REVIEW AND SYNTHESIS: CHAPTERS 6–8

Review Exercises

1. (a) Strategy Multiply the extension per mass by the mass to find the maximum extension required.

Solution

$$\left(\frac{1.0\text{ mm}}{25\text{ g}}\right)(5.0\text{ kg})\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right)=\boxed{0.20\text{ m}}$$

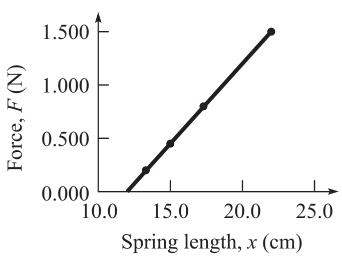
(b) Strategy Set the weight of the mass equal to the magnitude of the force due to the spring scale. Use Hooke's law.

Solution

$$\text{Weight} = mg = kx,\text{ so } k=\frac{mg}{x}=\frac{(5.0\text{ kg})(9.80\text{ N/kg})}{0.20\text{ m}}=\boxed{250\text{ N/m}}.$$

2. Strategy Plot force on the y-axis and the spring length on the x-axis. Use the graph to answer the questions.

Solution Graph the data.



(a) Determine the slope of the line to find k, since $F = kx$.

$$k=\frac{1.20\text{ N}-0\text{ N}}{20.0\text{ cm}-12.0\text{ cm}}=\frac{1.20\text{ N}}{8.0\text{ cm}}=\boxed{0.15\text{ N/cm}}$$

(b) The force on the spring is zero when the spring is relaxed, so from the figure, $x_0=\boxed{12\text{ cm}}$.

3. Strategy Use conservation of energy and Newton's second law.

Solution Relate the speed to the length of the cord.

$$\Delta K=\frac{1}{2}mv^2-0=\frac{1}{2}mv^2=-\Delta U=-mg\Delta y=mg\frac{L}{2},\text{ so } v^2=gL.$$

Use Newton's second law and solve for the tension.

$$\sum F_y=T-mg=ma_\text{r}=m\frac{v^2}{r}=m\frac{gL}{L}=mg,\text{ so } T=\boxed{2mg}.$$

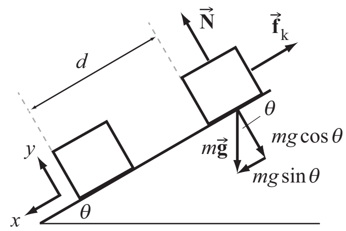
4. Strategy The work done by the muscles is 22% of the energy expended. The gravitational potential energy gained by the person is equal to the work done by the muscles.

Solution $0.22E = W = \Delta U = mgh,$ so

$$E = \frac{mgh}{0.22} = \frac{(80.0\text{ kg})(9.80\text{ N/kg})(15\text{ m})}{0.22} = \boxed{53\text{ kJ}}.$$

5. (a) Strategy Use the conservation of energy.

Solution Find the work done by friction.



$$W_{\text{total}} = W_{\text{friction}} + W_{\text{grav}} = W_{\text{friction}} + mgd\sin\theta = \Delta K = 0 - \frac{1}{2}mv_{\text{i}}^2, \text{ so}$$

$$W_{\text{friction}} = -\frac{1}{2}mv_{\text{i}}^2 - mgd\sin\theta = -m\left(\frac{1}{2}v_{\text{i}}^2 + gd\sin\theta\right)$$

$$= -(100\text{ kg})\left[\frac{1}{2}(2.00\text{ m/s})^2 + (9.80\text{ m/s}^2)(1.50\text{ m})\sin 30.0°\right]$$

$$= -940\text{ J}.$$

Thus, the energy dissipated by friction was $\boxed{940\text{ J}}$.

(b) Strategy Use Newton's second law.

Solution Find the normal force on the crate.
$\Sigma F_y = N - mg\cos\theta = 0,$ so $N = mg\cos\theta.$

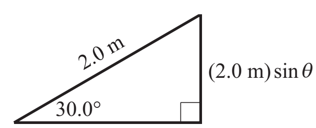
Since $v_{\text{fx}}^2 - v_{\text{ix}}^2 = 0 - v_{\text{i}}^2 = 2a_x\Delta x = 2a_x d,$ the acceleration of the crate is $-v_{\text{i}}^2/(2d)$.
Find the force of sliding friction.

$$\Sigma F_x = -f_{\text{k}} + mg\sin\theta = -\mu_{\text{k}}mg\cos\theta + mg\sin\theta = ma_x = -m\frac{v_{\text{i}}^2}{2d}, \text{ so}$$

$$\mu_{\text{k}} = \tan\theta + \frac{v_{\text{i}}^2}{2dg\cos\theta} = \tan 30.0° + \frac{(2.00\text{ m/s})^2}{2(1.50\text{ m})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.734}.$$

6. Strategy Use conservation of energy.

Solution Find the speed of the packing carton at the bottom of the inclined plane.



$$\Delta K = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = -\Delta U = -mg\Delta y, \text{ so}$$

$$v_{\text{f}} = \sqrt{v_{\text{i}}^2 - 2g\Delta y} = \sqrt{(4.0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)[0 - (2.0\text{ m})\sin 30.0°]}$$

$$= \boxed{6.0\text{ m/s}}.$$

7. Strategy Use conservation of energy.

Solution Find the maximum height of the swing.

$$\Delta K = 0 - \frac{1}{2}mv^2 = -\Delta U = mgh_{\text{i}} - mgh_{\text{max}}, \text{ so } h_{\text{max}} = \frac{v^2}{2g} + h_{\text{i}} = \frac{(6.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} + 0.50\text{ m} = \boxed{2.3\text{ m}}.$$

8. Strategy Use conservation of energy and Newton's second law.

Solution Find the normal force on the crate.

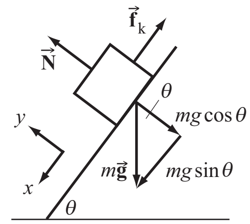
$\Sigma F_y = N - mg\cos\theta = 0,$ so $N = mg\cos\theta.$

Find the force of sliding friction.

$\Sigma F_x = -f_k + mg\sin\theta = -\mu_k mg\cos\theta + mg\sin\theta = ma_x,$ so

$a_x = -\mu_k g\cos\theta + g\sin\theta = -0.70(9.80\ \text{m/s}^2)\cos 53° + (9.80\ \text{m/s}^2)\sin 53° = 3.7\ \text{m/s}^2.$

Therefore, the acceleration of the block is $\boxed{3.7\ \text{m/s}^2\ \text{down the ramp}}$.



9. Strategy The collision is inelastic. Use conservation of momentum and energy.

Solution Write equations using conservation of momentum and energy.

momentum: $mv_i = (m+M)v_f$

energy: $\frac{1}{2}(m+M)v_f^2 = (m+M)g\Delta y$

Find the initial speed of the putty.

$$\frac{1}{2}(m+M)\left(\frac{mv_i}{m+M}\right)^2 = (m+M)g\Delta y$$

$$\left(\frac{m}{m+M}\right)^2 v_i^2 = 2g\Delta y$$

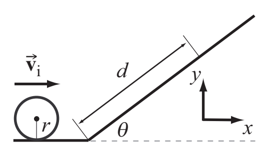
$$v_i = \sqrt{2g\Delta y\left(\frac{m+M}{m}\right)^2} = \sqrt{2(9.8\ \text{m/s}^2)(1.50\ \text{m})\left(\frac{0.50\ \text{kg}+2.30\ \text{kg}}{0.50\ \text{kg}}\right)^2} = \boxed{30\ \text{m/s}}$$

10. Strategy Use conservation of energy. The rotational inertia of a hollow cylinder is $I = mr^2$.

Solution Find d, the distance the cylinder travels up the incline.

$$0 = \Delta K + \Delta U = -\frac{1}{2}mv_i^2 - \frac{1}{2}I\omega^2 + mgd\sin\theta = -\frac{1}{2}mv_i^2 - \frac{1}{2}mr^2\left(\frac{v_i}{r}\right)^2 + mgd\sin\theta,$$

$$\text{so } d = \frac{v_i^2}{g\sin\theta} = \frac{(3.00\ \text{m/s})^2}{(9.80\ \text{m/s}^2)\sin 37.0°} = \boxed{1.53\ \text{m}}.$$



11. Strategy The rotational inertia of a wheel about its central axis is $I = \frac{1}{2}MR^2$. Use the rotational form of Newton's second law.

Solution

(a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(20.0\ \text{kg})(0.224\ \text{m})^2 = \boxed{0.502\ \text{kg}\cdot\text{m}^2}$

(b) The torque required to overcome the friction must be added to that necessary to accelerate the wheel to 1200 rpm in 4.00 s in the absence of friction to get the net torque necessary to accelerate the wheel to 1200 rpm in 4.00 s. Find the torque.

$$\Sigma\tau = I\alpha + I\alpha_f = I(\alpha + \alpha_f) = \frac{1}{2}MR^2\left[\frac{\Delta\omega}{\Delta t} + \left(\frac{\Delta\omega}{\Delta t}\right)_f\right]$$

$$= \frac{1}{2}(20.0\ \text{kg})(0.224\ \text{m})^2\left(\frac{1200\ \text{rpm}}{4.00\ \text{s}} + \frac{1200\ \text{rpm}}{60.0\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{17\ \text{N}\cdot\text{m}}$$

12. Strategy Use the work-kinetic energy theorem. The rotational inertia of a thin hoop is $I = mr^2$. The distance d the bike travels while slowing is equal to the distance the friction force is applied to each wheel.

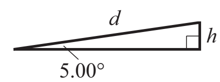
Solution The force of friction on one of the wheels due to one brake pad is $f_\text{k} = \mu_\text{k} N = 0.90N$. Assuming constant acceleration, the distance the bike travels in the time $\Delta t = 4.5\text{ s}$ is $\Delta x = d = (1/2)(v_{\text{f}x} + v_{\text{i}x})\Delta t = (1/2)v_\text{i}\Delta t$. Find the normal force on the wheel due to one brake pad.

$$W_\text{total} = W_\text{friction} = -4f_\text{k}d = -4(0.90)N\left(\frac{v_\text{i}\Delta t}{2}\right) = -1.8Nv_\text{i}\Delta t = \Delta K = 0 - \frac{1}{2}m_\text{b}v_\text{i}^2 - 2\left(\frac{1}{2}I_\text{w}\omega^2\right)$$

$$= -\frac{1}{2}m_\text{b}v_\text{i}^2 - m_\text{w}r^2\left(\frac{v_\text{i}}{r}\right)^2 = -\frac{1}{2}m_\text{b}v_\text{i}^2 - m_\text{w}v_\text{i}^2, \text{ so } N = \frac{\frac{1}{2}m_\text{b}v_\text{i} + m_\text{w}v_\text{i}}{1.8\Delta t} = \frac{(7.5\text{ m/s})[\frac{1}{2}(11\text{ kg}) + 1.3\text{ kg}]}{1.8(4.5\text{ s})} = \boxed{6.3\text{ N}}.$$

13. Strategy Use conservation of energy. Let $d = 2.05$ m. Then, the ramp rises $h = d\sin 5.00°$. The rotational inertia of a uniform sphere is $\frac{2}{5}mr^2$.

Solution Find the speed of the ball when it reaches the top of the ramp.



$$0 = \Delta K + \Delta U = \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}I\omega_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 - \frac{1}{2}I\omega_\text{i}^2 + mgh$$

$$= \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_\text{f}}{r}\right)^2 - \frac{1}{2}mv_\text{i}^2 - \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_\text{i}}{r}\right)^2 + mgh$$

$$= \frac{7}{10}mv_\text{f}^2 - \frac{7}{10}mv_\text{i}^2 + mgh, \text{ so}$$

$$v_\text{f} = \sqrt{v_\text{i}^2 - \frac{10}{7}gh} = \sqrt{(2.20\text{ m/s})^2 - \frac{10}{7}(9.80\text{ m/s}^2)(2.05\text{ m})\sin 5.00°} = \boxed{1.53\text{ m/s}}.$$

14. Strategy Use conservation of angular momentum, Eq. (8-1), Eq. (8-14), and the relationship between period and angular velocity.

Solution

(a) Since the angular momentum is conserved, the ratio is $\boxed{1}$.

(b) Since the rotational inertia is proportional to the square of the radius, $\omega = \frac{L}{I} \propto \frac{L}{r^2}$.

Find the ratio of the angular velocities.

$$\frac{\omega_\text{f}}{\omega_\text{i}} = \frac{r_\text{i}^2}{r_\text{f}^2} = \left(\frac{1}{1.0\times10^{-4}}\right)^2 = \boxed{1.0\times10^8}$$

(c) The rotational kinetic energy is $K_\text{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2 = \frac{1}{2}L\omega.$

Find the ratio of the rotational kinetic energies.

$$\frac{K_\text{f}}{K_\text{i}} = \frac{\omega_\text{f}}{\omega_\text{i}} = \boxed{1.0\times10^8}$$

(d) The period is related to the angular velocity by $T = \frac{2\pi}{\omega}$.

Find the period of the star after collapse.

$$\frac{T_\text{f}}{T_\text{i}} = \frac{\omega_\text{i}}{\omega_\text{f}}, \text{ so } T_\text{f} = \frac{\omega_\text{i}}{\omega_\text{f}}T_\text{i} = (1.0\times10^{-8})(1.0\times10^7\text{ s}) = \boxed{0.10\text{ s}}.$$

15. Strategy Assume the collision time between the dart and the block is short so that the block's motion during the collision can be neglected. Let the dart be fired to the right and let the positive *x*-direction be to the right. Let the origin be at the original position of the block. Use conservation of momentum during the collision and conservation of energy after.

Solution Find the speed v of the dart and block just after the collision.

$$p_i = m_d v_d = p_f = (m_d + m_b)v, \text{ so } v = \frac{m_d}{m_d + m_b} v_d = \frac{0.122\text{ kg}}{0.122\text{ kg} + 5.00\text{ kg}}(132\text{ m/s}) = 3.144\text{ m/s}.$$

Find the compression of the spring.

$$W_{\text{total}} = W_{\text{friction}} + W_{\text{spring}} = -f_k x - \frac{1}{2}kx^2 = -\mu_k Nx - \frac{1}{2}kx^2 = -\mu_k(m_d + m_b)gx - \frac{1}{2}kx^2 = \Delta K = 0 - \frac{1}{2}(m_d + m_b)v^2,$$

so $0 = kx^2 + 2\mu_k(m_d + m_b)gx - (m_d + m_b)v^2$. Solve for x.

$$x = \frac{-2\mu_k(m_d + m_b)g \pm \sqrt{[2\mu_k(m_d + m_b)g]^2 - 4k[-(m_d + m_b)v^2]}}{2k}$$

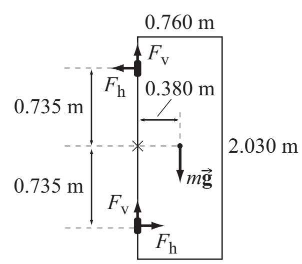
$$= \frac{-(0.630)(5.122\text{ kg})(9.80\text{ m/s}^2) \pm \sqrt{[(0.630)(5.122\text{ kg})(9.80\text{ m/s}^2)]^2 + (8.56\text{ N/m})(5.122\text{ kg})(3.144\text{ m/s})^2}}{8.56\text{ N/m}}$$

$$= -3.69\text{ m} \pm 4.42\text{ m} = 0.73\text{ m or } -8.11\text{ m}$$

Since $x > 0$, the maximum compression is $\boxed{0.73\text{ m}}$.

16. Strategy Use the conditions for equilibrium.

Solution Find the vertical components of the forces on each hinge.



$$\Sigma F_y = 2F_v - mg = 0, \text{ so } F_v = \frac{mg}{2} = \frac{(5.60\text{ kg})(9.80\text{ m/s}^2)}{2} = \boxed{27.4\text{ N}}.$$

Let the axis of rotation be a the midpoint of the left edge of the door. The only horizontal forces are the horizontal components of the forces on the hinges, therefore, these force are equal and opposite.

$$\Sigma\tau = (0.735\text{ m})F_h - (0.380\text{ m})(5.60\text{ kg})(9.80\text{ m/s}^2) + (0.735\text{ m})F_h = 0, \text{ so}$$

$$F_h = \frac{(0.380\text{ m})(5.60\text{ kg})(9.80\text{ m/s}^2)}{2(0.735\text{ m})} = 14.2\text{ N}.$$

The upper and lower horizontal forces on the hinges are 14.2 N away from the door and 14.2 N toward the door, respectively.

17. **Strategy** Use conservation of energy. The energy delivered to the fluid in the beaker plus the kinetic energies of the pulley, spool, axle, paddles, and the block are equal to the work done by gravity on the block, which is negative the change in the block's gravitational potential energy. The rotational inertia of the pulley (uniform solid disk) is $\frac{1}{2}m_p r^2$.

Solution Let the energy delivered to the fluid be E, the distance the block falls be h, and the rotational inertia of the spool, axle, and paddles be $I_s = 0.00140\ \text{kg}\cdot\text{m}^2$. Since the radii of the pulley and the spool are the same (r), their tangential speeds are the same, so let $v_p = v_s = v$.

$$m_b gh = \frac{1}{2}m_b v_b^2 + \frac{1}{2}I_p\omega_p^2 + \frac{1}{2}I_s\omega_s^2 + E = \frac{1}{2}m_b v_b^2 + \frac{1}{2}\left(\frac{1}{2}m_p r^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}I_s\left(\frac{v}{r}\right)^2 + E$$

The tangential speeds of the pulley and spool are equal to the speed of the block.

$$m_b gh = \frac{1}{2}m_b v_b^2 + \frac{1}{4}m_p v^2 + \frac{1}{2}I_s\frac{v^2}{r^2} + E = \frac{1}{2}m_b v^2 + \frac{1}{4}m_p v^2 + \frac{1}{2}I_s\frac{v^2}{r^2} + E, \text{ so}$$

$$E = m_b gh - \frac{v^2(2m_b + m_p + 2I_s/r^2)}{4}$$

$$= (0.870\ \text{kg})(9.80\ \text{m/s}^2)(2.50\ \text{m}) - \frac{(3.00\ \text{m/s})^2[2(0.870\ \text{kg}) + 0.0600\ \text{kg} + 2(0.00140\ \text{kg}\cdot\text{m}^2)/(0.0300\ \text{m})^2]}{4}$$

$$= \boxed{10.3\ \text{J}}.$$

18. **Strategy** Use conservation of linear momentum, the work-kinetic energy theorem, and Newton's second law.

Solution According to Newton's second law, the normal force of the ground on the players is $N = (m_1 + m_2)g$, where $m_1 = 85$ kg and $m_2 = 95$ kg. The force of friction is opposite the players direction of motion and has a magnitude of $f_k = \mu_k N = \mu_k(m_1 + m_2)g$. Find the initial speed v_2 of the two-player combination.

$$p_i = m_1 v_1 = p_f = (m_1 + m_2)v_2, \text{ so } v_2 = \frac{m_1 v_1}{m_1 + m_2}.$$

Find the distance d the players slide.

$$W_{\text{total}} = W_{\text{friction}} = -f_k d = -\mu_k(m_1 + m_2)gd = \Delta K = 0 - \frac{1}{2}(m_1 + m_2)v_2^2 = -\frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_1}{m_1 + m_2}\right)^2, \text{ so}$$

$$d = \frac{m_1^2 v_1^2}{2\mu_k(m_1 + m_2)^2 g} = \frac{(85\ \text{kg})^2(8.0\ \text{m/s})^2}{2(0.70)(85\ \text{kg} + 95\ \text{kg})^2(9.80\ \text{m/s}^2)} = \boxed{1.0\ \text{m}}.$$

19. **Strategy** Since the collision is elastic, kinetic energy is conserved. Use conservation of linear momentum and conservation of energy.

Solution The kinetic energy of bob A just before is strikes bob B is $\frac{1}{2}m_\text{A}v_\text{Ai}{}^2 = m_\text{A}gh,$ so $v_\text{Ai} = \sqrt{2gh},$ where h is the height fallen by bob A, 5.1 m. Since the mass of bob A is half that of bob B, let $m = m_\text{A}$ and $2m = m_\text{B}$. Since kinetic energy is conserved, we have

$m_\text{A}gh = \frac{1}{2}m_\text{A}v_\text{Ai}{}^2 = \frac{1}{2}m_\text{A}v_\text{A}{}^2 + \frac{1}{2}m_\text{B}v_\text{B}{}^2 = \frac{1}{2}mv_\text{A}{}^2 + mv_\text{B}{}^2 = mgh$ (1),

where v_A and v_B are the speeds of the bobs just after the collision.

Use conservation of linear momentum to find v_A in terms of v_B.

$p_\text{i} = mv_\text{Ai} = p_\text{f} = mv_\text{A} + 2mv_\text{B},$ so $v_\text{A} = v_\text{Ai} - 2v_\text{B} = \sqrt{2gh} - 2v_\text{B}$. Substitute this into (1) and solve for v_B.

$\frac{1}{2}m(\sqrt{2gh} - 2v_\text{B})^2 + mv_\text{B}{}^2 = \frac{1}{2}m(2gh - 4v_\text{B}\sqrt{2gh} + 4v_\text{B}{}^2) + mv_\text{B}{}^2 = mgh,$ so $v_\text{B} = \frac{2}{3}\sqrt{2gh}$. Thus, we have,

$v_\text{A} = \sqrt{2gh} - 2v_\text{B} = \sqrt{2gh} - \frac{4}{3}\sqrt{2gh} = -\frac{\sqrt{2gh}}{3}$. Now, we use conservation of energy to find how high each bob rises after the collision.

$$mgh_\text{A} = \frac{1}{2}mv_\text{A}{}^2 = \frac{1}{2}m\left(-\frac{\sqrt{2gh}}{3}\right)^2 = \frac{mgh}{9}, \text{ so } h_\text{A} = \frac{h}{9} = \frac{5.1\text{ m}}{9} = \boxed{0.57\text{ m}}.$$

$$2mgh_\text{B} = \frac{1}{2}(2m)v_\text{B}{}^2 = m\left(\frac{2\sqrt{2gh}}{3}\right)^2 = \frac{8mgh}{9}, \text{ so } h_\text{B} = \frac{4h}{9} = \frac{4(5.1\text{ m})}{9} = \boxed{2.3\text{ m}}.$$

20. Strategy Since the collision is elastic, kinetic energy is conserved. Use conservation of linear momentum. Let the positive y-direction be along the shooter's original velocity

Solution

(a) Let the mass of the marble be m, then the mass of the shooter is $3m$. After the collision, let the speed of the marble be v and the speed of the shooter be V.

$\frac{1}{2}(3m)V_\text{i}^2 = \frac{1}{2}(3m)V^2 + \frac{1}{2}mv^2$, so $3V_\text{i}^2 = 3V_x^2 + 3V_y^2 + v^2$ (1).

$p_\text{ix} = 0 = p_\text{fx} = 3mV_x + mv_x$, so $0 = 3V_x - v\sin 40°$ (2).

$p_\text{iy} = 3mV_\text{i} = p_\text{fy} = 3mV_y + mv_y$, so $3V_\text{i} = 3V_y + v\cos 40°$ (3).

We have three equations and three unknowns (V_x, V_y, and v). From (2), we have $v = \dfrac{3V_x}{\sin 40°}$ (4).

Substituting this into (3) and solving for V_y gives $V_y = V_\text{i} - V_x \cot 40°$ (5). Substitute (4) and (5) into (1) and solve for V_x.

$$3V_\text{i}^2 = 3V_x^2 + 3(V_\text{i} - V_x\cot 40°)^2 + \left(\frac{3V_x}{\sin 40°}\right)^2$$

$$3V_\text{i}^2 = 3V_x^2 + 3(V_\text{i}^2 - 2V_\text{i}V_x\cot 40° + V_x^2\cot^2 40°) + \frac{9V_x^2}{\sin^2 40°}$$

$$0 = V_x^2 - 2V_\text{i}V_x\cot 40° + V_x^2\cot^2 40° + \frac{3V_x^2}{\sin^2 40°}$$

$$0 = V_x\sin^2 40° - 2V_\text{i}\cos 40°\sin 40° + V_x\cos^2 40° + 3V_x$$

$$0 = V_x(\sin^2 40° + \cos^2 40°) + 3V_x - 2V_\text{i}\cos 40°\sin 40°$$

$$0 = V_x(1) + 3V_x - 2V_\text{i}\cos 40°\sin 40°$$

$$4V_x = 2V_\text{i}\cos 40°\sin 40° = V_\text{i}\sin 80°$$

$$V_x = \frac{V_\text{i}\sin 80°}{4}$$

Use this result and (5) to find V.

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{V_x^2 + (V_\text{i} - V_x\cot 40°)^2} = \sqrt{\left(\frac{V_\text{i}\sin 80°}{4}\right)^2 + \left[V_\text{i} - \left(\frac{V_\text{i}\sin 80°}{4}\right)\cot 40°\right]^2}$$

$$= V_\text{i}\sqrt{\frac{\sin^2 80°}{16} + \left[1 - \frac{\sin 80°\cot 40°}{4}\right]^2} = (3.2\text{ m/s})\sqrt{\frac{\sin^2 80°}{16} + \left[1 - \frac{\sin 80°\cot 40°}{4}\right]^2} = \boxed{2.4\text{ m/s}}$$

(b) Substituting the result for V_x in (4) gives

$$v = \frac{3V_x}{\sin 40°} = \frac{3V_\text{i}\sin 80°}{4\sin 40°} = \frac{3(3.2\text{ m/s})\sin 80°}{4\sin 40°} = \boxed{3.7\text{ m/s}}.$$

(c) According to the way we set up the coordinate system, the tangent of θ is equal to V_x divided by V_y instead of the usual V_y divided by V_x.

$$\theta = \tan^{-1}\frac{V_x}{V_y} = \tan^{-1}\frac{V_\text{i}\sin 80°/4}{V_\text{i} - (V_\text{i}\sin 80°/4)\cot 40°} = \tan^{-1}\frac{\sin 80°}{4 - \sin 80°\cot 40°} = \boxed{19°}$$

21. Strategy Use energy conservation to find the speed of Jones just before he grabs Smith. Then, use momentum conservation to find the speed of both just after. Finally, again use energy conservation to find the final height.

Solution Find Jones's speed, v_J.

$\frac{1}{2}m_J v_J{}^2 = m_J g h_J$, so $v_J = \sqrt{2gh_J}$.

Find the speed of both, v.

$p_i = m_J v_J = p_f = (m_J + m_S)v$, so $v = \frac{m_J v_J}{m_J + m_S} = \frac{m_J\sqrt{2gh_J}}{m_J + m_S}$.

Find the final height, h.

$(m_J + m_S)gh = \frac{1}{2}(m_J + m_S)\left(\frac{m_J\sqrt{2gh_J}}{m_J + m_S}\right)^2$, so $h = \frac{m_J{}^2 h_J}{(m_J + m_S)^2} = \frac{(78.0\text{ kg})^2(3.70\text{ m})}{(78.0\text{ kg} + 55.0\text{ kg})^2} = \boxed{1.27\text{ m}}$.

22. (a) Strategy Use the definition of angular acceleration.

Solution

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{11\text{ rad/s} - 0}{0.20\text{ s}} = \boxed{55\text{ rad/s}^2}$.

(b) Strategy Use Newton's second law for rotation.

Solution Find the torque.

$\Sigma\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = (1.5\text{ kg}\cdot\text{m}^2)\frac{11\text{ rad/s} - 0}{0.20\text{ s}} = \boxed{83\text{ N}\cdot\text{m}}$.

(c) Strategy Use Eq. (5-21).

Solution Let $\Delta\theta_1$ be the angle during spin-up and $\Delta\theta_2$ be the angle during spin-down.

$\omega_f{}^2 - \omega_i{}^2 = \omega^2 - 0 = 2\alpha_1\Delta\theta_1$, so $\Delta\theta_1 = \frac{\omega^2}{2\alpha_1}$. $\omega_f{}^2 - \omega_i{}^2 = 0 - \omega^2 = 2\alpha_2\Delta\theta_2$, so $\Delta\theta_2 = \frac{-\omega^2}{2\alpha_2}$.

Find $\Delta\theta_1 + \Delta\theta_2$.

$\Delta\theta_1 + \Delta\theta_2 = \frac{\omega^2}{2\alpha_1} + \frac{-\omega^2}{2\alpha_2} = \frac{\omega^2}{2}\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2}\right) = \frac{(11\text{ rad/s})^2}{2}\left(\frac{1}{55\text{ rad/s}^2} - \frac{1}{-9.8\text{ rad/s}^2}\right) = \boxed{7.3\text{ rad}}$

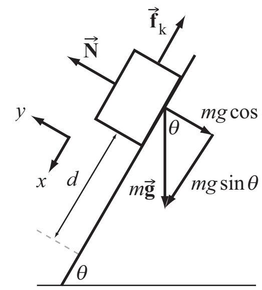
(d) Strategy Use Eq. (5-18) and the relationship between angular speed and linear speed.

Solution Find the speed of a point halfway along the radius of the disk 0.20 s after the accelerating torque is removed.

$\omega_f - \omega_i = \frac{v}{r} - \omega_i = \alpha\Delta t$, so

$v = r(\alpha\Delta t + \omega_i) = \frac{0.115\text{ m}}{2}\left[(-9.8\text{ rad/s}^2)(0.20\text{ s}) + (11\text{ rad/s})\right] = \boxed{0.52\text{ m/s}}$.

23. Strategy Use the work-kinetic energy theorem and Newton's second law.

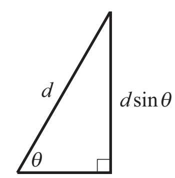


Solution $\Sigma F_y = N - mg\cos\theta = 0$, so $N = mg\cos\theta$. Thus, $f_k = \mu_k mg\cos\theta$.

$$W_{\text{total}} = W_{\text{grav}} + W_{\text{friction}} = mgd\sin\theta - \mu_k mg\cos\theta d = \Delta K = \frac{1}{2}mv_f^2, \text{ so}$$

$$v_f = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})(\sin 60.0° - 0.38\cos 60.0°)} = \boxed{2.0\text{ m/s}}.$$

24. Strategy The cylinder falls a vertical distance $h = d\sin\theta = (0.300\text{ m})\sin 60.0°$ as it rolls down the incline. The rotational inertia of a uniform solid cylinder is $\frac{1}{2}mr^2$. Use conservation of energy.

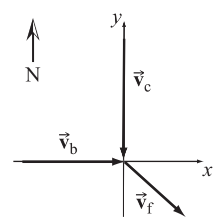


Solution Find the cylinder's final speed.

$$0 = \Delta K + \Delta U = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 - 0 + 0 - mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_f}{r}\right)^2 - mgd\sin\theta$$

$$= \frac{3}{4}mv_f^2 - mgd\sin\theta, \text{ so } v_f = \sqrt{\frac{4}{3}gd\sin\theta} = \sqrt{\frac{4}{3}(9.80\text{ m/s}^2)(0.300\text{ m})\sin 60.0°} = \boxed{1.84\text{ m/s}}.$$

25. Strategy Use conservation of linear momentum.



Solution

$$p_{ix} = m_b v_b = p_{fx} = (m_b + m_c)v_{fx}, \text{ so } v_{fx} = \frac{m_b v_b}{m_b + m_c}.$$

$$p_{iy} = m_c v_c = p_{fy} = (m_b + m_c)v_{fy}, \text{ so } v_{fy} = \frac{m_c v_c}{m_b + m_c}.$$

Compute the magnitude of the final velocity.

$$v = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(\frac{m_b v_b}{m_b + m_c}\right)^2 + \left(\frac{m_c v_c}{m_b + m_c}\right)^2} = \frac{\sqrt{(m_b v_b)^2 + (m_c v_c)^2}}{m_b + m_c}$$

$$= \frac{\sqrt{[(2.00\text{ kg})(2.70\text{ m/s})]^2 + [(1.50\text{ kg})(-3.20\text{ m/s})]^2}}{2.00\text{ kg} + 1.50\text{ kg}} = 2.06\text{ m/s}$$

Compute the angle.

$$\theta = \tan^{-1}\frac{v_{fy}}{v_{fx}} = \tan^{-1}\frac{\frac{m_c v_c}{m_b + m_c}}{\frac{m_b v_b}{m_b + m_c}} = \tan^{-1}\frac{m_c v_c}{m_b v_b} = \tan^{-1}\frac{(1.50\text{ kg})(-3.20\text{ m/s})}{(2.00\text{ kg})(2.70\text{ m/s})} = -41.6°$$

The velocity of the block and the clay after the collision is $\boxed{2.06\text{ m/s at } 41.6°\text{ S of E}}$.

26. (a) Strategy Use conservation of angular momentum.

Solution Find the tangential speed of the skaters after they grab the rods.

$L_i = I_{1i}\omega_{1i} + I_{2i}\omega_{2i} = m_1 r^2 \frac{v_1}{r} + m_2 r^2 \frac{v_2}{r} = m_1 r v_1 + m_2 r v_2 = L_f = I_f \omega_f = m_1 r v + m_2 r v$, so

$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v.$

Solve for the tangential speed.

$(m_1 + m_2)v = m_1 v_1 + m_2 v_2$, so $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(60.0\text{ kg})(6.0\text{ m/s}) + (30.0\text{ kg})(2.0\text{ m/s})}{60.0\text{ kg} + 30.0\text{ kg}} = \boxed{4.7\text{ m/s}}.$

(b) Strategy and Solution According to the RHR, the angular momentum is $\boxed{\text{upward, away from the ice}}$ before and after the collision. Angular momentum is conserved in magnitude and direction.

27. (a) Strategy Use conservation of angular momentum, since no external torques act on the two-disk system.

Solution Find the final angular velocity.

$L_f = I_f \omega_f = L_i = I_i \omega_i$, so $\omega_f = \frac{I_i \omega_i}{I_f} = \frac{I_i \omega_i}{I_i + \frac{1}{2}mr^2} = \frac{\omega_i}{1 + mr^2/[2(MR^2/2)]} = \boxed{\frac{\omega_i}{1 + mr^2/(MR^2)}}.$

(b) Strategy and Solution

$\boxed{\text{The total angular momentum does not change, since no external torques act on the system.}}$

(c) Strategy Compute the initial and final total kinetic energies and compare their values.

Solution

$$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2 = \frac{1}{4}MR^2\omega_i^2$$

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{1}{2}mr^2\right)\left(\frac{\omega_i}{1 + \frac{mr^2}{MR^2}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{1}{2}mr^2\right)\left(\frac{\frac{1}{2}MR^2\omega_i}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2}\right)^2$$

$$= \frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)^2\omega_i^2}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2} = \frac{\frac{1}{4}MR^2\omega_i^2}{1 + \frac{mr^2}{MR^2}} = \frac{K_i}{1 + \frac{mr^2}{MR^2}}$$

So, $K_f \neq K_i$. Therefore, the answer is $\boxed{\text{yes; the kinetic energy changes.}}$

28. (a) Strategy The candy is release with a horizontal speed equal to the tangential speed of the pocket of the rotating wheel. Use the relationship between angular and tangential speed and the equations of motion for a changing velocity.

Solution Find the time it takes for the candy to land.

$\Delta y = \frac{1}{2}a_y(\Delta t)^2 = \frac{1}{2}(-g)(\Delta t)^2$, so $\Delta t = \sqrt{-\frac{2\Delta y}{g}}.$

Find the candy's distance from its starting point.

$v = r|\omega|$, so $\Delta x = v\Delta t = r|\omega|\sqrt{-\frac{2\Delta y}{g}} = (0.120\text{ m})(1.60\text{ Hz})\left(\frac{2\pi\text{ rad}}{\text{cycle}}\right)\sqrt{-\frac{2(-0.240\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.267\text{ m}}.$

(b) Strategy Use the relationship between radial acceleration and angular speed.

Solution Find the radial acceleration of the candy.

$a_r = \omega^2 r = (1.60\text{ Hz})^2(2\pi\text{ rad/cycle})^2(0.120\text{ m}) = \boxed{12.1\text{ m/s}^2}$

29. (a) Strategy Consider the work-kinetic energy theorem and the impulse momentum theorem.

Solution Since the Romulan ship is twice as massive as the Vulcan ship, the Romulan ship will not travel as far as the Vulcan ship for the same engine force, since $\Delta x = (1/2)a(\Delta t)^2 = (1/2)(F/m)(\Delta t)^2$. Since $W = F\Delta x = \Delta K$, $\boxed{\text{the Vulcan ship will have the greater kinetic energy}}$. Since $\Delta p = F\Delta t$, $\boxed{\text{the ships will have the same momentum}}$.

(b) Strategy Consider the work-kinetic energy theorem and the impulse momentum theorem.

Solution Since the distances and the forces are the same, and since $W = F\Delta x = \Delta K$, $\boxed{\text{the ships will have the same kinetic energy}}$. Since $\Delta x = (1/2)a(\Delta t)^2 = (1/2)(F/m)(\Delta t)^2$, the more massive Romulan ship will have to fire its engines longer than the Vulcan ship to travel the same distance. Since $\Delta p = F\Delta t$ and the forces are the same, $\boxed{\text{the Romulan ship will have the greater momentum}}$.

(c) Strategy Refer to parts (a) and (b).

Solution For part (a), we have the following:
Vulcan:

$$\Delta K = W = F\Delta x = F\left[\frac{F}{2m}(\Delta t)^2\right] = \frac{(9.5\times10^6\ \text{N})^2(100\ \text{s})^2}{2(65{,}000\ \text{kg})} = 6.9\times10^{12}\ \text{J}$$

$$\Delta p = F\Delta t = (9.5\times10^6\ \text{N})(100\ \text{s}) = 9.5\times10^8\ \text{kg}\cdot\text{m/s}$$

Romulan:

$$\Delta K = W = F\Delta x = F\left[\frac{F}{2m}(\Delta t)^2\right] = \frac{(9.5\times10^6\ \text{N})^2(100\ \text{s})^2}{2(130{,}000\ \text{kg})} = 3.5\times10^{12}\ \text{J}$$

$$\Delta p = F\Delta t = (9.5\times10^6\ \text{N})(100\ \text{s}) = 9.5\times10^8\ \text{kg}\cdot\text{m/s}$$

$\boxed{\begin{array}{l}\text{In part (a), the momenta are the same, } 9.5\times10^8\ \text{kg}\cdot\text{m/s}\text{, but the kinetic energies differ:}\\ \text{Vulcan at } 6.9\times10^{12}\ \text{J and Romulan at } 3.5\times10^{12}\ \text{J.}\end{array}}$

For part (b), we have the following:
Vulcan:

$$\Delta K = W = F\Delta x = (9.5\times10^6\ \text{N})(100\ \text{m}) = 9.5\times10^8\ \text{J}$$

Since $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, $p = \sqrt{2mK} = \sqrt{2(65{,}000\ \text{kg})(9.5\times10^8\ \text{J})} = 1.1\times10^7\ \text{kg}\cdot\text{m/s}$.

Romulan:

$$\Delta K = W = F\Delta x = (9.5\times10^6\ \text{N})(100\ \text{m}) = 9.5\times10^8\ \text{J}$$

$$p = \sqrt{2mK} = \sqrt{2(2\times65{,}000\ \text{kg})(9.5\times10^8\ \text{J})} = 1.6\times10^7\ \text{kg}\cdot\text{m/s}.$$

$\boxed{\begin{array}{l}\text{In part (b), the kinetic energies are the same, } 9.5\times10^8\ \text{J, but the momenta differ:}\\ \text{Vulcan at } 1.1\times10^7\ \text{kg}\cdot\text{m/s and Romulan at } 1.6\times10^7\ \text{kg}\cdot\text{m/s.}\end{array}}$

30. **(a)** **Strategy** Use conservation of energy.

Solution Let d be the distance moved along the incline by m_2. Both masses move the same distance and have the same speed, since they are connected by a rope.

$$0=\Delta K+\Delta U=\frac{1}{2}m_1v^2+\frac{1}{2}m_2v^2-0-0+m_1g(-d\sin\theta)+m_2gd\sin\phi,\text{ so}$$

$$v=\sqrt{\frac{2gd(m_1\sin\theta-m_2\sin\phi)}{m_1+m_2}}=\sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})[(6.00\text{ kg})\sin 36.9°-(4.00\text{ kg})\sin 45.0°]}{6.00\text{ kg}+4.00\text{ kg}}}$$
$$=\boxed{1.7\text{ m/s}}.$$

(b) **Strategy** Use Newton's second law.

Solution Let the positive direction be along the inclines from left to right.
For m_1: $\Sigma F=-T+m_1g\sin\theta=m_1a$, so $T=m_1g\sin\theta-m_1a$.
For m_2: $\Sigma F=T-m_2g\sin\phi=m_2a$, so $T=m_2g\sin\phi+m_2a$.
Solve for the acceleration.
$m_2g\sin\phi+m_2a=m_1g\sin\theta-m_1a$, so

$$a=\frac{g(m_1\sin\theta-m_2\sin\phi)}{m_1+m_2}=\frac{(9.80\text{ m/s}^2)[(6.00\text{ kg})\sin 36.9°-(4.00\text{ kg})\sin 45.0°]}{6.00\text{ kg}+4.00\text{ kg}}=\boxed{0.76\text{ m/s}^2}.$$

Find the speed.

$$v_\text{f}^2-v_\text{i}^2=v^2-0=2ad,\text{ so }v=\sqrt{2ad}=\sqrt{2(0.76\text{ m/s}^2)(2.00\text{ m})}=\boxed{1.7\text{ m/s}}.$$

31. **Strategy** Refer to the figure and use conservation of energy.

Solution

(a) According to the graph, the particle's potential energy is $\boxed{-550\text{ J}}$. Since $E=K+U$, the kinetic energy of the particle is $K=E-U=-100\text{ J}-(-550\text{ J})=\boxed{450\text{ J}}$.

(b) The total energy is as given, $\boxed{-100\text{ J}}$. According to the graph, the potential energy is $\boxed{-100\text{ J}}$. The kinetic energy is $K=E-U=-100\text{ J}-(-100\text{ J})=\boxed{0}$.

(c) The kinetic energy is $K=E-U=-100\text{ J}-(-300\text{ J})=\boxed{200\text{ J}}$.

(d) The particle has a kinetic energy of 450 J at $t=0$, and we are told the motion is to the left. The particle will continue moving left but the kinetic energy will decrease by 450/4.5 J for every cm of travel until it reaches $x=1$ cm. At this point $K=0$, and the particle has stopped instantaneously. It will next move to the right with an increasing K until it reaches $x=5.5$ cm. At this point $K=450$ J, and this kinetic energy will be maintained as it continues moving right until it reaches $x=11$ cm. At this point, its kinetic energy will decrease by 450/2.5 J for every cm of travel until it reaches $x=13.5$ cm. At this point $K=0$, and the particle has again stopped instantaneously. It will then turn around again.

32. (a) Strategy Use conservation of angular momentum at the moment of impact.

Solution $L_i = I\omega_i = L_f = I\omega_f + rp$, where I is the rotational inertia of the blade, r is the distance from the center of the blade to the location of impact, and $p = mv$ is the momentum of the stone just after it is struck. Find the speed of the stone.

$$I\omega_i = I\omega_f + rp = I\omega_f + rmv_{tan}$$
$$I(\omega_i - \omega_f) = rmv_{tan}$$
$$v_{tan} = \frac{I(\omega_i - \omega_f)}{rm} = \frac{\frac{1}{12}ML^2(\omega_i - \omega_f)}{rm} = \frac{\frac{1}{12}M(2r)^2(\omega_i - \omega_f)}{rm} = \frac{Mr(\omega_i - \omega_f)}{3m}$$
$$= \frac{(2.0\text{ kg})(0.25\text{ m})[2\pi(60\text{ rev/s} - 55\text{ rev/s})]}{3(0.10\text{ kg})} = \boxed{52\text{ m/s}}$$

(b) Strategy Find time it takes for the stone to reach the house. Then use this time to find the distance the stone falls just before it reaches the window.

Solution Find the time.

$$\Delta x = v\Delta t,\text{ so } \Delta t = \frac{\Delta x}{v_{tan}} = \frac{10.0\text{ m}}{52.4\text{ m/s}} = 0.191\text{ s.}$$

Find the distance the stone falls.

$$\Delta y = -\frac{1}{2}g(\Delta t)^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.191\text{ s})^2 = -0.18\text{ m}$$

Since 0.18 m is less than half of 1.00 m (0.50 m), $\boxed{\text{the stone hits the window.}}$

33. Strategy Use conservation of energy. m is the mass of one wheel. M is the total mass of the system. v is the speed of the center of mass of the system (which is the same as the speed of a point on either wheel).

Solution

(a) $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 = K_{trans}$ for one wheel. $K_{rot,total} = 2\cdot\frac{1}{2}mv^2 = mv^2$ and $K_{trans,total} = \frac{1}{2}Mv^2$.

$$K_{total} = U_i$$
$$mv^2 + \frac{1}{2}Mv^2 = MgH$$
$$v^2(2m + M) = 2MgH$$
$$v = \sqrt{\frac{2MgH}{2m + M}} = \sqrt{\frac{2(80.0\text{ kg})(9.8\text{ m/s}^2)(20.0\text{ m})}{2(1.5\text{ kg}) + 80.0\text{ kg}}} = \boxed{19\text{ m/s}}$$

(b) Since the speed depends upon the combined total mass of the system, the speed at the bottom would not be the same for a less massive rider. The answer is $\boxed{\text{no}}$.

34. Strategy Find the change is height from the initial height to the height at which the vine breaks; then use the change in height to find Tarzan's speed when the vine breaks. Use Newton's second law to find the tension when the vine breaks.

Solution

(a) Find L, the length of the vine.

$$\sin\theta_\text{i}=\frac{\text{opposite}}{\text{hypotenuse}}=\frac{5.00\text{ m}}{L},\text{ so }L=\frac{5.00\text{ m}}{\sin\theta_\text{i}}=\frac{5.00\text{ m}}{\sin 60°}=5.77\text{ m}.$$

Find the change in height.

$$\Delta y=y_\text{f}-y_\text{i}=L\cos\theta_\text{f}-L\cos\theta_\text{i}=\frac{5.00\text{ m}}{\sin\theta_\text{i}}(\cos\theta_\text{f}-\cos\theta_\text{i})$$

Find Tarzan's speed when the vine breaks.

$$v_\text{f}^2-v_\text{i}^2=v^2-0=2g|\Delta y|=2gL(\cos\theta_\text{f}-\cos\theta_\text{i})=2g\frac{5.00\text{ m}}{\sin\theta_\text{i}}(\cos\theta_\text{f}-\cos\theta_\text{i})$$

$$v=\sqrt{2g\frac{5.00\text{ m}}{\sin\theta_\text{i}}(\cos\theta_\text{f}-\cos\theta_\text{i})}=\sqrt{2(9.80\text{ m/s}^2)\frac{5.00\text{ m}}{\sin 60.0°}(\cos 20.0°-\cos 60.0°)}=7.05\text{ m/s}$$

Find the tension.

$$\Sigma F=T-mg\cos\theta_\text{f}=\frac{mv^2}{r},\text{ so}$$

$$T=\frac{mv^2}{r}+mg\cos\theta_\text{f}=\frac{mgv^2}{gL}+mg\cos\theta_\text{f}=mg\left(\frac{v^2}{gL}+\cos\theta_\text{f}\right)$$

$$=(900.0\text{ N})\left[\frac{(7.05\text{ m/s})^2}{(9.80\text{ m/s}^2)(5.77\text{ m})}+\cos 20°\right]=\boxed{1.64\text{ kN}}$$

(b) At the moment of the vine breaking, the distance to ground level is

$8.00\text{ m}-L\cos\theta_\text{f}=8.00\text{ m}-(5.77\text{ m})\cos 20.0°=2.58\text{ m};$

and the distance to the river's edge is

$L\sin\theta_\text{f}=(5.77\text{ m})\sin 20.0°=1.97\text{ m}.$

The time it takes for Tarzan to reach ground level is given by

$$\Delta y=-v\sin\theta_\text{f}\Delta t-\frac{1}{2}g(\Delta t)^2,\text{ or }0=\frac{1}{2}g(\Delta t)^2+v\sin\theta_\text{f}\Delta t+\Delta y.$$

Using the quadratic formula, we find $\Delta t = 0.52$ s. The horizontal distance traveled in this time is $\Delta x=v\cos\theta_\text{f}\Delta t=(7.05\text{ m/s})\cos 20.0°(0.52\text{ s})=3.4\text{ m}$. Since 3.4 m > 1.97 m, Tarzan lands safely on the other side. The answer is $\boxed{\text{yes}}$.

35. Strategy Use the relationship between energy and work to find the boy's speed just before his friend lands on the sled and the speed when the two reach the bottom. Use conservation of momentum to find the speed of the boys just after the friend lands on the sled.

Solution Find the speed of the boy.

$$\Delta K = \frac{1}{2}m_1 v_1^2 - 0 = -\Delta U + W_{\text{friction}} = m_1 g h_1 - f d_1 = m_1 g d_1 \sin\theta - \mu m_1 g \cos\theta d_1, \text{ so}$$

$$v_1 = \sqrt{2gd_1(\sin\theta - \mu\cos\theta)} = \sqrt{2(9.8\text{ m/s}^2)(20\text{ m})(\sin 15° - 0.12\cos 15°)} = 7.48\text{ m/s}.$$

Find the initial speed of the two boys.

$$p_\text{i} = m_1 v_1 = p_\text{f} = (m_1 + m_2)v_2, \text{ so } v_2 = \frac{m_1 v_1}{m_1 + m_2} = \frac{(60\text{ kg})(7.48\text{ m/s})}{60\text{ kg} + 50\text{ kg}} = 4.08\text{ m/s}.$$

Find the final speed of the two boys.

$$\Delta K = -\Delta U + W_{\text{friction}}$$

$$\frac{1}{2}(m_1 + m_2)v_3^2 - \frac{1}{2}(m_1 + m_2)v_2^2 = (m_1 + m_2)gh_2 - fd_2 = (m_1 + m_2)gd_2 \sin\theta - \mu(m_1 + m_2)g\cos\theta d_2$$

$$v_3^2 = 2gd_2 \sin\theta - 2\mu g\cos\theta d_2 + v_2^2$$

$$v_3 = \sqrt{2gd_2(\sin\theta - \mu\cos\theta) + v_2^2}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(50\text{ m})(\sin 15° - 0.12\cos 15°) + (4.08\text{ m/s})^2} = \boxed{13\text{ m/s}}$$

36. Strategy Use conservation of momentum and the equations for motion with a constant acceleration.

Solution

(a) The banana will fall at the same rate as the monkey; therefore, you should throw the banana directly at the monkey.

$$\tan\theta = \frac{3.33\text{ m} + 1.67\text{ m}}{3.00\text{ m}}, \text{ so } \theta = \tan^{-1}\frac{5.00}{3.00} = \boxed{59.0° \text{ above the horizontal}}.$$

(b) $\boxed{\text{Since the banana will fall at the same rate as the monkey, regardless of the launch speed of the banana, the launch angle is the same for all launch speeds}}$. Relatively high launch speeds will reach the monkey relatively sooner (and higher); relatively low launch speeds will reach the monkey relatively later (and lower).

(c) Find the time it takes the banana to reach the monkey.

$$\Delta y = -\frac{1}{2}g(\Delta t)^2, \text{ so } \Delta t = \sqrt{-\frac{2\Delta y}{g}} = \sqrt{-\frac{2(-1.67\text{ m})}{9.80\text{ m/s}^2}} = 0.5838\text{ s} = t.$$

The banana reaches the monkey when it has traveled 3.00 m.

$$\Delta x = v_x \Delta t = v\cos\theta\Delta t, \text{ so } v = \frac{\Delta x}{\Delta t\cos\theta} = \frac{3.00\text{ m}}{(0.5838\text{ s})\cos 59.0°} = \boxed{9.98\text{ m/s}}.$$

(d) The speed of the monkey just before the collision is given by $v_{my} = -gt$ and $v_{mx} = 0$. The speed of the banana at this time is given by $v_{by} = v\sin\theta - gt$ and $v_{bx} = v\cos\theta$. Use conservation of momentum.

$mv_{bx} = (m+M)v_{fx}$, so

$$v_{fx} = \frac{m}{m+M}v_{bx} = \frac{m}{m+M}v\cos\theta = \frac{0.20\text{ kg}}{0.20\text{ kg}+3.00\text{ kg}}(9.98\text{ m/s})\cos 59.0° = 0.321\text{ m/s}.$$

$mv_{by} + Mv_{my} = (m+M)v_{fy}$, so

$$v_{fy} = \frac{mv_{by}+Mv_{my}}{m+M} = \frac{m(v\sin\theta - gt)+M(-gt)}{m+M} = \frac{m}{m+M}v\sin\theta - gt$$
$$= \frac{0.20\text{ kg}}{0.20\text{ kg}+3.00\text{ kg}}(9.98\text{ m/s})\sin 59.0° - (9.80\text{ m/s}^2)(0.5835\text{ s}) = -5.18\text{ m/s}.$$

The time it takes for the monkey to hit the ground is given by

$$\Delta y = v_{fy}t_2 - \frac{1}{2}gt_2^2,\text{ or } 0 = \frac{1}{2}gt_2^2 - v_{fy}t_2 + \Delta y = (4.90\text{ m/s}^2)t_2^2 + (5.18\text{ m/s})t_2 - 5.33\text{ m}.$$

Using the quadratic formula, we find $t = 0.64$ s. The horizontal distance is

$d = v_{fx}t = (0.321\text{ m/s})(0.64\text{ s}) = \boxed{0.21\text{ m}}$.

MCAT Review

1. Strategy Use conservation of momentum.

Solution

$p_i = mv_i = p_f = mv_f + p_{wall}$, so $p_{wall} = m(v_i - v_f) = (0.2\text{ kg})[2.0\text{ m/s} - (-1.0\text{ m/s})] = 0.6\text{ kg}\cdot\text{m/s}$.

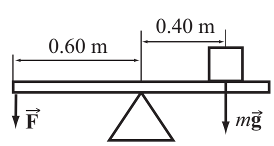
The correct answer is $\boxed{\text{D}}$.

2. Strategy Use Hooke's law.

Solution Let up be the positive direction. The gravitational force on the mass is $F = mg = (0.10\text{ kg})(-9.80\text{ m/s}^2) = -0.98\text{ N}$. Solving for the spring constant in Hooke's law, we have $k = -\frac{F}{x} = -\frac{-0.98\text{ N}}{0.15\text{ m}} = 6.5\text{ N/m}$. Thus, the correct answer is $\boxed{\text{D}}$.

3. Strategy The net torque is zero.

Solution



$\Sigma\tau = 0 = F(0.60\text{ m}) - (1.0\times10^{-7}\text{ kg})(9.80\text{ m/s}^2)(0.40\text{ m})$, so

$$F = \frac{(1.0\times10^{-7}\text{ kg})(9.80\text{ m/s}^2)(0.40\text{ m})}{0.60\text{ m}} = 6.5\times10^{-7}\text{ N}.$$

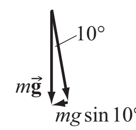
The correct answer is $\boxed{\text{B}}$.

4. **Strategy** Determine the speed of the first ball just before in collides with the second. The collision is completely inelastic; that is, the balls stick together. Use conservation of momentum to find the speed of the balls after the collision.

   Solution Find the speed of the first ball just before the collision.
   $v_{\text{fx}} - v_{\text{ix}} = v_1 - 0 = a_x \Delta t,$ so $v_1 = (10\ \text{m/s}^2)(2.0\ \text{s}) = 20\ \text{m/s}.$
   Find the speed v of the balls just after the collision.
   $$p_{\text{i}} = m_1 v_1 = p_{\text{f}} = (m_1 + m_2)v, \text{ so } v = \frac{m_1 v_1}{m_1 + m_2} = \frac{(0.50\ \text{kg})(20\ \text{m/s})}{0.50\ \text{kg} + 1.0\ \text{kg}} = 6.7\ \text{m/s}.$$
   The correct answer is $\boxed{\text{B}}$.

5. **Strategy** Use Newton's second law and Eq. (6-27).

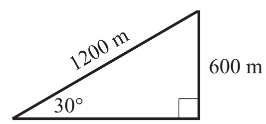
   Solution The gravitational force working against the motion of the car as it climbs the hill is $mg\sin 10°,$ so the additional power required is

   $$P_{\text{car}} = -P_{\text{grav}} = -Fv\cos 180° = (mg\sin 10°)v = (1000\ \text{kg})(10\ \text{m/s}^2)\sin 10°(15\ \text{m/s})$$
   $$= 1.5\times 10^5 \times \sin 10°\ \text{W}.$$
   The correct answer is $\boxed{\text{D}}$.

6. **Strategy** Find the vertical distance the patient would have climbed had the treadmill been stationary (and very long). Then, find the work done by the patient on the treadmill.

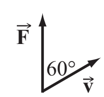
   Solution The "distance" walked along the incline is $(2\ \text{m/s})(600\ \text{s}) = 1200\ \text{m}.$
   Thus, the vertical distance climbed is $(1200\ \text{m})\sin 30° = 600\ \text{m}.$ The work done is
   $W = Fd = mgd = (90\ \text{kg})(10\ \text{m/s}^2)(600\ \text{m}) = 0.54\ \text{MJ}.$

   The correct answer is $\boxed{\text{C}}$.

7. **Strategy** Find the angle between the force exerted by the patient and the patient's velocity. Use Eq. (6-27).

   Solution The force due to gravity is down, so the force exerted by the patient is up. The velocity is directed at the angle of the incline, or 30° above the horizontal, so the angle between the force and the velocity is 60°. Compute the mechanical power output of the patient.

   $$P = Fv\cos\theta = mgv\cos\theta = (100\ \text{kg})(10\ \text{m/s}^2)(3\ \text{m/s})\cos 60° = 1500\ \text{W}$$
   The correct answer is $\boxed{\text{B}}$.

8. **Strategy and Solution** The force pushing each friction pad is normal to the wheel; that is, it is the normal force in $f_{\text{k}} = \mu_{\text{k}} N.$ Solve for the normal force.
   $$N = \frac{f_{\text{k}}}{\mu_{\text{k}}} = \frac{20\ \text{N}}{0.4} = 50\ \text{N}$$
   This is the total force. The force pushing each friction pad is half this, or 25 N. The correct answer is $\boxed{\text{B}}$.

9. **Strategy** Find the average tangential speed at the friction pads. Then, use the relationship between tangential speed and radial acceleration.

Solution

The average tangential speed is $v = \frac{4800 \text{ m}}{20 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 4.0 \text{ m/s}$. The radial acceleration is

$a_r = \frac{v^2}{r} = \frac{(4.0 \text{ m/s})^2}{0.3 \text{ m}} = 50 \text{ m/s}^2$. The correct answer is $\boxed{\text{D}}$.

10. **Strategy** Use the work-kinetic energy theorem.

Solution The work done by friction on the wheel is $W = -f_k d$, where *d* is the linear distance the wheel passes between the pads before it stops. Relate *d* to the kinetic energy of the wheel.

$W_{\text{total}} = -f_k d = \Delta K = 0 - K_i$, so $d = \frac{K_i}{f_k}$.

Divide *d* by the circumference of a circle with radius 0.3 m to find the number of rotations.

$$\frac{d}{2\pi r} = \frac{K_i}{2\pi r f_k} = \frac{30 \text{ J}}{2\pi(0.3 \text{ m})(20 \text{ N})} = 0.8 \text{ rotations}$$

Since 0.8 < 1, the correct answer is $\boxed{\text{A}}$.

11. **Strategy** Compute the average mechanical power output of the cyclist and compare it to the power consumed by the wheel at the friction pads.

Solution The metabolic power available for work is $535 \text{ W} - 85 \text{ W} = 450 \text{ W}$. Since the efficiency is 20%, the average mechanical power output of the cyclist is $0.20 \times 450 \text{ W} = 90 \text{ W}$. The average tangential speed of the wheel is $v = \frac{4800 \text{ m}}{20 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 4.0 \text{ m/s}$. Therefore, the power consumed by the friction pads is $P = f_k v = (20 \text{ N})(4.0 \text{ m/s}) = 80 \text{ W}$. Thus, the difference between the average mechanical power output of the cyclist and the power consumed by the wheel at the friction pads is $90 \text{ W} - 80 \text{ W} = 10 \text{ W}$.

The correct answer is $\boxed{\text{B}}$.

12. **Strategy and Solution** Increasing the force on the friction pads would increase the power consumed by the wheel at the friction pads (because $P = Fv$). So, if the cyclist is pedaling at the same rate and the power consumed by the friction pads increases, the difference between the two decreases and the fraction of mechanical power output of the cyclist consumed by the wheel at the friction pad increases. Thus, the correct answer is $\boxed{\text{D}}$.

13. **Strategy** Relate the cyclist's average metabolic rate to the energy released per volume of oxygen consumed, the time on the bike, and volume of oxygen consumed.

Solution The cyclist's average metabolic rate while riding is 535 W. The total energy used during 20 minutes is $(535 \text{ W})(20 \text{ min})\frac{60 \text{ s}}{1 \text{ min}} = 642{,}000 \text{ J}$. The total energy released by the consumption of oxygen is $(20{,}000 \text{ J/L})V$, where *V* is the volume of oxygen consumed. Equating these two expressions and solving for *V* gives the number of liters of oxygen the cyclist consumes.

$(20{,}000 \text{ J/L})V = 642{,}000 \text{ J}$, so $V = \frac{642{,}000 \text{ J}}{20{,}000 \text{ J/L}} = 32 \text{ L} \approx 30 \text{ L}$. The correct answer is $\boxed{\text{B}}$.

14. Strategy and Solution Since the force has been reduced by 50% and the distance has been doubled, the cyclist does the same amount of work $[W = 0.50F(2\Delta x) = F\Delta x]$. So, the energy transmitted in the second workout is equal to the energy transmitted in the first. The correct answer is $\boxed{\text{C}}$.

15. Strategy The circumference of a circle is $C = 2\pi r$. A wheel moves a distance equal to its circumference during each rotation. The wheel rotates twice during each rotation of the pedals.

Solution The circumference of a circle with a radius of 0.15 m is $2\pi(0.15 \text{ m})$. The circumference of a circle with a radius of 0.3 m is $2\pi(0.3 \text{ m})$. During each rotation of the pedals, a point on the wheel at a radius of 0.3 m moves a distance $2[2\pi(0.3 \text{ m})]$. The ratio of the distance moved by a pedal to the distance moved by a point on the wheel located at a radius of 0.3 m in the same amount of time is $\dfrac{2\pi(0.15 \text{ m})}{2[2\pi(0.3 \text{ m})]} = 0.25$.

The correct answer is $\boxed{\text{A}}$.

16. Strategy Use the definition of power.

Solution

$$P = \frac{\Delta E}{\Delta t}, \text{ so } \Delta t = \frac{\Delta E}{P} = \left(\frac{300 \text{ kcal}}{500 \text{ W}}\right)\left(\frac{4186 \text{ J}}{1 \text{ kcal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 41.9 \text{ min}.$$ The correct answer is $\boxed{\text{D}}$.

17. Strategy Consider the distance a point on the wheel travels for each situation.

Solution The circumference of a circle with a radius of 0.3 m is $2\pi(0.3 \text{ m})$. The circumference of a circle with a radius of 0.4 m is $2\pi(0.4 \text{ m})$. During each rotation, a point on a wheel travels a distance equal to the circumference. The force on the wheel is the same in each case, but the distance traveled by a point on the wheel is greater for a greater radius. In this case, the distance is $0.4 \text{ m}/(0.3 \text{ m}) = 1.33$ times farther or 33%. Since work is equal to the product of force times distance, the work done on the wheel per revolution is 33% more. Thus, the correct answer is $\boxed{\text{C}}$.