PHY2053 Summer 2012
Exam 3
Solutions

1. The cross-sectional area of the cable is

$$A = \pi r^2 = \pi(1.2\times10^{-2}\text{ m})^2 = 4.52\times10^{-4}\text{ m}^2$$

The force exerted by the mass is due to its weight

$$F = W = mg = (250\text{ kg})(9.80\text{ m/s}^2) = 2450\text{ N}$$

Young's modulus is

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$
$$\frac{\Delta L}{L} = \frac{F}{AY} = \frac{2450\text{ N}}{(4.52\times10^{-4}\text{ m}^2)(2.0\times10^{11}\text{ Pa})} = 2.7\times10^{-5}$$

2. The definition of bulk modulus

$$\Delta P = -B\frac{\Delta V}{V}$$
$$\Delta V = \frac{-PV}{B} = \frac{-(300\times1.01\times10^5\text{ Pa})(2.00\text{ m}^3)}{50\times10^9\text{ Pa}} = -1.2\times10^{-3}\text{ m}^3$$

3. The angular frequency can be found from the period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.3\text{ s}} = 20.9\text{ rad/s}$$

The angular frequency depends on the spring constant and mass

$$\omega = \sqrt{\frac{k}{m}}$$
$$k = \omega^2 m = (20.9\text{ rad/s})^2(3\text{ kg}) = 1300\text{ N/m}$$

4. The period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Since mass does not appear in the equation, the change of mass does not affect the period. Forming a ratio

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{\frac{L_2}{g}}}{2\pi\sqrt{\frac{L_1}{g}}} = \sqrt{\frac{L_2}{L_1}}$$

Solve for T_2,

$$T_2 = T_1\sqrt{\frac{L_2}{L_1}} = (3.6\text{ s})\sqrt{\frac{2L}{L}} = 5.1\text{ s}$$

5. Conservation of energy

$$\begin{gathered} K_1 + U_1 = K_2 + U_2 \\ \tfrac{1}{2}mv_1{}^2 + \tfrac{1}{2}kx_1{}^2 = \tfrac{1}{2}mv_2{}^2 + \tfrac{1}{2}kx_2{}^2 \\ mv_1{}^2 + kx_1{}^2 = mv_2{}^2 + kx_2{}^2 \end{gathered}$$

When the mass is at its maximum amplitude, its velocity is zero.

$$mv_1{}^2 + kx_1{}^2 = mv_2{}^2 + kx_2{}^2$$
$$mv_1{}^2 + kx_1{}^2 = m(0)^2 + kA^2$$
$$A = \sqrt{\left(\frac{m}{k}\right)v_1{}^2 + x_1{}^2} = \sqrt{\left(\frac{0.75\text{ kg}}{30\text{ N/m}}\right)(0.80\text{ m/s})^2 + (0.15\text{ m})^2} = 0.20\text{ m}$$

6. The angular frequency can be found from the period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.5\text{ s}} = 4.19\text{ rad/s}$$

The equation for the velocity is

$$x = -A\omega \sin \omega t = -(0.50\text{ m})(4.19\text{ rad/s})\sin[(4.19\text{ rad/s})(5\text{ s})] = -1.8\text{ m/s}$$

7. The intensity is defined as

$$I = \frac{P}{4\pi r^2}$$

Forming a ratio

$$\frac{I_2}{I_1} = \frac{\dfrac{P}{4\pi r_2^2}}{\dfrac{P}{4\pi r_1^2}} = \left(\frac{r_1}{r_2}\right)^2$$

Solving for I_2,

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{3\text{ m}}{4\text{ m}}\right)^2 (10\text{ W/m}^2) = 5.6\text{ W/m}^2$$

8. The speed of a wave in a string is given by

$$v = \sqrt{\frac{F}{\mu}}$$

The tension (F) in the string is due to the suspended mass,

$$F = Mg = (50\text{ kg})(9.8\text{ m/s}^2) = 490\text{ N}$$

and μ is the mass per unit length of the string

$$\mu = \frac{m}{L} = \frac{0.100\text{ kg}}{8\text{ m}} = 0.0125\text{ kg/m}$$

The speed of the wave is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{490\text{ N}}{0.0125\text{ kg/m}}} = 198\text{ m/s}$$

The time needed for the wave depends on the distance and the speed of the wave,

$$\Delta t = \frac{\Delta x}{v} = \frac{8\text{ m}}{198\text{ m/s}} = 0.040\text{ s}$$

9. In a transverse wave, the individual particles of the medium move perpendicularly to the direction of the wave's travel.

10. The speed of the wave is related to its frequency and wavelength,

$$v = f\lambda$$
$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{440\text{ Hz}} = 0.77\text{ m}$$

None of the choices are correct.

11. The general equation for a wave traveling along the +x-direction is

$$y(x,t) = A\cos(\omega t - kx)$$

The angular frequency can be found from the linear frequency

$$\omega = 2\pi f = 2\pi(25\text{ Hz}) = 157\text{ rad/s}$$

The wavenumber can be found from the speed of the wave and ω,

$$v = \frac{\omega}{k}$$
$$k = \frac{\omega}{v} = \frac{157\text{ rad/s}}{50\text{ m/s}} = 3.14\text{ rad/m}$$

Substituting into the general equation

$$y(x,t) = A\cos((157\text{ rad/s})t - (3.14\text{ rad/m})x)$$

12. When the pulse on the right reflects back from the wall, it will be inverted because the rope is a fixed point. The upright and inverted pulses cancel when they overlap. The combined amplitude is zero.

13. The general equation for a standing wave is

$$y = 2A\sin\omega t\cos kx$$

The wavelength is twice the distance between two nodes,

$$\lambda = 2\times(0.30\text{ m}) = 0.60\text{ m}$$

which can be used to find the wavenumber

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.60\text{ m}} = 10.5\text{ rad/m}$$

The angular frequency is related to the linear frequency

$$\omega = 2\pi f = 2\pi(60\text{ Hz}) = 377\text{ rad/s}$$

Substituting into the general equation

$$y = 2A\sin((377\text{ rad/s})t)\cos((10.5\text{ rad/m})x)$$

14. A wave can spread around an obstacle when the wavelength is similar to the size of the obstacle. This is called **diffraction**.

15. The speed of the wave in the rope is related to the tension in the rope and its linear density,

$$v = \sqrt{\frac{F}{\mu}}$$

The linear density is

$$\mu = \frac{m}{L} = \frac{0.160\text{ kg}}{4\text{ m}} = 0.040\text{ kg/m}$$

The speed of the wave is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{400\ \text{N}}{0.040\ \text{kg/m}}} = 100\ \text{m/s}$$

The equation for the frequencies in a rope is

$$f_n = \frac{nv}{2L}$$

For the second harmonic, $n = 2$

$$f_n = \frac{nv}{2L} = \frac{2(100\ \text{m/s})}{2(4\ \text{m})} = 25\ \text{Hz}$$

16. The decibel scale is defined as

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

Taking the difference of two dB readings

$$\begin{aligned}
\beta_2 - \beta_1 &= (10\ \text{dB})\log\frac{I_2}{I_0} - (10\ \text{dB})\log\frac{I_2}{I_0} \\
&= (10\ \text{dB})\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right) \\
&= (10\ \text{dB})\left(\log\frac{I_2/I_0}{I_1/I_0}\right) \\
&= (10\ \text{dB})\left(\log\frac{I_2}{I_1}\right)
\end{aligned}$$

Solving for I_2/I_1,

$$\beta_2 - \beta_1 = (10\ \text{dB})\left(\log\frac{I_2}{I_1}\right)$$

$$\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10} = 10^{(90-74)/10} = 40$$

17. The possible wavelengths in an open pipe are related to the length of the pipe

$$\lambda_n = \frac{2L}{n}$$

For the fundamental frequency, $\lambda = 2L = 2(0.077 \text{ m}) = 0.154 \text{ m}$. The speed of the wave is related to its frequency and wavelength

$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{0.154 \text{ m}} = 2200 \text{ Hz}$$

18. For a pipe with one end open, the wavelength of the fundamental frequency is related to the length of the pipe, $\lambda = 4L$. The corresponding frequency is

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

Forming a ratio

$$\frac{f_2}{f_1} = \frac{v/4L_2}{v/4L_1} = \frac{L_1}{L_2} = \frac{L_1}{2L_1/3} = \frac{3}{2}$$

We have $f_2 = 3f_1/2$.

19. The lighter tuning fork has frequency f_2. Since it has lighter mass, it vibrates quicker and it will have the higher frequency.

$$f_{beat} = f_2 - f_1$$

$$f_2 = f_1 + f_{beat} = 440 \text{ Hz} + 6 \text{ Hz} = 446 \text{ Hz}$$

20. Since the source moves in the same direction as the sound, $v_S > 0$.The Doppler shift for a moving source is

$$f_O = \left(\frac{1}{1 - v_S/v}\right) f_S = \left(\frac{1}{1 - (31 \text{ m/s})/(340 \text{ m/s})}\right)(1000 \text{ Hz}) = 1100 \text{ Hz}$$