PHY2053
Summer C 2013
Exam 2 Solutions

1. Since the normal force is perpendicular to the incline and motion is along the incline, the angle between the force and displacement is 90º.

$$W = F\Delta r\cos\theta = F\Delta r\cos 90° = 0$$

The normal force does no work.

2. The acceleration due to the force is

$$F = ma$$
$$a = \frac{F}{m} = \frac{4\,\text{N}}{2\,\text{kg}} = 2\,\text{m}/\text{s}^2$$

The final velocity of the 2-kg block is

$$v_f - v_i = a\Delta t$$
$$v_f = v_i + a\Delta t = 0 + (2\,\text{m/s})(6\,\text{s}) = 12\,\text{m/s}$$

The 2-kg block has kinetic energy,

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\,\text{kg})(12\,\text{m/s})^2 = 144\,\text{J}$$

The potential energy of the 3-kg block will be 144 J. Its height is

$$U = mgy$$
$$y = \frac{U}{mg} = \frac{144\,\text{J}}{(3\,\text{kg})(9.8\,\text{m/s}^2)} = 4.9\,\text{m}$$

3. The initial mechanical energy of the ball is

$$\begin{aligned} ME_1 &= K_1 + U_1 \\ &= \tfrac{1}{2}mv_1^2 + mgy_1 \\ &= \tfrac{1}{2}(0.1\,\text{kg})(8\,\text{m/s})^2 + (0.1\,\text{kg})(9.8\,\text{m/s}^2)(10\,\text{m}) \\ &= 13\,\text{J} \end{aligned}$$

When the ball is 5 m above the ground its potential energy will be

$$U_2 = mgy_2 = (0.1\,\text{kg})(9.8\,\text{m/s}^2)(5\,\text{m}) = 4.9\,\text{J}$$

Since mechanical energy is conserved, $ME_2 = ME_1 = 13$ J. The kinetic energy when the ball is 5 m above the ground is

$$ME_2 = K_2 + U_2$$
$$K_2 = ME_2 - U_2 = 13\,\text{J} - 4.9\,\text{J} = 8.1\,\text{J}$$

The velocity is

$$K_2 = \tfrac{1}{2} m v_2^{\;2}$$
$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(8.1\,\text{J})}{0.1\,\text{kg}}} = 12.7\,\text{m/s}$$

4. Use the work-energy theorem

$$W_{NC} = \Delta K + \Delta U$$

The initial kinetic energy is

$$K_1 = \tfrac{1}{2} m v_1^{\;2} = \tfrac{1}{2}(75\,\text{kg})(2\,\text{m/s})^2 = 150\,\text{J}$$

The final kinetic energy is $K_2 = 0$ since the bicycle stops. The initial potential energy is

$$U_1 = mgy_1 = (75\,\text{kg})(9.8\,\text{m/s}^2)(10\,\text{m}) = 7350\text{ J}$$

The final potential energy is $U_2 = 0$ since the bicycle is at the bottom of the hill. The work done by nonconservative forces is

$$W_{NC} = \Delta K + \Delta U = (K_2 - K_1) + (U_2 - U_1) = (0 - 150\text{ J}) + (0 - 7350\text{ J}) = -7500\text{ J}$$

Work is defined as

$$W = F\,\Delta r \cos\theta$$

Since the frictional forces opposes the displacement, $\theta = 180^\circ$. Solving for Δr,

$$\Delta r = \frac{W}{F\cos\theta} = \frac{-7500\text{ J}}{(150\text{ N})\cos 180^\circ} = 50\,\text{m}$$

5. When in equilibrium, the force of the spring pulling up is equal to the weight of the mass,

$$F_s = W_1$$

$$kx = m_1 g$$
$$k = \frac{m_1 g}{x} = \frac{(1.4\,\text{kg})(9.8\,\text{m/s}^2)}{7.0\,\text{cm}} = 1.96\,\text{N/cm}$$

When the mass is added, the total displacement from equilibrium is

$$x_2 = x_1 + \Delta x = 7\,\text{cm} + 10\,\text{cm} = 17\,\text{cm}$$

The new mass m_2, can be found using the same reasoning as above

$$kx_2 = m_2 g$$
$$m_2 = \frac{kx_2}{g} = \frac{(1.96\,\text{N/cm})(17\,\text{cm})}{9.8\,\text{m/s}^2} = 3.4\,\text{kg}$$

The mass added is

$$\Delta m = m_2 - m_1 = 3.4\,\text{kg} - 1.4\,\text{kg} = 2.0\,\text{kg}$$

6. The spring constant is

$$F = kx$$
$$k = \frac{F}{x} = \frac{80\,\text{N}}{0.196\,\text{m}} = 408\,\text{N/m}$$

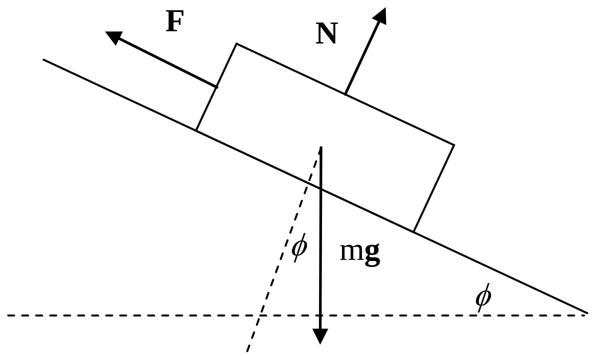
As the ball is shot up, only conservative forces act and mechanical energy is conserved.

$$U_1 + K_1 = U_2 + K_2$$

Initially, the ball is at rest. At the highest point it is a rest again. Therefore, $K_1 = K_2 = 0$. There are two types of potential energy. The elastic potential energy stored into the spring turns into gravitational potential energy.

$$U_1 + K_1 = U_2 + K_2$$
$$\tfrac{1}{2}kx^2 + 0 = mgy + 0$$
$$y = \frac{kx^2}{2mg} = \frac{(408\,\text{N/m})(0.196\,\text{m})^2}{2(0.20\,\text{kg})(9.8\,\text{m/s}^2)} = 4.0\,\text{m}$$

7. The car rolls at constant speed. By Newton's first law, the net force must equal to zero. The component of the weight along the incline must be cancelled by air resistance and friction. Call the net resistive force **F**. The free body diagram is



Taking the *x*-axis along the incline,

$$\sum F_x = 0$$

$$mg\sin\theta - F = 0$$

$$F = mg\sin\phi = (1500\text{ kg})(9.8\text{ m/s}^2)\sin 2° = 513\text{ N}$$

When the car is driven at 20 m/s on level ground, the engine must supply a force equal to the resistive force of 513 N. The power needed is

$$P = Fv\cos\theta = (513\text{ N})(20\text{ m/s})\cos 0 = 10{,}000\text{ W}$$

8. The impulse-momentum theorem states that the change in linear momentum is caused by the impulse.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$$

To work with vectors, we must take components

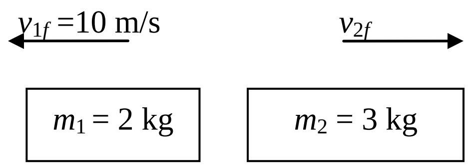
$$\begin{aligned} \Delta p_x &= F_x\Delta t \\ p_{2x} - p_{1x} &= F_x\Delta t \\ p_{2x} &= p_{1x} + F_x\Delta t \\ &= mv_{1x} + F_x\Delta t \end{aligned}$$

The initial velocity is to the right, which we take to be the positive direction. The force acts to the left, which has a negative component,

$$p_{2x} = mv_{1x} + F_x\Delta t = (2\text{ kg})(5\text{ m/s}) + (-20\text{ N})(4\text{ s}) = -70\text{ kg}\cdot\text{m/s}$$

Since the *x*-component is negative, the momentum is 70 kg·m/s to the left.

9. Linear momentum is conserved in an explosion. Since the block is initially at rest, $p_i = 0$. After the explosion,



The final momentum is

$$\vec{\mathbf{p}}_f = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Taking components

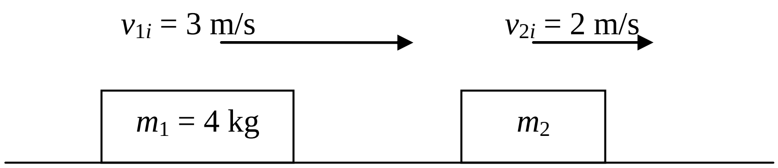
$$p_{fx} = -m_1 v_{1f} + m_2 v_{2f}$$

Since m_1 moves to the left, its velocity has a negative component. Using conservation of linear momentum,

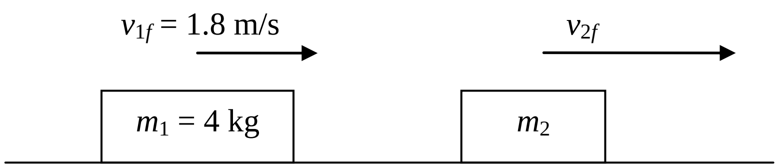
$$p_{ix} = p_{fx}$$
$$0 = -m_1 v_{1f} + m_2 v_{2f}$$
$$v_{2f} = \frac{m_1 v_{1f}}{m_2} = \frac{(2\,\text{kg})(10\,\text{m/s})}{3\,\text{kg}} = 6.7\,\text{m/s}$$

The positive sign means the 3-kg mass moves to the right.

10. Before the collision,



After the collision,



In class, we derived the general equations for a one dimensional elastic collision,

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Solve the first equation for m_2 since we are not given its final speed.

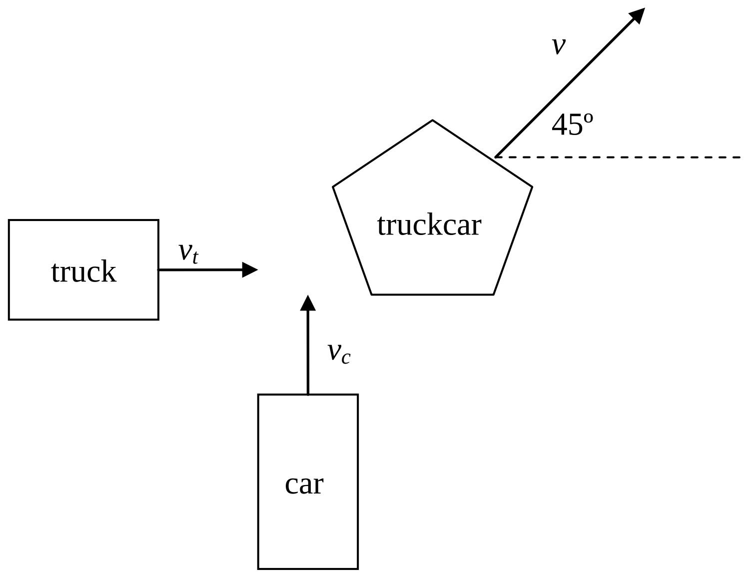
$$\begin{aligned}
v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \\
(m_1 + m_2)v_{1f} &= (m_1 - m_2)v_{1i} + 2m_2 v_{2i} \\
m_1 v_{1f} + m_2 v_{1f} &= m_1 v_{1i} - m_2 v_{1i} + 2m_2 v_{2i} \\
m_2(v_{1f} + v_{1i} - 2v_{2i}) &= m_1(v_{1i} - v_{1f}) \\
m_2 &= m_1\left(\frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i} - 2v_{2i}}\right) \\
&= (4\,\text{kg})\left(\frac{3.0\,\text{m/s} - 1.8\,\text{m/s}}{1.8\,\text{m/s} + 3.0\,\text{m/s} - 2(2.0\,\text{m/s})}\right) \\
&= 6.0\,\text{kg}
\end{aligned}$$

11. Linear momentum is conserved in the collision,

$$\begin{aligned}
p_{ix} &= p_{fx} \\
m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\
m_2(v_{2i} - v_{2f}) &= m_1(v_{1f} - v_{1i}) \\
m_2 &= m_1\left(\frac{v_{1f} - v_{1i}}{v_{2i} - v_{2f}}\right) \\
&= (10\,\text{kg})\left(\frac{0 - 5\,\text{m/s}}{-3\,\text{m/s} - 2\,\text{m/s}}\right) \\
&= 10\,\text{kg}
\end{aligned}$$

12. Linear momentum is conserved in the collision. Since the collision is two dimensional, we must use components. For the x-component (along the east direction),

$$\begin{aligned}
p_{ix} &= p_{fx} \\
m_t v_t &= (m_c + m_t)v\cos 45^\circ
\end{aligned}$$



For the *y*-component (along the north direction)

$$p_{iy} = p_{fy}$$
$$m_c v_c = (m_c + m_t) v \sin 45°$$

Divide the two equations,

$$\frac{m_c v_c}{m_t v_t} = \frac{(m_c + m_t) v \sin 45°}{(m_c + m_t) v \cos 45°}$$
$$= 1$$
$$m_c v_c = m_t v_t$$

Solve for v_t

$$m_c v_c = m_t v_t$$
$$v_t = \frac{m_c v_c}{m_t} = \frac{(1200 \text{ kg})(15 \text{ m/s})}{1500 \text{ kg}} = 12 \text{ m/s}$$

13. The center of mass is defined as

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{x_A m_A + x_B m_B}{m_A + m_B}$$

In this problem, the mass are equal. Let's call them *m*. Substituting above gives

$$x_{cm} = \frac{x_A m_A + x_B m_B}{m_A + m_B} = \frac{x_A m + x_B m}{m + m} = \frac{m(x_A + x_B)}{2m} = \tfrac{1}{2}(x_A + x_B)$$

The velocity is related to the position by $x = v\,\Delta t$. Substituting above,

$$\begin{aligned} x_{cm} &= \tfrac{1}{2}(x_A + x_B) \\ v_{cm}\Delta t &= \tfrac{1}{2}(v_A \Delta t + v_B \Delta t) \\ v_{cm} &= \tfrac{1}{2}(v_A + v_B) \end{aligned}$$

We have $v_A = 0$. The velocity of the center of mass is

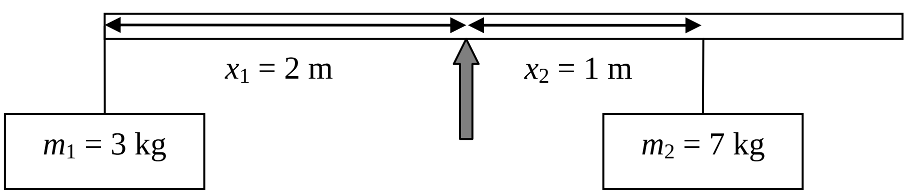
$$v_{cm} = \tfrac{1}{2}(v_A + v_B) = \tfrac{1}{2}(0 + v_B) = \tfrac{1}{2} v_B$$

Since *B* moves away from *A*, the center of mass moves away from *A* at half the speed of *B*.

14. The rotational kinetic energy is given by

$$\begin{aligned} K &= \tfrac{1}{2} I \omega^2 \\ I &= \frac{2K}{\omega^2} = \frac{2(50\,\text{J})}{(30\,\text{rad/s})^2} = 0.11\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

15. The figure is



The net torque is

$$\sum \tau = \tau_1 + \tau_2 = m_1 g x_1 - m_2 g x_2 = (3\,\text{kg})(9.8\,\text{m/s}^2)(2\,\text{m}) - (7\,\text{kg})(9.8\,\text{m/s}^2)(1\,\text{m}) = -9.8\,\text{N}\cdot\text{m}$$

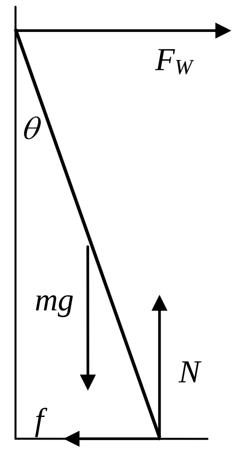
Since the torque is negative, it is in the clockwise direction.

16. For a system in equilibrium, we have the net force equal to zero,

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

And the net torque must vanish as well,

$$\sum \tau = 0$$



Using the rotational equilibrium condition and taking torques at the foot of the ladder,

$$\begin{aligned}
\sum \tau &= 0 \\
\tau_W + \tau_{mg} &= 0 \\
-F_W L\cos\theta + mg\frac{L}{2}\sin\theta &= 0 \\
F_W &= \frac{mg\tan\theta}{2} \\
&= \frac{(10\,\text{kg})(9.8\,\text{m/s}^2)\tan 25°}{2} \\
&= 23\,\text{N}
\end{aligned}$$

The x-component equation gives,

$$\begin{aligned}
\sum F_x &= 0 \\
F_W - f &= 0 \\
f &= 23\,\text{N}
\end{aligned}$$

17. The rotational inertia for a solid sphere is

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(1.5\,\text{kg})(0.30\,\text{m})^2 = 0.054\,\text{kg}\cdot\text{m}^2$$

Newton’s second law for rotational motion is

$$\sum \tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{8.1\,\text{N}\cdot\text{m}}{0.054\,\text{kg}\cdot\text{m}^2} = 150\,\text{rad/s}^2$$

18. Using conservation of energy

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + U_1 &= K_2 + 0 \end{aligned}$$

The ball starts from rest and ends up at the bottom of the hill.

$$\begin{aligned} 0 + U_1 &= K_2 + 0 \\ mgy_1 &= \tfrac{1}{2}mv_2{}^2 + \tfrac{1}{2}I\omega_2{}^2 \end{aligned}$$

For a spherical shell,

$$I = \tfrac{2}{3}mR^2$$

The condition for rolling is $v = \omega R$. Substituting into the energy equation,

$$\begin{aligned} mgy_1 &= \tfrac{1}{2}mv_2{}^2 + \tfrac{1}{2}I\omega_2{}^2 \\ &= \tfrac{1}{2}mv_2{}^2 + \tfrac{1}{2}(\tfrac{2}{3}mR^2)(v_2/R)^2 \\ &= \tfrac{1}{2}mv_2{}^2 + \tfrac{1}{3}mv_2{}^2 \\ &= \tfrac{5}{6}mv_2{}^2 \\ v_2 &= \sqrt{\frac{6gy_1}{5}} = \sqrt{\frac{6(9.8\,\text{m/s}^2)(20\,\text{m})}{5}} = 15\,\text{m/s} \end{aligned}$$

19. The definition of angular momentum is

$$\begin{aligned} L &= I\omega \\ I &= \frac{L}{\omega} = \frac{50\,\text{kg}\cdot\text{m}^2\text{/s}}{300\,\text{rad/s}} = 0.166\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

For a ring, $I = MR^2$. Solving for R,

$$R = \sqrt{\frac{I}{M}} = \sqrt{\frac{0.166\,\text{kg}\cdot\text{m}^2}{0.5\,\text{kg}}} = 0.58\,\text{m}$$

20. Angular momentum is conserved, $L_1 = L_2$. Using the definition of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

For a solid sphere,

$$I = \frac{2}{5} MR^2$$

The angular velocity is the angular displacement divided by the time,

$$\omega = \frac{\Delta\theta}{\Delta t}$$

When the time interval is equal to the period of the rotation (*T*) the angular displacement is 2π radians

$$\omega = \frac{2\pi \text{ rad}}{T}$$

The conservation of angular momentum relation becomes,

$$\begin{aligned} I_1\omega_1 &= I_2\omega_2 \\ \left(\frac{2}{5}MR_1^{\,2}\right)\left(\frac{2\pi \text{ rad}}{T_1}\right) &= \left(\frac{2}{5}MR_2^{\,2}\right)\left(\frac{2\pi \text{ rad}}{T_2}\right) \\ \frac{T_2}{T_1} &= \left(\frac{R_2}{R_1}\right)^2 \\ T_2 &= T_1\left(\frac{2R_1}{R_1}\right)^2 \\ &= (30 \text{ days})(4) \\ &= 120 \text{ days} \end{aligned}$$