PHY2053
Summer 2012
Exam 1 Solutions

1. The period is proportional to the square root of its length

$$T \propto \sqrt{L}$$

Forming a ratio:

$$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

We want $T_2 = 1.5\ T_1$. Substituting

$$\frac{1.5T_1}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

$$\sqrt{\frac{L_2}{L_1}} = 1.5$$

$$\frac{L_2}{L_1} = (1.5)^2 = 2.25$$

2. $$60\,\frac{\text{miles}}{\text{hour}} = 60\,\frac{\text{miles}}{\text{hour}} \times \frac{1\,\text{hour}}{60\,\text{minutes}} \times \frac{1\,\text{minute}}{60\,\text{seconds}} \times \frac{5280\,\text{feet}}{1\,\text{mile}} = 88\,\frac{\text{feet}}{\text{second}}$$

3. The definition of average velocity is

$$v_{x,av} = \frac{\Delta x}{\Delta t}$$

where Δx is the total distance traveled. For the first part of the trip,

$$\Delta x_1 = v_1 \Delta t = 70\,\frac{\text{miles}}{\text{hour}} \times \frac{3}{4}\,\text{hour} = 52.5\,\text{miles}\,.$$

For the second part,

$$\Delta x_2 = v_2 \Delta t = 50\,\frac{\text{miles}}{\text{hour}} \times \frac{1}{2}\,\text{hour} = 25\,\text{miles}$$

Overall,

$$v_{x,av} = \frac{\Delta x}{\Delta t} = \frac{52.5\text{ miles} + 25\text{ miles}}{0.75\text{ hour} + 0.5\text{ hour}} = 62\text{ miles/hour}$$

4. Use

$$v_{fx}^{\ 2} - v_{ix}^{\ 2} = 2a\Delta x$$

$$a = \frac{v_{fx}^{\ 2} - v_{ix}^{\ 2}}{2\Delta x} = \frac{(15\text{ m/s})^2 - (9\text{ m/s})^2}{2(20\text{ m})} = 3.6\text{ m/s}^2$$

5. First find the time it takes for the cage to hit the ground.

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$= 0 - \tfrac{1}{2}g(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-25\text{ m})}{9.8\text{ m/s}^2}} = 2.26\text{ s}$$

Princess needs to run 3 m in that time.

$$\Delta x = v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$= 0 + \tfrac{1}{2}a_x(\Delta t)^2$$

$$a_x = \frac{2\Delta x}{(\Delta t)^2} = \frac{2(3\text{ m})}{(2.26\text{ s})^2} = 1.18\text{ m/s}^2$$

6. Solve for the initial speed.

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$= v_{iy}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$v_{iy} = \frac{\Delta y}{\Delta t} + \frac{g\Delta t}{2} = \frac{-37\text{ m}}{3.7\text{ s}} + \frac{(9.8\text{ m/s}^2)(3.7\text{ s})}{2} = 8.13\text{ m/s}$$

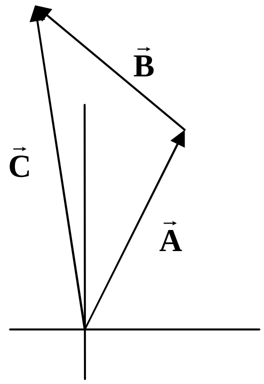
7. Find the components of each vector. For the tee shot

$$A_x = A\sin 30^\circ = (150\text{ yards})\sin 30^\circ = 75\text{ yards}$$

$$A_y = A\cos 30^\circ = (150\text{ yards})\cos 30^\circ = 130\text{ yards}$$

For the second shot

$$B_x = -B\sin 45^\circ = -(120 \text{ yards})\sin 45^\circ = -84.9 \text{ yards}$$
$$B_y = B\cos 45^\circ = (120 \text{ yards})\cos 45^\circ = 84.9 \text{ yards}$$



Add like components

$$C_x = A_x + B_x = (75 \text{ yards}) + (-84.9 \text{ yards}) = -9.9 \text{ yards}$$
$$C_y = A_y + B_y = (130 \text{ yards}) + (84.9 \text{ yards}) = 215 \text{ yards}$$

The calculator gives

$$\theta = \tan^{-1}\left(\frac{215}{-9.9}\right) = -87^\circ$$

Since the x-component is negative, we add 180° to get 93° or 3° W of N.

8. We have

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$$

Taking components

$$A_x + B_x + C_x = 0 \qquad A_y + B_y + C_y = 0$$

Solve for C_x and C_y

$$C_x = -A_x - B_x \qquad C_y = -A_y - B_y$$

We need the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$:

$$A_x = A\cos\theta_A = (50 \text{ N})\cos 30^\circ = 43.3 \text{ N}$$
$$A_y = A\sin\theta_A = (50 \text{ N})\sin 30^\circ = 25.0 \text{ N}$$

$$B_x = B\cos\theta_B = (70\text{ N})\cos 90^\circ = 0$$
$$B_y = B\sin\theta_B = (70\text{ N})\sin 90^\circ = 70.0\text{ N}$$

Finding the components of $\vec{\mathbf{C}}$,

$$C_x = -A_x - B_x = -(43.3\text{ N}) - 0 = -43.3\text{ N}$$
$$C_y = -A_y - B_y = -(25.0\text{ N}) - (70.0\text{ N}) = -95.0\text{ N}$$

The magnitude of $\vec{\mathbf{C}}$

$$C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{(-43.3\text{ N})^2 + (-95.0\text{ N})^2} = 104\text{ N}$$

From the calculator

$$\tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-95.0\text{ N}}{-43.3\text{ N}}\right) = 65^\circ$$

But since C_x is negative, add 180^o to get 245^o.

9. The relative velocity equation for this situation is

$$\vec{\mathbf{v}}_{TC} = \vec{\mathbf{v}}_{TG} + \vec{\mathbf{v}}_{GC}$$

The velocity of the ground relative to the car is the negative of the car relative to the ground, and

$$\vec{\mathbf{v}}_{TC} = \vec{\mathbf{v}}_{TG} - \vec{\mathbf{v}}_{CG}$$

The x-component equation

$$(v_{TC})_x = (v_{TG})_x - (v_{CG})_x = -v_{TG}\sin 35^\circ - 0 = -(85\text{ km/hr})\sin 35^\circ = -48.8\text{ km/hr}$$

The y-component equation

$$(v_{TC})_y = (v_{TG})_y - (v_{CG})_y = v_{TG}\cos 35^\circ - v_{CG} = (85\text{ km/hr})\sin 35^\circ - (110\text{ km/hr})$$
$$= 40.4\text{ km/hr}$$

The velocity is

$$v_{TC} = \sqrt{[(v_{TC})_x]^2 + [(v_{TC})_y]^2} = \sqrt{[-48.8\text{ km/hr}]^2 + [40.4\text{ km/hr}]^2} = 63\text{ km/hr}$$

10. If a projectile is shot horizontally, $v_{iy} = 0$. All objects thrown horizontally will take the same amount of time to reach the ground. None of these choices will make stone stay in the air any longer.

11. The motion in the x direction gives the initial velocity

$$\Delta x = v_{ix}\Delta t$$
$$v_{ix} = \frac{\Delta x}{\Delta t} = \frac{105\text{ m}}{4.2\text{ s}} = 25\text{ m/s}$$

$$v_{ix} = v_i \cos 25^\circ$$
$$v_i = \frac{v_{ix}}{\cos 25^\circ} = \frac{25.0\text{ m/s}}{\cos 25^\circ} = 27.6\text{ m/s}$$

The x-component of the velocity is unchanged since $a_x = 0$. The y-component of the velocity is found

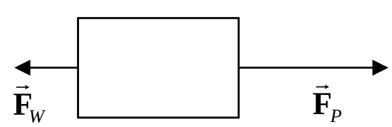
$$v_{fy} - v_{iy} = a_y \Delta t$$
$$v_{fy} = v_{iy} - g\Delta t = v_i \sin 25^\circ - g\Delta t = (27.6\text{ m/s})\sin 25^\circ - (9.8\text{ m/s}^2)(4.2\text{ s})$$
$$= -29.5\text{ m/s}$$

The final speed

$$v_f = \sqrt{{v_{fx}}^2 + {v_{fy}}^2} = \sqrt{(25.0\text{ m/s})^2 + (-29.5\text{ m/s})^2} = 38.6\text{ m/s}$$

12. The forces act on different systems. The chair starts to move because my force is greater than friction.

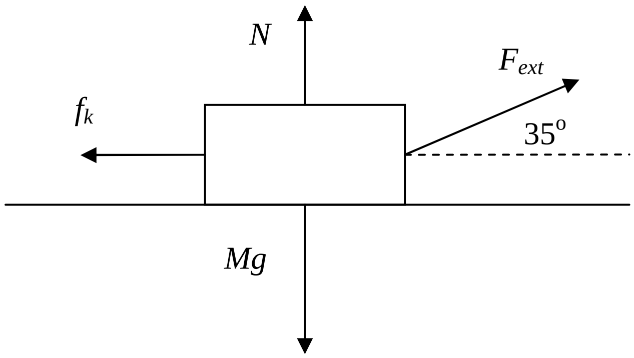
13. The free-body diagram is



Newton's second law gives

$$\sum F_x = ma_x$$
$$F_P - F_W = ma$$
$$a = \frac{F_P - F_W}{m} = \frac{600\text{ N} - 200\text{ N}}{100\text{ kg}} = 4\text{ m/s}^2$$

14. The free-body diagram is



The x-component

$$\sum F_x = Ma_x$$
$$F_{ext}\cos 35^\circ - f_k = Ma$$

The y-component

$$\sum F_y = Ma_y$$
$$N + F_{ext}\sin 35^\circ - Mg = 0$$

There are three unknowns, N, F_{ext}, and f_k. We need another equation.

$$f_k = \mu_k N$$

Solve the x-component equation for f_k

$$f_k = F_{ext}\cos 35^\circ - Ma$$

and the y-component equation for N

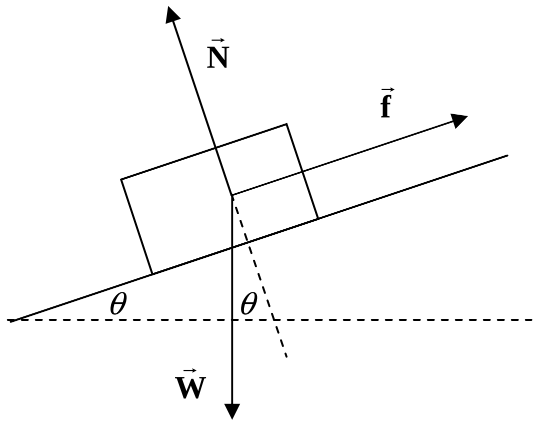
$$N = Mg - F_{ext}\sin 35^\circ$$

Now substitute into the definition of μ_k

$$\begin{aligned} F_{ext}\cos 35^\circ - Ma &= \mu_k(Mg - F_{ext}\sin 35^\circ) \\ &= \mu_k Mg - \mu_k F_{ext}\sin 35^\circ \end{aligned}$$
$$F_{ext}\cos 35^\circ + \mu_k F_{ext}\sin 35^\circ = Ma + \mu_k Mg$$

$$
\begin{aligned}
F_{ext} &= \frac{M(a+\mu_k g)}{\cos 35^\circ + \mu_k \sin 35^\circ} \\
&= \frac{(3\text{ kg})[1\text{ m/s}^2 + (0.4)(9.8\text{ m/s}^2)]}{\cos 35^\circ + (0.4)\sin 35^\circ} \\
&= 14.1\text{ N}
\end{aligned}
$$

15. The free-body diagram is (assuming the block slides down the incline)



Taking the *x*-axis along the incline

$$
\begin{aligned}
\sum F_x &= ma_x \\
f - W\sin\theta &= 0 \\
f &= W\sin\theta
\end{aligned}
$$

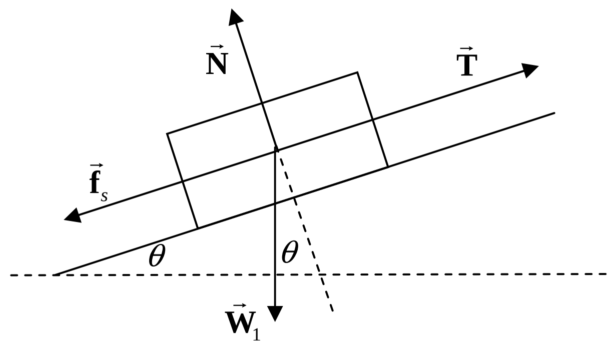
and the *y*-axis perpendicular to the incline

$$
\begin{aligned}
\sum F_y &= ma_y \\
N - W\cos\theta &= 0 \\
N &= W\cos\theta
\end{aligned}
$$

Since the block is sliding, use the coefficient of kinetic friction

$$
\begin{aligned}
f_k &= \mu_k N \\
W\sin\theta &= \mu_k W\cos\theta \\
\tan\theta &= \mu_k \\
\theta &= \tan^{-1}\mu_k \\
&= \tan^{-1}(0.4) \\
&= 21.8^\circ
\end{aligned}
$$

16. The free-body diagram for the left block is



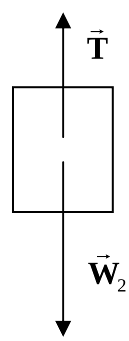
Again, take the x-axis along the incline

$$\sum F_x = m_1 a_x$$
$$T - f_s - W_1 \sin\theta = 0$$
$$f_s = T - W_1 \sin\theta$$

Along the y-axis,

$$\sum F_y = m_1 a_y$$
$$N - W_1 \cos\theta = 0$$
$$N = W_1 \cos\theta$$

The free-body diagram for the other block is



Using Newton's second law with this free-body diagram

$$\sum F_y = m_2 a_y$$
$$T - W_2 = 0$$
$$T = W_2$$

The three equations are

$$\begin{aligned} f_s &= T - W_1 \sin\theta \\ N &= W_1 \cos\theta \\ T &= W_2 \end{aligned}$$

Plug the last equation into the first

$$\begin{aligned} f_s &= W_2 - W_1 \sin\theta \\ N &= W_1 \cos\theta \end{aligned}$$

The definition of the coefficient of static friction

$$\begin{aligned} f_s &\le \mu_s N \\ W_2 - W_1 \sin\theta_2 &\le \mu_s W_1 \cos\theta \end{aligned}$$

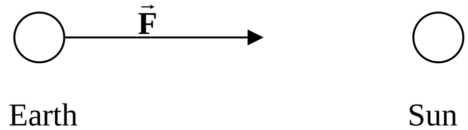
Since the blocks have the same mass, $W_1 = W_2$ and they cancel

$$\begin{aligned} 1 - \sin\theta &\le \mu_s \cos\theta \\ \mu_s &\ge \frac{1-\sin\theta}{\cos\theta} \\ \mu_s &\ge \frac{1-\sin 37^\circ}{\cos 37^\circ} \\ \mu_s &\ge 0.50 \end{aligned}$$

17. The angular speed is defined as

$$\begin{aligned} \omega &= \frac{\Delta\theta}{\Delta t} \\ &= \frac{2\pi\text{ rad}}{1\text{ day}} \times \frac{1\text{ day}}{24\text{ hours}} \times \frac{1\text{ hour}}{60\text{ minutes}} \times \frac{1\text{ minute}}{60\text{ seconds}} \\ &= 7.27\times10^{-5}\text{ rad/s} \end{aligned}$$

18. The free-body diagram is



Applying Newton's second law

$$\sum F_r = M_E a_r$$
$$F = M_E \frac{v^2}{r}$$

The force is found from Newton's law of gravity

$$F = \frac{GM_E M_S}{r^2}$$

The velocity can be found from the time it takes to complete one orbit

$$\begin{aligned} v &= \frac{\text{distance}}{\text{time}} \\ &= \frac{2\pi r}{T} \\ &= \frac{2\pi(1.50\times10^{11}\text{ m})}{1\text{ year}} \times \frac{1\text{ year}}{365\text{ days}} \times \frac{1\text{ day}}{24\text{ hours}} \times \frac{1\text{ hour}}{3600\text{ s}} \\ &= 29{,}900\text{ m/s} \end{aligned}$$

The Sun's mass can be found from Newton's second law

$$\begin{aligned} F &= M_E \frac{v^2}{r} \\ \frac{GM_E M_S}{r^2} &= M_E \frac{v^2}{r} \\ M_S &= \frac{v^2 r}{G} \\ &= \frac{(29{,}900\text{ m/s})^2(1.50\times10^{11}\text{ m})}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} \\ &= 2.0\times10^{30}\text{ kg} \end{aligned}$$

19. After 60 revolutions, the angular speed is 16 rev/s. Convert all to radians

$$\Delta\theta = 60\text{ rev}\times\frac{2\pi\text{ rad}}{\text{rev}} = 120\pi\text{ rad}$$

$$\omega = 16\,\frac{\text{rev}}{\text{s}}\times\frac{2\pi\text{ rad}}{\text{rev}} = 32\pi\,\frac{\text{rad}}{\text{s}}$$

Find the angular acceleration

$$\omega_f{}^2 - \omega_i^2 = 2\alpha\Delta\theta$$

$$\alpha = \frac{\omega_f{}^2 - \omega_i^2}{2\Delta\theta} = \frac{(32\pi \text{ rad/s})^2 - (0)^2}{2(120\pi \text{ rad/s})} = 13.4 \text{ rad/s}^2$$

The time is found

$$\Delta\theta = \omega_i\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$
$$= 0 + \tfrac{1}{2}\alpha(\Delta t)^2$$
$$\Delta t = \sqrt{\frac{2\Delta\theta}{\alpha}} = \sqrt{\frac{2(68.2 \text{ rev})}{13.4 \text{ rad/s}^2} \times \frac{2\pi \text{ rad}}{\text{rev}}} = 8.0 \text{ s}$$

20. The radial acceleration is

$$a_r = \frac{v^2}{r} = \frac{(0.5 \text{ m/s})^2}{0.3 \text{ m}} = 0.833 \text{ m/s}^2$$

The tangential acceleration is

$$a_t = \alpha r = (2.5 \text{ rad/s}^2)(0.3 \text{ m}) = 0.750 \text{ m/s}^2$$

The net acceleration is

$$a = \sqrt{a_r{}^2 + a_t{}^2} = \sqrt{(0.833 \text{ m/s}^2)^2 + (0.750 \text{ m/s}^2)^2} = 1.12 \text{ m/s}^2$$