divide the length of the roadway by the distance between reflectors. There are 1760 yards in one mile.

Solution Find the distance between reflectors.
$\frac{2.20 \mathrm{mi}}{17.6 \frac{\mathrm{yd}}{\text { marker }}} \times \frac{1760 \mathrm{yd}}{1 \mathrm{mi}}=220$ markers
(b) Strategy Divide the length of the roadway by the distance between reflectors. There are 1000 meters in one kilometer.

Solution Find the distance between reflectors.
$\frac{3.54 \mathrm{~km}}{16.0 \frac{\mathrm{~m}}{\text { marker }}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}}=221$ markers
4. Strategy Use the conversion between mL and tsp.

Solution Find the factor by which the baby was overmedicated.
$\frac{(3 / 4 \mathrm{tsp}) \frac{4.9 \mathrm{~mL}}{1 \mathrm{tsp}}}{0.75 \mathrm{~mL}}=4.9$
The baby was overmedicated by a factor of 4.9.
5. (a) Strategy Find the total distance traveled and the time of travel. Then divide the distance by the time to obtain the average speed.

Solution Find Mike's average speed.
$\Delta x=50.0 \mathrm{~m}+34.0 \mathrm{~m}=84.0 \mathrm{~m}$ and $\Delta t=\frac{50.0 \mathrm{~m}}{1.84 \mathrm{~m} / \mathrm{s}}+\frac{34.0 \mathrm{~m}}{1.62 \mathrm{~m} / \mathrm{s}}=48.2 \mathrm{~s}$, so the average speed is
$v_{\mathrm{av}}=\frac{\Delta x}{\Delta t}=\frac{84.0 \mathrm{~m}}{48.2 \mathrm{~s}}=1.74 \mathrm{~m} / \mathrm{s}$.
(b) Strategy Find Mike's total displacement and divide it by the time found in part (a) to obtain his average velocity. Let his initial direction be positive.

Solution Mike's total displacement is $\Delta \overrightarrow{\mathbf{r}}=50.0 \mathrm{~m}$ forward -34.0 m back $=16.0 \mathrm{~m}$ forward. So, his average velocity is $\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{16.0 \mathrm{~m} \text { forward }}{48.2 \mathrm{~s}}=0.332 \mathrm{~m} / \mathrm{s}$ forward, or $0.332 \mathrm{~m} / \mathrm{s}$ in his original direction of motion.
6. Strategy Assume constant acceleration. Use ${v_{\mathrm{fx}}}^{2}-v_{\mathrm{ix}}^{2}=v=2 a_{x} \Delta x$ and Newton's second law.

Solution Find the acceleration of the jet.
$v_{\mathrm{fx}}^{2}-v_{\mathrm{ix}}^{2}=v_{\mathrm{fx}}^{2}-0=2 a_{x} \Delta x$, so $a_{x}=\frac{v_{\mathrm{f} x}^{2}}{2 \Delta x}$.
Estimate the average force on the jet due to the catapult.
$\Sigma F_{x}=F_{\text {eng }}+F_{\text {cata }}=m a_{x}$, so

$$
\begin{aligned}
F_{\text {cata }} & =m a_{x}-F_{\mathrm{eng}}=\frac{m v_{\mathrm{f} x}^{2}}{2 \Delta x}-F_{\mathrm{eng}}=\frac{(33,000 \mathrm{~kg})(160 \mathrm{mi} / \mathrm{h})^{2}}{2(90 \mathrm{~m})}\left(\frac{0.4470 \mathrm{~m} / \mathrm{s}}{1 \mathrm{mi} / \mathrm{h}}\right)^{2}-2(27,000 \mathrm{lb})\left(\frac{1 \mathrm{~N}}{0.2248 \mathrm{lb}}\right) \\
& =700 \mathrm{kN} .
\end{aligned}
$$

7. Strategy Use the conversion between feet and meters.

Solution Find the difference between the incorrect and correct altitudes.
$1500 \mathrm{~m}-(1500 \mathrm{ft}) \frac{1 \mathrm{~m}}{3.281 \mathrm{ft}}=1000 \mathrm{~m}=3300 \mathrm{ft}$
The captain thought they were 3300 ft or 1000 m above the correct altitude.
8. Strategy To reach the other side of the river in as short a time as possible, Paula must swim in the direction perpendicular to the river's flow. Find the time it takes for Paula to cross. Then use this time and the speed of the river to find how far downstream she travels while crossing.

Solution It takes Paula a time $\Delta t=\Delta x / v_{\text {swim }}$ to cross the river. During this time, she travels $\Delta y=v_{\text {river }} \Delta t=v_{\text {river }}\left(\frac{\Delta x}{v_{\text {swim }}}\right)=\frac{(1.43 \mathrm{~m} / \mathrm{s})(10.2 \mathrm{~m})}{0.833 \mathrm{~m} / \mathrm{s}}=17.5 \mathrm{~m}$ downstream.

9. Strategy Let north be up. Using a ruler and a protractor, draw the force vectors to scale; then, find the sum of the force vectors graphically.

Solution Draw the diagram and measure the length and angle of the sum of the force vectors.


The net force has a magnitude of about 2.3F, where F is the magnitude of the force with which Sandy pulls. The net force is at an angle of about $12^{\circ}$ south of east. So, the cart will go off the road toward south.
10. Strategy Use Newton's second law.

## Solution

(a) Find the magnitude of the horizontal force exerted on the tire by the wind. $\Sigma F_{x}=F-T \sin \theta=0$, so $F=T \sin \theta ; \Sigma F_{y}=T \cos \theta-W=0$, so $T=\frac{W}{\cos \theta}$.
Thus, $F=\frac{W}{\cos \theta} \sin \theta=W \tan \theta=W \tan 12^{\circ}$.

(b) From part (a), $T=\frac{W}{\cos \theta}=\frac{W}{\cos 12^{\circ}}$.
11. (a) Strategy According to Newton's third law, when the astronaut exerts a force of 250 N on the asteroid, the asteroid exerts an equal and opposite force on the astronaut. Use Newton's second law and the equations for constant acceleration.

Solution Let the astronaut be 1 and the asteroid be 2 . Find the accelerations of the astronaut and the asteroid. $F=m a$, so $a=F / m$. The acceleration of the astronaut during the 0.35 -s time interval is $a_{1}=\frac{250 \mathrm{~N}}{60.0 \mathrm{~kg}}=4.167 \mathrm{~m} / \mathrm{s}^{2}$ and that of the asteroid is $a_{2}=\frac{250 \mathrm{~N}}{40.0 \mathrm{~kg}}=6.25 \mathrm{~m} / \mathrm{s}^{2}$.
The distances traveled by the astronaut and the asteroid during the initial 0.35 -s time interval are
$d_{1 \mathrm{i}}=\frac{1}{2} a_{1}\left(\Delta t_{\mathrm{i}}\right)^{2}=\frac{1}{2}\left(4.167 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~s})^{2}=0.255 \mathrm{~m}$ and
$d_{2 \mathrm{i}}=\frac{1}{2} a_{2}\left(\Delta t_{\mathrm{i}}\right)^{2}=\frac{1}{2}\left(6.25 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~s})^{2}=0.383 \mathrm{~m}$. The speeds of the astronaut and the asteroid after the acceleration are $v_{1}=a_{1} \Delta t_{\mathrm{i}}=\left(4.167 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~s})=1.458 \mathrm{~m} / \mathrm{s}$ and $v_{2}=a_{2} \Delta t_{\mathrm{i}}=\left(6.25 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~s})=2.188 \mathrm{~m} / \mathrm{s}$. The distances traveled by the astronaut and the asteroid during the final $5.00-\mathrm{s}$ time interval are $d_{1 \mathrm{f}}=v_{1} \Delta t_{\mathrm{f}}=(1.458 \mathrm{~m} / \mathrm{s})(5.00 \mathrm{~s})=7.29 \mathrm{~m}$ and $d_{2 \mathrm{f}}=v_{2} \Delta t_{\mathrm{f}}=(2.188 \mathrm{~m} / \mathrm{s})(5.00 \mathrm{~s})=10.94 \mathrm{~m}$. The sum of all the computed distances is the total distance between the astronaut and the asteroid.
$0.255 \mathrm{~m}+0.383 \mathrm{~m}+7.29 \mathrm{~m}+10.94 \mathrm{~m}=19 \mathrm{~m}$ (rounded to two significant figures)
(b) Strategy and Solution The relative speed between the astronaut and the asteroid is the sum of the two speeds found in part (a): $1.458 \mathrm{~m} / \mathrm{s}+2.188 \mathrm{~m} / \mathrm{s}=3.6 \mathrm{~m} / \mathrm{s}$.
12. (a) Strategy and Solution Answers will vary, but a reasonable magnitude of the force required to pull out a single hair is 1 N .
(b) Strategy The total force exerted on all of the hairs is the weight of the prince, $W=m g$. Dividing his weight by the number of hairs gives the average force pulling on each strand of hair.

## Solution Will Rapunzel be bald?

$$
F_{\text {per hair }}=\frac{W}{100,000}=\frac{(60 \mathrm{~kg})(9.80 \mathrm{~N} / \mathrm{kg})}{100,000}=6 \mathrm{mN}
$$

Since $6 \mathrm{mN} \ll 1 \mathrm{~N}$, Rapunzel will most certainly not be made bald by the prince climbing up her hair.
13. Strategy Use Newton's second law to find the acceleration due to friction. Then use the acceleration and distance the plate must travel to determine the necessary initial speed.

Solution According to Newton's second law, $\Sigma F_{y}=N-m g=0$ and $\Sigma F_{x}=-f_{\mathrm{k}}=m a$.
So, $a=-\frac{f_{\mathrm{k}}}{m}=-\frac{\mu_{\mathrm{k}} N}{m}=-\frac{\mu_{\mathrm{k}} m g}{m}=-\mu_{\mathrm{k}} g$. Find the initial speed.
$v_{\mathrm{f}}^{2}-v_{\mathrm{i}}^{2}=0-v_{\mathrm{i}}^{2}=2 a \Delta x=-2 \mu_{\mathrm{k}} g \Delta x$, so

$v_{\mathrm{i}}=\sqrt{2 \mu_{\mathrm{k}} g \Delta x}=\sqrt{2(0.32)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.44 \mathrm{~m})}=1.7 \mathrm{~m} / \mathrm{s}$.
14. Strategy Draw a free-body diagram for each crate. Use Newton's second law.

Solution Let the $9.00-\mathrm{kg}$ crate be (1) and the $14.0-\mathrm{kg}$ crate be (2). The crates have the same acceleration.
Crate 1:
$\Sigma F_{x}=T-f_{1}=T-\mu N_{1}=m_{1} a$ and
$\Sigma F_{y}=N_{1}-m_{1} g=0$, so $T=\mu m_{1} g+m_{1} a$ (I).


Crate 2:
$\Sigma F_{x}=F \cos \theta-T-f_{2}=F \cos \theta-T-\mu N_{2}=m_{2} a$ and
$\Sigma F_{y}=N_{2}+F \sin \theta-m_{2} g=0$, so $F \cos \theta-T-\mu\left(m_{2} g-F \sin \theta\right)=m_{2} a$ (II).
Substitute I into II and solve for $a$.

$$
\begin{aligned}
F \cos \theta-\mu m_{1} g-m_{1} a-\mu\left(m_{2} g-F \sin \theta\right) & =m_{2} a \\
F \cos \theta-\mu m_{1} g-\mu m_{2} g+\mu F \sin \theta & =\left(m_{1}+m_{2}\right) a \\
a & =F(\cos \theta+\mu \sin \theta) /\left(m_{1}+m_{2}\right)-\mu g
\end{aligned}
$$

So, the magnitude of the acceleration is

$$
a=\frac{(195 \mathrm{~N})\left(\cos 20.0^{\circ}+0.550 \sin 20.0^{\circ}\right)}{9.00 \mathrm{~kg}+14.0 \mathrm{~kg}}-0.550\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=4.2 \mathrm{~m} / \mathrm{s}^{2} .
$$

Compute the tension.
$T=m_{1}(\mu g+a)=(9.00 \mathrm{~kg})\left[0.550\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+4.172 \mathrm{~m} / \mathrm{s}^{2}\right]=86 \mathrm{~N}$
15. (a) Strategy Since the coefficient of static friction between the blocks is greater than that between the bottom block and the floor, the two blocks will just begin to slide as a unit $(a \approx 0)$. The boy must push with a horizontal force equal to the maximum force of static friction on the bottom block due to the floor. Let the top block be 1 and the bottom block be 2. Use Newton's second law.

## Solution

$\Sigma F_{y}=N-\left(m_{1}+m_{2}\right) g=0$ and
$\Sigma F_{x}=F_{\text {boy }}-f_{\mathrm{s}, \text { max }}=F_{\text {boy }}-\mu_{\mathrm{s}} N=F_{\text {boy }}-\mu_{\mathrm{s}}\left(m_{1}+m_{2}\right) g=0$, so
$F_{\text {boy }}=\mu_{\mathrm{s}}\left(m_{1}+m_{2}\right) g=0.220(5.00 \mathrm{~kg}+2.00 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=15.1 \mathrm{~N}$.

(b) Strategy The block is now sliding and the boy is still pushing. There are two forces acting on the system of blocks, the push of the boy and the force of kinetic friction between the bottom block and the floor. There are also two forces acting on each block alone; for the top block, the forces are the push of the boy and the force of static friction due to the bottom block; for the bottom block, the force of static friction due to the top block and the force of kinetic friction due to the floor. Use Newton's second law.

Solution For the two-block system:
$\Sigma F_{x}=F_{\text {boy }}-\mu_{\mathrm{k}}\left(m_{1}+m_{2}\right) g=\left(m_{1}+m_{2}\right) a$, so $a=\frac{F_{\text {boy }}}{m_{1}+m_{2}}-\mu_{\mathrm{k}} g$.
For the top block alone:

$\Sigma F_{y}=N-m_{1} g=0$ and $\Sigma F_{x}=F_{\text {boy }}-f_{\mathrm{s}}=F_{\text {boy }}-\mu_{\mathrm{s}} N=F_{\text {boy }}-\mu_{\mathrm{s}} m_{1} g=m_{1} a$.
Substitute the expression for the acceleration $a$ found above and solve for the maximum force.
$F_{\text {boy }}-\mu_{\mathrm{s}} m_{1} g=m_{1} a=m_{1}\left(\frac{F_{\text {boy }}}{m_{1}+m_{2}}-\mu_{\mathrm{k}} g\right)$, so
$F_{\text {boy }}=\frac{\left(\mu_{\mathrm{s}}-\mu_{\mathrm{k}}\right) m_{1} g\left(m_{1}+m_{2}\right)}{m_{2}}=\frac{(0.400-0.200)(5.00 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.00 \mathrm{~kg}+2.00 \mathrm{~kg})}{2.00 \mathrm{~kg}}=34.3 \mathrm{~N}$.
16. Strategy The point at which the gravitational field is zero is somewhere along the line between the centers of the two stars. Use Newton's law of universal gravitation.

Solution The distance between the stars is $d=d_{1}+d_{2}$, where $d_{1}$ is the distance to the $F=0$ point from the star with mass $M_{1}$ and $d_{2}$ is the distance to the $F=0$ point from the other star. The forces are equal in magnitude and opposite in direction at the $F=0$ point. Let $m$ be a test mass at the $F=0$ point.
$\frac{G M_{1} m}{d_{1}^{2}}=\frac{G\left(4.0 M_{1}\right) m}{d_{2}^{2}}$, so $d_{2}=2.0 d_{1}$.
$d=d_{1}+d_{2}=d_{1}+2.0 d_{1}=3.0 d_{1}$, so $d_{1}=\frac{d}{3.0}=0.33 d$.

The gravitational field is zero approximately one third (0.33) of the distance between the stars as measured from the star with mass $M_{1}$.
17. Strategy and Solution Consider a cord attached to a wall at one end and pulled by one of the boys at the other end. The cord does not accelerate when the boy pulls it; thus, the force on the cord from the wall must be equal in magnitude to the pulling force. This situation is identical to the one in which the two boys pull from opposite ends of the cord-the tension in the cord is the same as the case when only one boy is pulling. However, if both pull from one end, the tension is doubled, so Stefan's plan is superior and thus more likely to work.
18. Strategy Use the definition of average acceleration and Newton's second law.

## Solution

(a) Compute the trout's average acceleration.

$$
a_{\mathrm{av}}=\frac{\Delta v}{\Delta t}=\frac{2 \mathrm{~m} / \mathrm{s}}{0.05 \mathrm{~s}}=40 \mathrm{~m} / \mathrm{s}^{2}, \text { so } \overrightarrow{\mathbf{a}}_{\mathrm{av}}=40 \mathrm{~m} / \mathrm{s}^{2} \text { in the direction of motion. }
$$

(b) Find the average net force on the trout.

$$
F_{\mathrm{av}}=m a_{\mathrm{av}}=\frac{W}{g} a_{\mathrm{av}}=W \frac{40 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=4 W, \text { so } \overrightarrow{\mathbf{F}}_{\mathrm{av}}=4 W \text { in the direction of motion. }
$$

(c) The trout pushes backward on the water, which then pushes the trout forward.
19. Strategy Use components of the displacement and the Pythagorean theorem to find the distance the trawler must travel to catch up to the tuna. (This is the same as the distance the tuna is from the trawler's initial position in 4.0 hours.) Let $+y$ be north and $+x$ be west.

Solution Find the distance.
$y=100.0 \mathrm{~km}+(5.00 \mathrm{~km} / \mathrm{h})(4.0 \mathrm{~h}) \cos 45^{\circ}$ and $x=(5.00 \mathrm{~km} / \mathrm{h})(4.0 \mathrm{~h}) \sin 45^{\circ}$, so
$r=\sqrt{x^{2}+y^{2}}=\sqrt{(5.00 \mathrm{~km} / \mathrm{h})^{2}(4.0 \mathrm{~h})^{2} \sin ^{2} 45^{\circ}+\left[100.0 \mathrm{~km}+(5.00 \mathrm{~km} / \mathrm{h})(4.0 \mathrm{~h}) \cos 45^{\circ}\right]^{2}}=115 \mathrm{~km}$.
So, $v_{\text {trawler }}=\frac{115 \mathrm{~km}}{4.0 \mathrm{~h}}=29 \mathrm{~km} / \mathrm{h}$.
Find the direction.

$$
\theta=\tan ^{-1} \frac{y}{x}=\tan ^{-1} \frac{100.0 \mathrm{~km}+(5.00 \mathrm{~km} / \mathrm{h})(4.0 \mathrm{~h}) \cos 45^{\circ}}{(5.00 \mathrm{~km} / \mathrm{h})(4.0 \mathrm{~h}) \sin 45^{\circ}}=83^{\circ} \mathrm{N} \text { of } \mathrm{W}
$$

Thus, $\overrightarrow{\mathbf{v}}=29 \mathrm{~km} / \mathrm{h}$ at $83^{\circ} \mathrm{N}$ of W.
20. Strategy Find the time it takes for the newspaper to hit the ground; then use the time to find the horizontal distance the newspaper travels before it hits the ground. Finally, use the speed of the car and the coefficient of kinetic friction to find the distance the newspaper slides.

Solution Find the time it takes for the newspaper to hit the ground.
$\Delta y=\frac{1}{2} g t^{2}$, so $t=\sqrt{\frac{2 \Delta y}{g}}=\sqrt{\frac{2(1.00 \mathrm{~m})}{9.8 \mathrm{~m} / \mathrm{s}^{2}}}=0.45 \mathrm{~s}$.
Find the horizontal distance traveled by the newspaper as it travels through the air.
$\Delta x_{1}=v t=(15 \mathrm{~m} / \mathrm{s})(0.45 \mathrm{~s})=6.8 \mathrm{~m}$
Use Newton's second law to find the acceleration due to friction that stops the newspaper.
$\Sigma F_{x}=-f_{\mathrm{k}}=-\mu_{\mathrm{k}} m g=m a$, so $a=-\mu_{\mathrm{k}} g$.
Find the horizontal distance traveled by the newspaper after it hits the ground.

$$
\Delta v=v_{\mathrm{f}}^{2}-v_{\mathrm{i}}^{2}=0-v_{\mathrm{i}}^{2}=2 a \Delta x_{2}=-2 \mu_{\mathrm{k}} g \Delta x_{2}, \text { so } \Delta x_{2}=\frac{v_{\mathrm{i}}^{2}}{2 \mu_{\mathrm{k}} g}=\frac{(15 \mathrm{~m} / \mathrm{s})^{2}}{2(0.40)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}=29 \mathrm{~m} .
$$

Julia should drop the paper a distance of $6.8 \mathrm{~m}+29 \mathrm{~m}=36 \mathrm{~m}$.
21. (a) Strategy Use $v_{\mathrm{f} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{f}}{ }^{2}-v_{\mathrm{i}}{ }^{2}=2 g h$. Let the $+y$-direction be down and neglect air resistance.

Solution Let the initial speed for all three rocks be $v_{\mathrm{i}}$ and the vertical distance from the cliff to the ground be $h$. For the first rock (thrown straight down):
$v_{\mathrm{f} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{f}}^{2}-v_{\mathrm{i}}^{2}=2 g h$, so $v_{\mathrm{f}}=\sqrt{v_{\mathrm{i}}^{2}+2 g h}$.
For the second rock (thrown straight up):
$v_{\mathrm{f} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{f}}{ }^{2}-v_{\mathrm{i}}^{2}=2 g h$, so $v_{\mathrm{f}}=\sqrt{v_{\mathrm{i}}^{2}+2 g h}$.
For the third rock (thrown horizontally):
$v_{\mathrm{f} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{f} y}{ }^{2}-0=v_{\mathrm{f} y}{ }^{2}=2 g h$ and $v_{\mathrm{f} x}=v_{\mathrm{i} x}=v_{\mathrm{i}}$, so $v_{\mathrm{f}}=\sqrt{v_{\mathrm{f} x}{ }^{2}+v_{\mathrm{f} y}{ }^{2}}=\sqrt{v_{\mathrm{i}}^{2}+2 g h}$.
Therefore, just before the rocks hit the ground at the bottom of the cliff, all three have the same final speed.
(b) Strategy Use the result obtained for the final speed in part (a).

Solution Compute the final speed.

$$
v_{\mathrm{f}}=\sqrt{v_{\mathrm{i}}^{2}+2 g h}=\sqrt{(10.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15.00 \mathrm{~m})}=19.8 \mathrm{~m} / \mathrm{s}
$$

22. Strategy The question is: Will the ball be 10 ft above the ground when it has traveled $45 \mathrm{yd}(135 \mathrm{ft})$ horizontally? Use the equations of motion with a changing velocity.

Solution The ball will travel 135 ft in the time $\Delta t=\Delta x / v_{x}=\Delta x /\left(v_{\mathrm{i}} \cos \theta\right)$.
At this time, the ball will be

$$
\begin{aligned}
\Delta y & =v_{\mathrm{i}} \sin \theta \Delta t-\frac{1}{2} g(\Delta t)^{2}=v_{\mathrm{i}} \sin \theta\left(\frac{\Delta x}{v_{\mathrm{i}} \cos \theta}\right)-\frac{1}{2} g\left(\frac{\Delta x}{v_{\mathrm{i}} \cos \theta}\right)^{2} \\
& =\Delta x \tan \theta-\frac{1}{2} g\left(\frac{\Delta x}{v_{\mathrm{i}} \cos \theta}\right)^{2}=(135 \mathrm{ft}) \tan 35^{\circ}-\frac{1}{2}\left(32.2 \mathrm{ft} / \mathrm{s}^{2}\right)\left[\frac{135 \mathrm{ft}}{(21 \mathrm{~m} / \mathrm{s})(3.281 \mathrm{ft} / \mathrm{m}) \cos 35^{\circ}}\right]^{2}
\end{aligned}
$$

$=2.4 \mathrm{ft}$ above the ground.
So, the ball will not clear the goal post and your favorite team will win.
23. Strategy Use Newton's second law to find the maximum force of static friction. Then, use the equations for circular motion.

## Solution

$\Sigma F_{\mathrm{r}}=f_{\mathrm{s}}=\mu_{\mathrm{s}} N=m a_{\mathrm{r}}=m \frac{v^{2}}{r}$ and $\Sigma F_{y}=N-m g=0$, so $\mu_{\mathrm{s}} g=\frac{v^{2}}{r}$, or $v=\sqrt{\mu_{\mathrm{s}} g r}$.


Now, $v=r \omega$ and $\omega=\alpha \Delta t$.
Substitute for $v$ and solve for $\Delta t$ to find the time it takes for the coin to slide.
$\sqrt{\mu_{\mathrm{s}} g r}=v=r \omega=r \alpha \Delta t$, so $\Delta t=\frac{1}{\alpha} \sqrt{\frac{\mu_{\mathrm{s}} g}{r}}=\frac{1}{1.20 \mathrm{rad} / \mathrm{s}^{2}} \sqrt{\frac{0.110\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.130 \mathrm{~m}}}=2.40 \mathrm{~s}$
24. (a) Strategy Use Newton's second law to find the acceleration down the slope.

Then find Carlos's speed when Shannon starts from rest. Finally, find the time it takes for Shannon to catch up to Carlos, and use this time to find the distance traveled.

Solution Find the acceleration down the slope.

$\Sigma F_{x}=-f_{\mathrm{k}}+m g \sin \theta=-\mu_{\mathrm{k}} N+m g \sin \theta=m a_{x}$ and $\Sigma F_{y}=N-m g \cos \theta=0$,
so $N=m g \cos \theta$. Thus, $a_{x}=-\mu_{\mathrm{k}} g \cos \theta+g \sin \theta=g\left(\sin \theta-\mu_{\mathrm{k}} \cos \theta\right)$.
Find Carlos's speed when Shannon starts from rest.
$\Delta v=v_{\mathrm{f}}^{2}-v_{\mathrm{i}}^{2}=v_{\mathrm{c}}^{2}-0=2 a_{\mathrm{c}} \Delta x_{1}$, so $v_{\mathrm{c}}=\sqrt{2 a_{\mathrm{c}} \Delta x_{1}}$, where $\Delta x_{1}=5 \mathrm{~m}$.
Find the time when Shannon and Carlos meet.

$$
\Delta x_{\mathrm{s}}=\Delta x_{1}+\Delta x_{\mathrm{c}}
$$

$$
\frac{1}{2} a_{\mathrm{s}} t^{2}=\Delta x_{1}+v_{\mathrm{c}} t+\frac{1}{2} a_{\mathrm{c}} t^{2}
$$

$$
a_{\mathrm{s}} t^{2}=2 \Delta x_{1}+2 t \sqrt{2 a_{\mathrm{c}} \Delta x_{1}}+a_{\mathrm{c}} t^{2}
$$

$$
0=\left(a_{\mathrm{s}}-a_{\mathrm{c}}\right) t^{2}-t \sqrt{8 a_{\mathrm{c}} \Delta x_{1}}-2 \Delta x_{1}
$$

We can solve for $t$ using the quadratic formula.
$t=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$, where
$a=a_{\mathrm{s}}-a_{\mathrm{c}}=g\left(\sin \theta-\mu_{\mathrm{s}} \cos \theta\right)-g\left(\sin \theta-\mu_{\mathrm{c}} \cos \theta\right)=g\left(\mu_{\mathrm{c}}-\mu_{\mathrm{s}}\right) \cos \theta$ $=\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.10-0.010) \cos 12^{\circ} \approx 0.8627 \mathrm{~m} / \mathrm{s}^{2}$
$b=-\sqrt{8 a_{\mathrm{c}} \Delta x_{1}}=-\sqrt{8 g\left(\sin \theta-\mu_{\mathrm{c}} \cos \theta\right) \Delta x_{1}}=-\sqrt{8\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 12^{\circ}-0.10 \cos 12^{\circ}\right)(5 \mathrm{~m})} \approx-6.569 \mathrm{~m} / \mathrm{s}$
$c=-2 \Delta x_{1}=-2(5 \mathrm{~m})=-10 \mathrm{~m}$
$t \approx 8.915 \mathrm{~s}$ (The other answer, -1.3 s , is extraneous.)
The distance traveled is

$$
\Delta x_{\mathrm{s}}=\frac{1}{2} a_{\mathrm{s}} t^{2}=\frac{1}{2} g\left(\sin \theta-\mu_{\mathrm{s}} \cos \theta\right) t^{2}=\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 12^{\circ}-0.010 \cos 12^{\circ}\right)(8.915 \mathrm{~s})^{2}=77 \mathrm{~m} .
$$

(b) Strategy Shannon is moving faster than Carlos when the two meet. Shannon's speed relative to Carlos is the difference of the two speeds.

Solution Compute the relative speed.

$$
\begin{aligned}
v_{\mathrm{s}}-v_{\mathrm{c}} & =a_{\mathrm{s}} t-\left(v_{\mathrm{c}}+a_{\mathrm{c}} t\right)=\left(a_{\mathrm{s}}-a_{\mathrm{c}}\right) t-\sqrt{2 a_{\mathrm{c}} \Delta x_{1}}=\left(a_{\mathrm{s}}-a_{\mathrm{c}}\right) t-\sqrt{2 g\left(\sin \theta-\mu_{\mathrm{c}} \cos \theta\right) \Delta x_{1}} \\
& =\left(0.8627 \mathrm{~m} / \mathrm{s}^{2}\right)(8.915 \mathrm{~s})-\sqrt{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 12^{\circ}-0.10 \cos 12^{\circ}\right)(5 \mathrm{~m})}=4.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

25. (a) Strategy Use Kepler's third law. Ignore the mass of the cable.

Solution Find the height $H$.

$$
\begin{aligned}
R & =\left(\frac{G m_{\mathrm{E}} T^{2}}{4 \pi^{2}}\right)^{1 / 3}=R_{\mathrm{E}}+H, \text { so } \\
H & =\left(\frac{G m_{\mathrm{E}} T^{2}}{4 \pi^{2}}\right)^{1 / 3}-R_{\mathrm{E}}=\left(\frac{\left[6.674 \times 10^{-11} \mathrm{~m}^{3} /\left(\mathrm{kg} \cdot \mathrm{~s}^{2}\right)\right]\left(5.974 \times 10^{24} \mathrm{~kg}\right)(86,400 \mathrm{~s})^{2}}{4 \pi^{2}}\right)^{1 / 3}-6.371 \times 10^{6} \mathrm{~m} \\
& \cong 3.6 \times 10^{7} \mathrm{~m}
\end{aligned}
$$

(b) Strategy Use Newton's second law and law of universal gravitation.

Solution Find the tension in the cable.

$$
\begin{aligned}
& R=R_{\mathrm{E}}+\frac{H}{2}=6.371 \times 10^{6} \mathrm{~m}+\frac{3.5874 \times 10^{7} \mathrm{~m}}{2}=2.4308 \times 10^{7} \mathrm{~m} \\
& \Sigma F_{y}=F_{\mathrm{g}}-T=m a=\frac{m v^{2}}{R}=m \omega^{2} R, \text { so } T=\frac{G m_{\mathrm{E}} m}{R^{2}}-m\left(\frac{4 \pi^{2}}{T^{2}}\right) R . \\
& T=\frac{\left[6.674 \times 10^{-11} \mathrm{~m}^{3} /\left(\mathrm{kg} \cdot \mathrm{~s}^{2}\right)\right]\left(5.974 \times 10^{24} \mathrm{~kg}\right)(100 \mathrm{~kg})}{\left(2.4308 \times 10^{7} \mathrm{~m}\right)^{2}}-\frac{4 \pi^{2}(100 \mathrm{~kg})\left(2.4308 \times 10^{7} \mathrm{~m}\right)}{(86,400 \mathrm{~s})^{2}}=55 \mathrm{~N}
\end{aligned}
$$

26. Strategy Draw a free-body diagram for the package and use Newton's second law.

Solution Find the maximum acceleration the truck can have without the package falling of the back.
$\Sigma F_{x}=f_{\mathrm{s}}-m g \sin 10^{\circ}=\mu_{\mathrm{s}} N-m g \sin 10^{\circ}=m a_{x}$ and $\Sigma F_{y}=N-m g \cos 10^{\circ}=0$, so $N=m g \cos 10^{\circ}$.
The maximum acceleration allowed for the truck is $a_{x}=g\left(\mu_{\mathrm{s}} \cos 10^{\circ}-\sin 10^{\circ}\right)$


$$
=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(0.380 \cos 10^{\circ}-\sin 10^{\circ}\right)=2.0 \mathrm{~m} / \mathrm{s}^{2} \text {. }
$$

27. Strategy Draw a diagram. Use Newton's second law and the relationship between radial acceleration and linear speed. Refer to Example 5.7.

Solution From Example 5-7, the banking angle is given by $\theta=\tan ^{-1}\left[v_{\mathrm{b}}^{2} /(r g)\right]$, where $v_{\mathrm{b}}$ is the speed that a car can navigate the curve without any friction. Find the relationship for the slowest speed the car can go around the curve without sliding down the bank.
$\Sigma F_{y}=N_{y}-m g+f_{y}=0$ and $\Sigma F_{x}=N_{x}-f_{x}=m a_{r}=m \frac{v^{2}}{r}$, so

$m \frac{v^{2}}{r}=N_{x}-f_{x}=N \sin \theta-\mu_{\mathrm{s}} N \cos \theta$, or $v=\sqrt{\frac{r N}{m}\left(\sin \theta-\mu_{\mathrm{s}} \cos \theta\right)}$.
Find $N$.
$N_{y}-m g+f_{y}=N \cos \theta-m g+\mu_{\mathrm{s}} N \sin \theta=0$, so $N\left(\cos \theta+\mu_{\mathrm{s}} \sin \theta\right)=m g$, or $N=\frac{m g}{\cos \theta+\mu_{\mathrm{s}} \sin \theta}$.
Substitute for $N$ in $v$ and substitute for $\theta$.

$$
\begin{aligned}
v & =\sqrt{\frac{r}{m}\left(\frac{m g}{\cos \theta+\mu_{\mathrm{s}} \sin \theta}\right)\left(\sin \theta-\mu_{\mathrm{s}} \cos \theta\right)}=\sqrt{\frac{g r\left(\sin \theta-\mu_{\mathrm{s}} \cos \theta\right)}{\cos \theta+\mu_{\mathrm{s}} \sin \theta}}=\sqrt{\frac{g r\left(\tan \theta-\mu_{\mathrm{s}}\right)}{1+\mu_{\mathrm{s}} \tan \theta}} \\
& =\sqrt{\frac{g r\left[\tan \tan ^{-1} v_{\mathrm{b}}^{2} /(r g)-\mu_{\mathrm{s}}\right]}{1+\mu_{\mathrm{s}} \tan ^{2} \tan ^{-1}{v_{\mathrm{b}}^{2} /(r g)}}=\sqrt{\frac{g r\left[v_{\mathrm{b}}^{2} /(r g)-\mu_{\mathrm{s}}\right]}{1+\mu_{\mathrm{s}}\left[v_{\mathrm{b}}^{2} /(r g)\right]}}=\sqrt{\frac{v_{\mathrm{b}}^{2}-\mu_{\mathrm{s}} g r}{1+\mu_{\mathrm{s}} v_{\mathrm{b}}^{2} /(r g)}}}
\end{aligned}
$$

Thus, the slowest allowed speed is $v=\sqrt{\frac{(15.0 \mathrm{~m} / \mathrm{s})^{2}-0.120\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(75.0 \mathrm{~m})}{1+0.120(15.0 \mathrm{~m} / \mathrm{s})^{2} /\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(75.0 \mathrm{~m})\right]}}=11.5 \mathrm{~m} / \mathrm{s}$.
28. Strategy The rope has the same tension throughout its length. Use Newton's second law.

Solution Since the box is to be lifted with constant velocity, the acceleration must be zero. For the pulley on the left, we have $\Sigma F_{y}=2 T-m g=0$, so $T=m g / 2$. Since the tension is the same along the length of the rope, the minimum force required is equal to the tension. Thus, the pull force is
$F=T=\frac{m g}{2}=\frac{(98.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2}=480 \mathrm{~N}$.
29. Strategy Use Newton's second law to determine the acceleration of each block. Then, use the accelerations to describe the motion of each block.

Solution Let the positive $x$-direction be to the right.
The accelerations of each block are $a_{A}=\frac{F_{A}}{m_{A}}$ and $a_{B}=\frac{F_{B}}{m_{B}}$. In the time $\Delta t$, the blocks are displaced by $\Delta x_{A 1}=\frac{1}{2} a_{A}(\Delta t)^{2}=\frac{1}{2}\left(\frac{F_{A}}{m_{A}}\right)(\Delta t)^{2}$ and $\Delta x_{B 1}=\frac{1}{2} a_{B}(\Delta t)^{2}=\frac{1}{2}\left(\frac{F_{B}}{m_{B}}\right)(\Delta t)^{2}$. At the end of $\Delta t$, the blocks move with constant velocities until they meet. These velocities are $v_{A}=a_{A} \Delta t=\frac{F_{A}}{m_{A}} \Delta t$ and $v_{B}=a_{B} \Delta t=\frac{F_{B}}{m_{B}} \Delta t$. Let the time between $\Delta t$ and the moment when the two blocks meet be $\Delta t_{2}$. Then, the displacements of each block during $\Delta t_{2}$ are $\Delta x_{A 2}=v_{A} \Delta t_{2}=\frac{F_{A}}{m_{A}} \Delta t \Delta t_{2}$ and $\Delta x_{B 2}=v_{B} \Delta t_{2}=\frac{F_{B}}{m_{B}} \Delta t \Delta t_{2}$. Let $d=3.40 \mathrm{~m}$. Then, we have the equation $d=\frac{1}{2}\left(\frac{F_{A}}{m_{A}}\right)(\Delta t)^{2}+\frac{F_{A}}{m_{A}} \Delta t \Delta t_{2}-\frac{1}{2}\left(\frac{F_{B}}{m_{B}}\right)(\Delta t)^{2}-\frac{F_{B}}{m_{B}} \Delta t \Delta t_{2}$, where the displacements of block $B$ are subtracted because they are negative. Solving this equation for $\Delta t_{2}$ gives
$\Delta t_{2}=\frac{d-\frac{(\Delta t)^{2}}{2}\left(\frac{F_{A}}{m_{A}}-\frac{F_{B}}{m_{B}}\right)}{\left(\frac{F_{A}}{m_{A}}-\frac{F_{B}}{m_{B}}\right) \Delta t}=\frac{3.40 \mathrm{~m}-\frac{(0.100 \mathrm{~s})^{2}}{2}\left(\frac{2.00 \mathrm{~N}}{0.225 \mathrm{~kg}}-\frac{-5.00 \mathrm{~N}}{0.600 \mathrm{~kg}}\right)}{\left(\frac{2.00 \mathrm{~N}}{0.225 \mathrm{~kg}}-\frac{-5.00 \mathrm{~N}}{0.600 \mathrm{~kg}}\right)(0.100 \mathrm{~s})}=1.92 \mathrm{~s}$. So, the elapsed time from $t=0$ until the blocks meet is $\Delta t+\Delta t_{2}=0.100 \mathrm{~s}+1.92 \mathrm{~s}=2.02 \mathrm{~s}$. The distance from block $B$ 's initial position to where the two blocks meet is negative the displacement of $B$.

$$
-\frac{1}{2}\left(\frac{F_{B}}{m_{B}}\right)(\Delta t)^{2}-\frac{F_{B}}{m_{B}} \Delta t \Delta t_{2}=-\frac{1}{2}\left(\frac{-5.00 \mathrm{~N}}{0.600 \mathrm{~kg}}\right)(0.100 \mathrm{~s})^{2}-\frac{-5.00 \mathrm{~N}}{0.600 \mathrm{~kg}}(0.100 \mathrm{~s})(1.924 \mathrm{~s})=1.65 \mathrm{~m}
$$

The blocks meet at 1.65 m to the left of $B$ 's initial position.
30. (a) Strategy Use the equations for circular motion.

Solution Find the angular acceleration.
$\Delta \omega=\omega_{\mathrm{f}}-\omega_{\mathrm{i}}=\omega_{\mathrm{f}}-0=\alpha \Delta t$, so $\alpha=\frac{\omega_{\mathrm{f}}}{\Delta t}$.
Find the tangential acceleration.
$a=r \alpha=r \frac{\omega_{\mathrm{f}}}{\Delta t}=(0.100 \mathrm{~m}) \frac{1.00 \mathrm{~Hz}}{0.800 \mathrm{~s}}\left(\frac{2 \pi \mathrm{rad}}{\text { cycle }}\right)=0.785 \mathrm{~m} / \mathrm{s}^{2}$
(b) Strategy While the hamster is running, the forces on it are due to gravity and the normal force due to the wheel. Use Newton's second law.

Solution Find the normal force on the hamster.
$\Sigma F_{\mathrm{r}}=N-m g=m a_{r}=m \frac{v^{2}}{r}=m \frac{(r \omega)^{2}}{r}=m r \omega^{2}$, so
$N=m\left(g+r \omega^{2}\right)=(0.100 \mathrm{~kg})\left[9.80 \mathrm{~m} / \mathrm{s}^{2}+(0.100 \mathrm{~m})(1.00 \mathrm{~Hz})^{2}(2 \pi \mathrm{rad} / \text { cycle })^{2}\right]=1.37 \mathrm{~N}$.
31. Strategy Use the equations of motion for changing velocity.

Solution $\Delta t=\Delta t_{1}+\Delta t_{2}=0.10 \mathrm{~s}+0.15 \mathrm{~s}=0.25 \mathrm{~s}$ has elapsed since the first pellet was fired from the toy gun. The components of its velocity are given by $v_{1 x}=v_{\mathrm{i}} \cos \theta$ and $v_{1 y}=v_{\mathrm{i}} \sin \theta-g \Delta t$. The components of the second pellet's velocity are given by $v_{2 x}=v_{\mathrm{i}} \cos \theta$ and $v_{2 y}=v_{\mathrm{i}} \sin \theta-g \Delta t_{2}$. The horizontal components of the two velocities are the same, but the vertical components differ. Compute this difference.
$v_{1 y}-v_{2 y}=v_{\mathrm{i}} \sin \theta-g \Delta t-\left(v_{\mathrm{i}} \sin \theta-g \Delta t_{2}\right)=-g\left(\Delta t-\Delta t_{2}\right)=-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.25 \mathrm{~s}-0.15 \mathrm{~s})=-0.98 \mathrm{~m} / \mathrm{s}$ So, the velocity of the first pellet with respect to the second after the additional 0.15 s have passed is $0.98 \mathrm{~m} / \mathrm{s}$ directed downward.
32. Strategy Use Newton's second law and the equations of motion for changing velocity.

## Solution

(a) Find the acceleration of the crate.

$$
\Sigma F_{x}=m g \sin 35.0^{\circ}=m a_{x}, \text { so } a_{x}=g \sin 35.0^{\circ} .
$$

Find the distance traveled by the crate.

$$
\begin{aligned}
\Delta x & =\frac{1}{2} a_{x}(\Delta t)^{2}=\frac{1}{2}\left(g \sin 35.0^{\circ}\right)(\Delta t)^{2} \\
& =\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 35.0^{\circ}(2.50 \mathrm{~s})^{2}=17.6 \mathrm{~m}
\end{aligned}
$$


(b) The speed of the crate after 2.50 s of travel is

$$
v=a_{x} \Delta t=g \sin 35.0^{\circ} \Delta t=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 35.0^{\circ}(2.50 \mathrm{~s})=14.1 \mathrm{~m} / \mathrm{s} .
$$

33. Strategy Use the equations of motion for constant vertical acceleration.

## Solution

(a) The vertical and horizontal components of the projectile are given by

$$
\Delta y=\left(v_{\mathrm{i}} \sin \theta\right) \Delta t-\frac{1}{2} g(\Delta t)^{2} \text { and } \Delta x=\left(v_{\mathrm{i}} \cos \theta\right) \Delta t .
$$

When a projectile returns to its original height, $\Delta y=0$.

$$
0=\left(v_{\mathrm{i}} \sin \theta\right) \Delta t-\frac{1}{2} g(\Delta t)^{2}=v_{\mathrm{i}} \sin \theta-\frac{1}{2} g \Delta t, \text { so } \Delta t=\frac{2 v_{\mathrm{i}} \sin \theta}{g} .
$$

Substitute this value for $\Delta t$ into $R=\Delta x=\left(v_{\mathrm{i}} \cos \theta\right) \Delta t$ to find the range.

$$
R=\left(v_{\mathrm{i}} \cos \theta\right) \Delta t=v_{\mathrm{i}} \cos \theta\left(\frac{2 v_{\mathrm{i}} \sin \theta}{g}\right)=\frac{2 v_{\mathrm{i}}^{2} \sin \theta \cos \theta}{g}
$$

(b) Using the equation for the range found in part (a), we find that the range of the projectile if it is not intercepted by the wall is $R=\frac{2(50.0 \mathrm{~m} / \mathrm{s})^{2} \sin 30.0^{\circ} \cos 30.0^{\circ}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=221 \mathrm{~m}$.
(c) Find the time of flight in terms of $v_{\mathrm{i}}, \Delta x$, and $\theta$.

$$
\Delta x=\left(v_{\mathrm{i}} \cos \theta\right) \Delta t, \text { so } \Delta t=\frac{\Delta x}{v_{\mathrm{i}} \cos \theta} .
$$

Find the height at which the cannonball strikes.

$$
\begin{aligned}
y & =y_{\mathrm{i}}+\left(v_{\mathrm{i}} \sin \theta\right) \Delta t-\frac{1}{2} g(\Delta t)^{2}=y_{\mathrm{i}}+\left(v_{\mathrm{i}} \sin \theta\right)\left(\frac{\Delta x}{v_{\mathrm{i}} \cos \theta}\right)-\frac{1}{2} g\left(\frac{\Delta x}{v_{\mathrm{i}} \cos \theta}\right)^{2}=y_{\mathrm{i}}+\Delta x \tan \theta-\frac{g(\Delta x)^{2}}{2 v_{\mathrm{i}}^{2} \cos ^{2} \theta} \\
& =1.10 \mathrm{~m}+(215 \mathrm{~m}) \tan 30.0^{\circ}-\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(215 \mathrm{~m})^{2}}{2(50.0 \mathrm{~m} / \mathrm{s})^{2} \cos ^{2} 30.0^{\circ}}=4 \mathrm{~m}
\end{aligned}
$$

34. (a) Strategy Let $+y$ be down for $m_{1}$ and up for $m_{2}$, since $m_{1} \gg m_{2}$. Use Newton's second law.

Solution Find the acceleration of each block.
For $m_{1}: \sum F_{y}=m_{1} g-T=m_{1} a_{y}$, so $T=m_{1} g-m_{1} a_{y}$.
For $m_{2}: \sum F_{y}=T-m_{2} g=m_{2} a_{y}$, so $T=m_{2} g+m_{2} a_{y}$.
$T$ and $a_{y}$ are identical in these two equations. Eliminate $T$.

$$
\begin{aligned}
m_{1} g-m_{1} a_{y} & =m_{2} g+m_{2} a_{y} \\
m_{2} a_{y}+m_{1} a_{y} & =m_{1} g-m_{2} g \\
\left(m_{1}+m_{2}\right) a_{y} & =\left(m_{1}-m_{2}\right) g \\
a_{y} & =\frac{m_{1}-m_{2}}{m_{1}+m_{2}} g
\end{aligned}
$$



Since $m_{1} \gg m_{2}, m_{1}-m_{2} \approx m_{1}$ and $m_{1}+m_{2} \approx m_{1}$, so $a_{y} \approx g$.
(b) Strategy Use the results from part (a) for the tension and the vertical component of the acceleration.

Solution Find the tension.
$T=m_{2} g+m_{2} a_{y}=m_{2} g+m_{2} \frac{m_{1}-m_{2}}{m_{1}+m_{2}} g=m_{2} g\left(1+\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right)=m_{2} g \frac{m_{1}+m_{2}+m_{1}-m_{2}}{m_{1}+m_{2}}=\frac{2 m_{1} m_{2}}{m_{1}+m_{2}} g$
Since $m_{1} \gg m_{2}, m_{1}+m_{2} \approx m_{1}$, so $T \approx \frac{2 m_{1} m_{2}}{m_{1}} g=2 m_{2} g$.
35. (a) Strategy The circumference of the track is given by $C=2 \pi r$. The distance traveled is three-fourths of this.

Solution Compute the distance traveled by the runner before the collision.
distance traveled $=\frac{3}{4}(2 \pi r)=\frac{3}{2} \pi(60.0 \mathrm{~m})=283 \mathrm{~m}$
(b) Strategy Let the center of the circle be the origin, and let the runner begin at $\theta=0^{\circ}$ and collide at $\theta=270^{\circ}$.

Solution Find the components of the runner's displacement.
$r_{\mathrm{i} x}=60.0 \mathrm{~m}, r_{\mathrm{i} y}=0, r_{\mathrm{f} x}=0$ and $r_{\mathrm{f} y}=-60.0 \mathrm{~m}$.
Find the magnitude of the displacement.

$|\Delta \overrightarrow{\mathbf{r}}|=\sqrt{\left(r_{\mathrm{f} x}-r_{\mathrm{ix}}\right)^{2}+\left(r_{\mathrm{f} y}-r_{\mathrm{i} y}\right)^{2}}=\sqrt{(-60.0 \mathrm{~m})^{2}+(-60.0 \mathrm{~m})^{2}}=84.9 \mathrm{~m}$
36. Strategy Let the $+x$-direction be along the dashed line away from the sun, and let the $+y$-direction be perpendicular to the dashed line away from the Earth. Use Newton's second law.

## Solution

(a) Find the net force acting on the sailplane.

$$
\begin{aligned}
& \sum F_{x}=\left(8.00 \times 10^{2} \mathrm{~N}\right) \cos 30.0^{\circ}-173 \mathrm{~N}=m a_{x}=F_{x} \\
& \sum F_{y}=\left(8.00 \times 10^{2} \mathrm{~N}\right) \sin 30.0^{\circ}-1.00 \times 10^{2} \mathrm{~N}=m a_{y}=F_{y}
\end{aligned}
$$

Compute the magnitude of the net force.

$$
\begin{aligned}
F & =\sqrt{F_{x}^{2}+F_{y}^{2}}=\sqrt{\left[\left(8.00 \times 10^{2} \mathrm{~N}\right) \cos 30.0^{\circ}-173 \mathrm{~N}\right]^{2}+\left[\left(8.00 \times 10^{2} \mathrm{~N}\right) \sin 30.0^{\circ}-1.00 \times 10^{2} \mathrm{~N}\right]^{2}} \\
& =6.00 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

Compute the direction of the net force.
$\theta=\tan ^{-1} \frac{F_{y}}{F_{x}}=\tan ^{-1} \frac{\left(8.00 \times 10^{2} \mathrm{~N}\right) \sin 30.0^{\circ}-1.00 \times 10^{2} \mathrm{~N}}{\left(8.00 \times 10^{2} \mathrm{~N}\right) \cos 30.0^{\circ}-173 \mathrm{~N}}=30.0^{\circ}$
So, $\overrightarrow{\mathbf{F}}=6.00 \times 10^{2} \mathrm{~N}$ directed along the $8.00 \times 10^{2}-\mathrm{N}$ vector.
(b) Find the acceleration of the sailplane.
$a=\frac{F}{m}=\frac{6.00 \times 10^{2} \mathrm{~N}}{14,500 \mathrm{~kg}}=0.0414 \mathrm{~m} / \mathrm{s}^{2}$, so $\overrightarrow{\mathbf{a}}=0.0414 \mathrm{~m} / \mathrm{s}^{2}$ in the same direction as the force.
37. (a) Strategy Use Newton's law of universal gravitation. Assume the mass of the galaxy is concentrated at its center.

Solution Estimate the mass of the galaxy.

$$
\begin{aligned}
& \frac{G M m}{R^{2}}=\frac{m v^{2}}{R}, \text { so } \\
& M
\end{aligned}=\frac{v^{2} R}{G}=\frac{\left(2.75 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}(40,000 \mathrm{ly})}{6.674 \times 10^{-11} \mathrm{~m}^{3} /\left(\mathrm{kg} \cdot \mathrm{~s}^{2}\right)}\left(\frac{9.461 \times 10^{15} \mathrm{~m}}{11 \mathrm{y}}\right) .
$$

(b) Strategy Compute the ratio of the visible mass to the estimated mass.

## Solution

$\frac{10^{11}}{2.16 \times 10^{11}}=0.46$
38. Strategy Draw and analyze the vector diagrams. The velocity of the water relative to the sailboat is opposite to the velocity of the sailboat relative to the water.

## Solution

(a) a = air, w = water, s = sailboat

(b) According to the vector diagrams, the apparent wind speed is greater than the true wind speed in cases 1 and 2 .
(c) According to the vector diagrams, the apparent wind direction is forward of the true wind in all three cases.

## MCAT Review

1. Strategy and Solution Gravity contributes an acceleration of $-g$. Air resistance is always opposite an object's direction of motion, so the vertical component of the acceleration contributed by air resistance is negative as well. According to Newton's second law, $F=m a$, so the magnitude of the acceleration due to air resistance is $a_{\mathrm{R}}=F_{\mathrm{R}} / m=b v^{2} / m$. Since we want the vertical component of acceleration, the correct answer is D , $-g-\left(b v v_{y}\right) /(0.5 \mathrm{~kg})$.
2. Strategy Use the result for the range derived in Problem 4.48 b.

Solution Assuming air resistance is negligible, the horizontal distance the projectile travels before returning to the elevation from which it was launched is $R=\frac{v_{\mathrm{i}}^{2} \sin 2 \theta}{g}=\frac{(30 \mathrm{~m} / \mathrm{s})^{2} \sin \left[2\left(40^{\circ}\right)\right]}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=90 \mathrm{~m}$. Thus, the correct answer is C .
3. Strategy and Solution The magnitude of the horizontal component of air resistance is
$F_{\mathrm{R}} \cos \theta=b v^{2} \cos \theta=b v(v \cos \theta)=b v v_{x}$. Thus, the correct answer is D .
4. Strategy Use Newton's second law to analyze each case. For simplicity, consider only vertical motion.

Solution Let the positive $y$-direction be up.
On the way up:
$\Sigma F_{y}=-m g-b v^{2}=m a_{y}$, so $a_{y}=-g-\frac{b v^{2}}{m}$.
On the way down:
$\Sigma F_{y}=-m g+b v^{2}=m a_{y}$, so $a_{y}=-g+\frac{b v^{2}}{m}$.
The magnitude of the acceleration is greater on the way up than on the way down. On the way up, the magnitude of the acceleration is never less than $g$. On the way down, it may be as small as zero. The projectile must travel the same distance in each case. So, when a projectile is rising, it begins with an initial speed which is reduced to zero relatively quickly due to the relatively large negative acceleration it experiences. When a projectile is falling, it begins with zero speed and is accelerated toward the ground by a smaller acceleration relative to when it is rising. Thus, it must take the projectile longer to reach the ground than to reach its maximum height; therefore, the correct answer is C .
5. Strategy Find the time it takes to cross the river. Use this time and the speed of the river to find how far downstream the raft travels while crossing. Then use the Pythagorean theorem to find the total distance traveled.

Solution Let $x$ be the width of the river and $y$ be the distance traveled down the river during the crossing. The raft takes the time $\Delta t=x / v_{\text {raft }}$ to cross the river. During this time, the raft travels the distance $y=v_{\text {river }} \Delta t=v_{\text {river }}\left(x / v_{\text {raft }}\right)$ down the river. Compute the distance traveled.
$\sqrt{x^{2}+y^{2}}=\sqrt{x^{2}+\left(\frac{v_{\text {river }}}{v_{\text {raft }}} x\right)^{2}}=x \sqrt{1+\left(\frac{v_{\text {river }}}{v_{\text {raft }}}\right)^{2}}=(200 \mathrm{~m}) \sqrt{1+\left(\frac{2 \mathrm{~m} / \mathrm{s}}{2 \mathrm{~m} / \mathrm{s}}\right)^{2}}=283 \mathrm{~m}$
The correct answer is C .
6. Strategy To row directly across the river, the component of the raft's velocity that is antiparallel to the current of the river must equal the speed of the current, $2 \mathrm{~m} / \mathrm{s}$.

Solution Since the angle is relative to the shore, the antiparallel component of the raft's velocity is $(3 \mathrm{~m} / \mathrm{s}) \cos \theta$. Set this equal to the speed of the current and solve for $\theta$.
$(3 \mathrm{~m} / \mathrm{s}) \cos \theta=2 \mathrm{~m} / \mathrm{s}$, so $\cos \theta=\frac{2}{3}$, or $\theta=\cos ^{-1} \frac{2}{3}$. The correct answer is D .
7. Strategy Use $\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}$.

Solution Find the time it takes the rock to reach the ground.
$\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}=(0) \Delta t-\frac{1}{2} g(\Delta t)^{2}$, so $\Delta t=\sqrt{-\frac{2 \Delta y}{g}}=\sqrt{-\frac{2(0-100 \mathrm{~m})}{10 \mathrm{~m} / \mathrm{s}^{2}}}=4.5 \mathrm{~s}$.
The correct answer is A .

