

PHY2053  
Summer C 2013  
Exam 1 Solutions

1. The force is

$$F = \frac{Gm_1m_2}{r^2}$$

Forming a ratio

$$\begin{aligned}\frac{F'}{F} &= \frac{\frac{Gm'_1m'_2}{r'^2}}{\frac{Gm_1m_2}{r^2}} \\ &= \frac{r^2}{r'^2} \frac{m'_1m'_2}{m_1m_2}\end{aligned}$$

The only combination that gives  $F'/F = 2$  is doubling  $m_1$ .

2. The speed of light is

$$c = 186,282 \frac{\text{miles}}{\text{s}} \times \frac{8 \text{ furlongs}}{\text{mile}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{14 \text{ days}}{\text{fortnight}} = 1.8 \times 10^{12} \text{ furlongs/fortnight}$$

3. The distance traveled for the entire trip is

$$\begin{aligned}v_{av,x} &= \frac{\Delta x}{\Delta t} \\ \Delta x &= v_{av,x} \Delta t \\ &= \left(36 \frac{\text{mi}}{\text{hr}}\right) (75 \text{ min}) \left(\frac{1 \text{ hr}}{60 \text{ min}}\right) \\ &= 45 \text{ mi}\end{aligned}$$

The distance covered during the first part is

$$\begin{aligned}v_{av,x} &= \frac{\Delta x}{\Delta t} \\ \Delta x_1 &= v_{av,x} \Delta t \\ &= \left(30 \frac{\text{mi}}{\text{hr}}\right) (30 \text{ min}) \left(\frac{1 \text{ hr}}{60 \text{ min}}\right)\end{aligned}$$

$$\Delta x_1 = 15 \text{ mi}$$

The distance for the second part is

$$\begin{aligned}\Delta x &= \Delta x_1 + \Delta x_2 \\ \Delta x_2 &= \Delta x - \Delta x_1 \\ &= 45 \text{ mi} - 15 \text{ mi} \\ &= 30 \text{ mi}\end{aligned}$$

The duration of the second part is

$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2 \\ \Delta t_2 &= \Delta t - \Delta t_1 \\ &= 75 \text{ min} - 30 \text{ min} \\ &= 45 \text{ min}\end{aligned}$$

The average velocity for the second part is

$$\begin{aligned}v_{av,x} &= \frac{\Delta x}{\Delta t} \\ &= \left( \frac{30 \text{ mi}}{45 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) \\ &= 40 \text{ mph}\end{aligned}$$

4. For the first 10 s, the distance is 120 m. This gives,

$$\begin{aligned}\Delta x &= v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\ \Delta x_1 &= v_{ix} \Delta t_1 + \frac{1}{2} a_x (\Delta t_1)^2 \\ 120 \text{ m} &= v_{ix} (10 \text{ s}) + \frac{1}{2} a_x (10 \text{ s})^2 \\ 12 \frac{\text{m}}{\text{s}} &= v_{ix} + (5 \text{ s}) a_x\end{aligned}$$

Two seconds later,

$$\begin{aligned}\Delta x_2 &= \Delta x_1 + 48 \text{ m} = 120 \text{ m} + 48 \text{ m} = 168 \text{ m} \\ \Delta t_2 &= \Delta t_1 + 2 \text{ s} = 10 \text{ s} + 2 \text{ s} = 12 \text{ s}\end{aligned}$$

$$\begin{aligned}\Delta x_2 &= v_{ix} \Delta t_2 + \frac{1}{2} a_x (\Delta t_2)^2 \\ 168 \text{ m} &= v_{ix} (12 \text{ s}) + \frac{1}{2} a_x (12 \text{ s})^2\end{aligned}$$

$$14 \frac{\text{m}}{\text{s}} = v_{ix} + (6\text{s})a_x$$

There are two equations and two unknowns,

$$12 \frac{\text{m}}{\text{s}} = v_{ix} + (5\text{s})a_x$$

$$14 \frac{\text{m}}{\text{s}} = v_{ix} + (6\text{s})a_x$$

Solve the first equation for  $a_x$ ,

$$12 \frac{\text{m}}{\text{s}} = v_{ix} + (5\text{s})a_x$$

$$a_x = 2.4 \frac{\text{m}}{\text{s}^2} - \frac{v_{ix}}{5\text{s}}$$

Substitute into the second equation,

$$\begin{aligned} 14 \frac{\text{m}}{\text{s}} &= v_{ix} + (6\text{s})a_x \\ &= v_{ix} + (6\text{s}) \left( 2.4 \frac{\text{m}}{\text{s}^2} - \frac{v_{ix}}{5\text{s}} \right) \\ &= v_{ix} + 14.4 \frac{\text{m}}{\text{s}} - 1.2v_{ix} \end{aligned}$$

$$14 \frac{\text{m}}{\text{s}} - 14.4 \frac{\text{m}}{\text{s}} = -0.2v_{ix}$$

$$-0.4 \frac{\text{m}}{\text{s}} = -0.2v_{ix}$$

$$v_{ix} = 2.0 \frac{\text{m}}{\text{s}}$$

5. The acceleration is

$$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$\Delta x = 0 + \frac{1}{2}a_x(\Delta t)^2$$

$$a_x = \frac{2\Delta x}{(\Delta t)^2} = \frac{2(100\text{m})}{(10\text{s})^2} = 2\text{m/s}^2$$

The total distance is  $\Delta x = 100\text{ m} + 300\text{ m} = 400\text{ m}$ . The final velocity is

$$\begin{aligned}
 v_{fx}^2 - v_{ix}^2 &= 2a_x \Delta x \\
 v_{fx}^2 &= v_{ix}^2 + 2a_x \Delta x \\
 &= 0 + 2a_x \Delta x \\
 v_{fx} &= \sqrt{2(2 \text{ m/s}^2)(400 \text{ m})} = 40 \text{ m/s}
 \end{aligned}$$

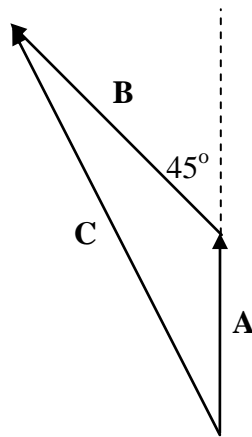
6. The time it takes for the first stone to hit the water is

$$\begin{aligned}
 \Delta y &= v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \\
 \Delta y &= 0 + \frac{1}{2} a_y (\Delta t)^2 \\
 \Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-19.6 \text{ m})}{-9.8 \text{ m/s}^2}} = 2.0 \text{ s}
 \end{aligned}$$

The second stone must hit the water in  $\Delta t = 2.0 \text{ s} - 1.0 \text{ s} = 1.0 \text{ s}$ .

$$\begin{aligned}
 \Delta y &= v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \\
 v_{iy} \Delta t &= \Delta y - \frac{1}{2} a_y (\Delta t)^2 \\
 v_{iy} &= \frac{\Delta y}{\Delta t} - \frac{1}{2} a_y \Delta t \\
 &= \frac{-19.6 \text{ m}}{1.0 \text{ s}} - \frac{1}{2} (-9.8 \text{ m/s}^2)(1 \text{ s}) \\
 &= -14.7 \text{ m/s}
 \end{aligned}$$

7. The run looks like,



To add vectors we take components. By inspection for **A**

$$A_x = 0$$

$$A_y = 20 \text{ yards}$$

For **B**,

$$B_x = -B \sin 45^\circ = -(30 \text{ yards}) \sin 45^\circ = -21.2 \text{ yards}$$

$$B_y = B \cos 45^\circ = (30 \text{ yards}) \cos 45^\circ = 21.2 \text{ yards}$$

The components of **C**

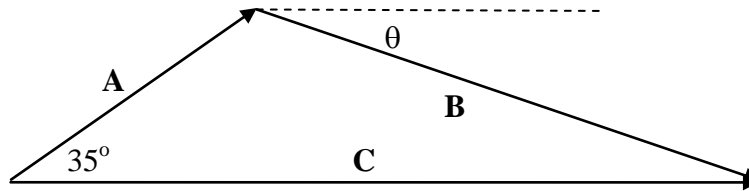
$$C_x = A_x + B_x = 0 + (-21.2 \text{ yards}) = -21.2 \text{ yards}$$

$$C_y = A_y + B_y = 20.0 \text{ yards} + 21.2 \text{ yards} = 41.2 \text{ yards}$$

The magnitude of **C**

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-21.2 \text{ yards})^2 + (41.2 \text{ yards})^2} = 46.3 \text{ yards}$$

8. The picture looks like



The vector relation is

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{B} = \vec{C} - \vec{A}$$

The components of **A** and **C** are

$$A_x = A \cos 35^\circ = (30 \text{ N}) \cos 35^\circ = 24.6 \text{ N}$$

$$A_y = A \sin 35^\circ = (30 \text{ N}) \sin 35^\circ = 17.2 \text{ N}$$

$$C_x = C \cos 0 = (75 \text{ N}) \cos 0 = 75 \text{ N}$$

$$C_y = C \sin 0 = (75 \text{ N}) \sin 0 = 0$$

The components of **B** are

$$B_x = C_x - A_x = 75\text{N} - 24.6\text{N} = 50.4\text{N}$$

$$B_y = C_y - A_y = 0 - 17.2\text{N} = -17.2\text{N}$$

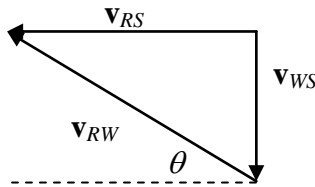
The magnitude of **B** is

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(50.4\text{N})^2 + (-17.2\text{N})^2} = 53\text{N}$$

The angle is

$$\theta = \arctan\left|\frac{B_y}{B_x}\right| = \arctan\left|\frac{-17.2\text{N}}{50.4\text{N}}\right| = 19^\circ$$

9. The figure is



The velocity of the rowboat relative to the shore is

$$v_{RS} = \frac{\Delta x}{\Delta t} = \frac{250\text{m}}{4.2\text{min}} \times \frac{1\text{min}}{60\text{s}} = 0.99\text{m/s}$$

The vector equation is

$$\vec{v}_{RS} = \vec{v}_{RW} + \vec{v}_{WS}$$

From the diagram

$$\theta = \arctan\left|\frac{v_{WS}}{v_{RS}}\right| = \arctan\left|\frac{-0.61\text{m/s}}{-0.99\text{m/s}}\right| = 32^\circ$$

10. The time it takes for the ball to hit the ground is

$$\begin{aligned} \Delta y &= v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\ &= 0 + \frac{1}{2}a_y(\Delta t)^2 \\ \Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-78.4\text{m})}{-9.8\text{m/s}^2}} = 4.0\text{s} \end{aligned}$$

The ball falls 80 m from the bottom of the building

$$\begin{aligned}\Delta x &= v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \\ &= v_i\Delta t + 0 \\ v_i &= \frac{\Delta x}{\Delta t} = \frac{80\text{m}}{4\text{s}} = 20\text{m/s}\end{aligned}$$

Since the ball is thrown horizontally, the ball is in the air for the same amount of time regardless of the magnitude of the throw. So again  $\Delta t = 4.0\text{ s}$ . The new range is 160 m,

$$\begin{aligned}\Delta x &= v_{ix}\Delta t \\ v_i &= \frac{\Delta x}{\Delta t} = \frac{160\text{m}}{4\text{s}} = 40\text{m/s}\end{aligned}$$

11. The range equation is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

The time of flight equation is

$$\Delta t_F = \frac{2v_i \sin \theta}{g}$$

Solve the time of flight equation for  $v_i$  and substitute into the range equation,

$$\begin{aligned}\Delta t_F &= \frac{2v_i \sin \theta}{g} \\ v_i &= \frac{g\Delta t_F}{2\sin \theta}\end{aligned}$$

The range equation becomes,

$$\begin{aligned}R &= \frac{v_i^2 \sin 2\theta}{g} \\ &= \left(\frac{g\Delta t_F}{2\sin \theta}\right)^2 \frac{2\sin \theta \cos \theta}{g} \\ &= \frac{g^2(\Delta t_F)^2}{4\sin^2 \theta} \frac{2\sin \theta \cos \theta}{g}\end{aligned}$$

$$R = \frac{g(\Delta t_F)^2 \cos \theta}{2 \sin \theta}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{g(\Delta t_F)^2}{2R}$$

$$\tan \theta = \frac{g(\Delta t_F)^2}{2R}$$

$$\theta = \arctan\left(\frac{g(\Delta t_F)^2}{2R}\right) = \arctan\left(\frac{9.8 \text{ m/s}^2 (5.85 \text{ s})^2}{2(240 \text{ m})}\right) = 35^\circ$$

Use the time of flight equation to find  $v_i$

$$\Delta t_F = \frac{2v_i \sin \theta}{g}$$

$$v_i = \frac{g\Delta t_F}{2 \sin \theta} = \frac{(9.8 \text{ m/s}^2)(5.85 \text{ s})}{2 \sin 35^\circ} = 50 \text{ m/s}$$

Since the field is level, the ball hits the ground with the same velocity.

Easier solution: The golf ball travels 240 m in the  $x$ -direction. It takes 5.85 s to travel that distance. The  $x$ -component of the velocity is constant for a projectile and is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{240 \text{ m}}{5.85 \text{ s}} = 41.0 \text{ m/s}$$

The ball reaches its highest point at  $\Delta t_h = \Delta t_F/2 = (5.85 \text{ s})/2 = 2.93 \text{ s}$ . The  $y$ -component of the initial velocity is

$$v_{fy} - v_{iy} = a_y \Delta t$$

$$0 - v_{iy} = -g\Delta t_h$$

$$v_{iy} = g\Delta t_h = (9.8 \text{ m/s}^2)(2.93 \text{ s}) = 28.7 \text{ m/s}$$

The initial velocity is

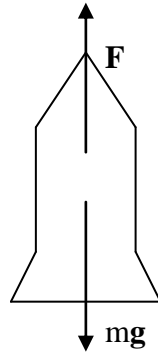
$$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(41.0 \text{ m/s})^2 + (28.7 \text{ m/s})^2} = 50.0 \text{ m/s}$$

The final velocity equals the initial velocity since the motion is symmetric.

12. The weight is the Earth pulling down on the block. The interaction partner is the block pulling up on the Earth.



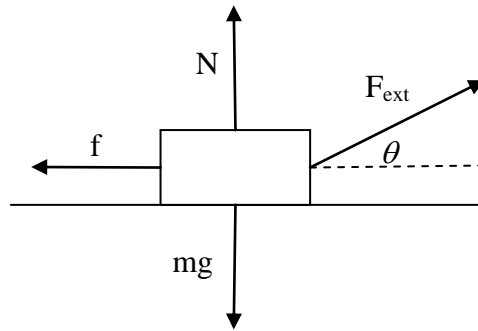
13. The free body diagram is



Using Newton's second law,

$$\begin{aligned}\sum F_y &= ma_y \\ F - mg &= ma_y \\ a_y &= \frac{F - mg}{m} = \frac{49000\text{N} - (1000\text{kg})(9.8\text{m/s}^2)}{1000\text{kg}} = 39\text{m/s}^2\end{aligned}$$

14. The free body diagram is



Using Newton's second law

$$\begin{aligned}\sum F_x &= ma_x \\ -f + F_{ext} \cos\theta &= 0 \\ f &= F_{ext} \cos\theta \\ \sum F_y &= ma_y \\ N + F_{ext} \sin\theta - mg &= 0 \\ N &= mg - F_{ext} \sin\theta\end{aligned}$$

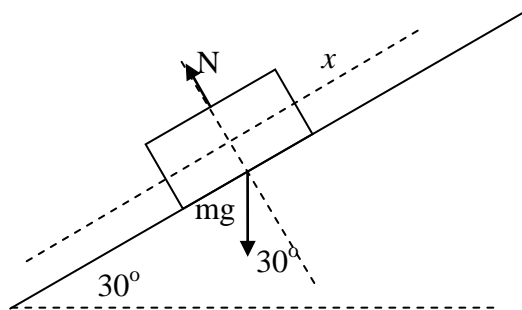
The coefficient of friction is defined in

$$f = \mu N$$

Substituting for  $f$  and  $N$ ,

$$\begin{aligned} f &= \mu N \\ F_{ext} \cos \theta &= \mu(mg - F_{ext} \sin \theta) \\ &= \mu mg - \mu F_{ext} \sin \theta \\ F_{ext} \cos \theta + \mu F_{ext} \sin \theta &= \mu mg \\ F_{ext} &= \frac{\mu mg}{\cos \theta + \mu \sin \theta} \\ &= \frac{(0.3)(4 \text{ kg})(9.8 \text{ m/s}^2)}{\cos 25^\circ + (0.3) \sin 25^\circ} \\ &= 11.4 \text{ N} \end{aligned}$$

15. The free body diagram is

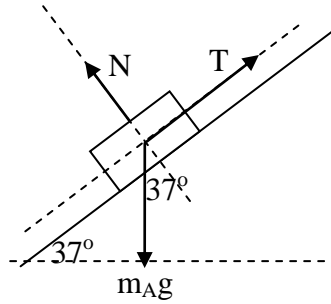


Using Newton's second law,

$$\begin{aligned} \sum F_x &= ma_x \\ mg \sin \theta &= ma_x \\ a_x &= g \sin \theta = (9.8 \text{ m/s}^2) \sin 30^\circ = 4.9 \text{ m/s}^2 \end{aligned}$$

Since the acceleration is independent of the mass, so are all the other kinematic variables. The final velocity is still 10 m/s when the mass of the roller coaster is doubled.

16. The free body diagram for  $m_A$  is

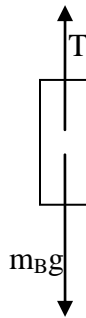


Using Newton's second law,

$$\sum F_x = m_A a_x$$

$$T - m_A g \sin \theta = m_A a_{Ax}$$

For  $m_B$ ,



Again using Newton's second law,

$$\sum F_y = m_B a_{By}$$

$$T - m_B g = m_B a_{By}$$

Since blocks A and B are connected they move together. Suppose we call their acceleration  $a$ . If A moves up the incline, B will descend. This gives,

$$a_{Ax} = a \quad \text{and} \quad a_{By} = -a$$

The two equations from Newton's second law become

$$T - m_A g \sin \theta = m_A a_{Ax}$$

$$T - m_A g \sin \theta = m_A a$$

and

$$\begin{aligned}T - m_B g &= m_B a_{By} \\T - m_B g &= m_B (-a) \\m_B g - T &= m_B a\end{aligned}$$

We have two equations with two unknowns ( $T$  and  $a$ )

$$\begin{aligned}T - m_A g \sin \theta &= m_A a \\m_B g - T &= m_B a\end{aligned}$$

Adding the two equations cancels the  $T$ ,

$$\begin{aligned}m_B g - m_A g \sin \theta &= m_A a + m_B a \\(m_B - m_A \sin \theta) g &= (m_A + m_B) a \\a &= \frac{(m_B - m_A \sin \theta) g}{(m_A + m_B)} = \frac{(5 \text{ kg} - 10 \text{ kg} \sin 37^\circ)(9.8 \text{ m/s}^2)}{(5 \text{ kg} + 10 \text{ kg})} = -0.67 \text{ m/s}^2\end{aligned}$$

The acceleration is positive if B ascends. Since the acceleration is negative, B descends with an acceleration of  $0.67 \text{ m/s}^2$ .

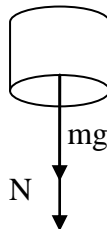
17. The angular velocity of the wheel is

$$\omega = 36 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 3.77 \frac{\text{rad}}{\text{s}}$$

The angular displacement is

$$\begin{aligned}\omega &= \frac{\Delta \theta}{\Delta t} \\ \Delta \theta &= \omega \Delta t = (3.77 \text{ rad/s})(45 \text{ s}) = 170 \text{ rad}\end{aligned}$$

18. A diagram would be



There are two forces acting on the water in the bucket, its weight and the normal force of the bucket pushing on the water. At the top of the circle, both forces point down. When we apply Newton's second law,

$$\sum F_r = ma_r$$

$$N + mg = m \frac{v^2}{r}$$

(Since the radial direction is downward, forces downward are positive.) When the water just barely stays in the bucket, the bucket is not really pushing on the water. That means  $N = 0$ ,

$$N + mg = m \frac{v^2}{r}$$

$$0 + mg = m \frac{v^2}{r}$$

$$v = \sqrt{rg} = \sqrt{(1\text{m})(9.8\text{m/s}^2)} = 3.1\text{m/s}$$

19. The operating speed of the Eye is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{30\text{min}} \times \frac{1\text{min}}{60\text{s}} = 3.49 \times 10^{-3} \text{ rad/s}$$

The angular displacement while it accelerates is

$$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t = \frac{1}{2}(3.49 \times 10^{-3} \text{ rad/s} + 0)(20\text{s}) = 3.49 \times 10^{-2} \text{ rad}$$

20. The net acceleration is the sum of the radial and tangential accelerations,

$$\vec{\mathbf{a}}_{net} = \vec{\mathbf{a}}_r + \vec{\mathbf{a}}_t$$

Since the radial and tangential directions are perpendicular to each other,

$$a_{net}^2 = a_r^2 + a_t^2$$

From the radius and the angular speed, the radial acceleration can be found,

$$a_r = \omega^2 r = (2\text{rad/s})^2 (0.25\text{m}) = 1.0\text{m/s}^2$$

Since the net acceleration is also known, the tangential acceleration can be determined,

$$\begin{aligned}a_{net}^2 &= a_r^2 + a_t^2 \\a_t^2 &= a_{net}^2 - a_r^2 \\a_t &= \sqrt{a_{net}^2 - a_r^2} = \sqrt{(1.6 \text{ m/s}^2)^2 - (1.0 \text{ m/s}^2)^2} = 1.25 \text{ m/s}^2\end{aligned}$$

The tangential acceleration is related to the angular acceleration,

$$\begin{aligned}a_t &= \alpha r \\ \alpha &= \frac{a_t}{r} = \frac{1.25 \text{ m/s}^2}{0.25 \text{ m}} = 5.0 \text{ rad/s}^2\end{aligned}$$