PHY2053
Summer 2013
Exam 3 Solutions

1. By Pascal's principle, the pressure needed to lift the mass is distributed throughout the hydraulic fluid. The pressure is

$$P_{out} = \frac{F_{out}}{A_{out}} = \frac{mg}{\pi (r_{out})^2} \frac{(1250\text{kg})(9.8\,\text{m/s}^2)}{\pi[(6\,\text{in})(2.54\times10^{-2}\,\text{m/in})]^2} = 1.68\times10^5\,\text{Pa}$$

By Pascal's principle, $P_{out} = P_{in}$.

$$P_{in} = \frac{F_{in}}{A_{in}}$$
$$F_{in} = P_{in}A_{in} = (1.68\times10^5\,\text{Pa})(\pi[(1\text{in})(2.54\times10^{-2}\,\text{m/in})]^2 = 340\,\text{N}$$

2. The pressure at the bottom of the tube is due to the weight of the water, the weight of the oil, and the atmosphere.

$$\begin{aligned} P_2 &= P_1 + \rho_O g h_O + \rho_W g h_W \\ &= 1.01\times10^5\,\text{Pa} + (800\text{kg/m}^3)(9.8\,\text{m/s}^2)(15\,\text{m}) + (1000\text{kg/m}^3)(9.8\,\text{m/s}^2)(12.5\,\text{m}) \\ &= 3.4\times10^5\,\text{Pa} \end{aligned}$$

3. There two ways to solve the problem. One is to use the density of steel to find the volume of the bolt. First find the mass of the bolt

$$W = mg$$
$$m = \frac{W}{g} = \frac{0.770\,\text{N}}{9.8\,\text{m/s}^2} = 0.0786\text{kg}$$

Use the density to find the volume,

$$\rho = \frac{m}{V}$$
$$V = \frac{m}{\rho} = \frac{0.0786\text{kg}}{7860\text{kg/m}^3} = 1.0\times10^{-5}\,\text{m}^3$$

The other way to solve this problem is to use Archimedes' principle. The buoyant force is the difference in the actual weight and the apparent weight,

$$F_B = W - W_{app} = 0.770\,\text{N} - 0.672\,\text{N} = 0.098\,\text{N}$$

Now use Archimedes' principle

$$F_B = \rho g V$$
$$V = \frac{F}{\rho g} = \frac{0.098\,\text{N}}{(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.0\times10^{-5}\,\text{m}^3$$

4. The volume flow rate is related to the speed of the water,

$$\frac{\Delta V}{\Delta t} = Av$$
$$v = \frac{1}{A}\frac{\Delta V}{\Delta t} = \frac{1}{\pi r^2}\frac{\Delta V}{\Delta t} = \frac{1}{\pi((0.0254\,\text{m})/2)^2}\frac{0.080\,\text{m}^3}{12\,\text{s}} = 13.2\,\text{m/s}$$

This velocity is created as the water flows from down from the tower. Using Bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho {v_1}^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho {v_2}^2$$

Associate the 1 subscript with the tower and the 2 subscript with the faucet. Since the tower and the faucet are open to the atmosphere, $P_1 = P_2 = P_{atm}$. The tower has such a large volume and its level falls very slowly, $v_1 = 0$.

$$P_{atm} + \rho g y_1 + \tfrac{1}{2}\rho(0)^2 = P_{atm} + \rho g y_2 + \tfrac{1}{2}\rho {v_2}^2$$
$$\rho g(y_1 - y_2) = \tfrac{1}{2}\rho {v_2}^2$$
$$y_1 - y_2 = \frac{{v_2}^2}{2g} = \frac{(13.2\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 8.8\,\text{m}$$

5. Use Poiseuille's law

$$\frac{\Delta V}{\Delta t} = \frac{\pi}{8}\frac{\Delta P / L}{\eta} r^4$$
$$\Delta P = \frac{\Delta V}{\Delta t}\frac{8}{\pi}\frac{L\eta}{r^4} = (4.5\times10^{-4}\,\text{m}^3/\text{s})\frac{8}{\pi}\frac{(1900\,\text{m})(0.20\,\text{Pa}\cdot\text{s})}{(0.15\,\text{cm})^4} = 860\,\text{Pa}$$

6. The stress in the cable is

$$\text{stress} = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(250\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.012\,\text{m})^2} = 5.42\times10^6\,\text{Pa}$$

Young's modulus is

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\text{strain} = \frac{\text{stress}}{Y} = \frac{5.42\times10^{6}\ \text{Pa}}{2.0\times10^{11}\ \text{Pa}} = 2.7\times10^{-5}$$

Since strain = $\Delta L/L$ this is the fractional increase in length.

7. The volume compresses 0.10% = 0.001. The relationship defining the bulk modulus,

$$\Delta P = -B\frac{\Delta V}{V} = (90\times10^{9}\ \text{Pa})(0.001) = 9.0\times10^{7}\ \text{Pa}$$

8. The spring constant is

$$k = \frac{mg}{x} = \frac{(3\,\text{kg})(9.8\,\text{m/s}^2)}{0.25\,\text{m}} = 118\,\text{N/m}$$

The period of a simple harmonic oscillator is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(3\,\text{kg})}{(118\,\text{N/m})}} = 1.0\,\text{s}$$

9. The period of a pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Forming a ratio

$$\frac{T_M}{T_E} = \frac{2\pi\sqrt{\frac{L}{g_M}}}{2\pi\sqrt{\frac{L}{g_E}}} = \sqrt{\frac{g_E}{g_M}}$$

$$T_M = T_E\sqrt{\frac{g_E}{g_M}} = (2.0\,\text{s})\sqrt{\frac{9.8\,\text{m/s}^2}{3.8\,\text{m/s}^2}} = 3.2\,\text{s}$$

10. Use the conservation of energy

$$U_1 + K_1 = U_2 + K_2$$

$$\tfrac{1}{2}kA^2 + 0 = \tfrac{1}{2}k{x_2}^2 + K_2$$

The amplitude is A = 0.25 cm. Solving for K_2,

$$K_2 = \tfrac{1}{2}kA^2 - \tfrac{1}{2}k{x_2}^2 = \tfrac{1}{2}k(A^2 - {x_2}^2) = \tfrac{1}{2}(50\,\text{N/m})((0.25\text{m})^2 - (0.10\text{m})^2) = 1.3\text{J}$$

11. Since $v_m = A\omega$, the amplitude can be found,

$$A = \frac{v_m}{\omega} = \frac{4\,\text{m/s}}{6\,\text{rad/s}} = 0.667\,\text{m}$$

The position function for the given velocity function is

$$y = A\sin\omega t = (0.667m)\sin[(6\,\text{rad/s})(3\text{s})] = -0.50\,\text{m}$$

Your calculator must be in radian mode to get this result.

12. The intensity is a function of distance,

$$I = \frac{P}{4\pi r^2}$$

Forming a ratio

$$\frac{I_2}{I_1} = \frac{\dfrac{P}{4\pi {r_2}^2}}{\dfrac{P}{4\pi {r_1}^2}} = \left(\frac{r_1}{r_2}\right)^2$$

Solving for r_2

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}}$$

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (2\,\text{m})\sqrt{\frac{0.1\,\text{W/m}^2}{0.01\,\text{W/m}^2}} = 6.3\,\text{m}$$

13. The speed of a wave in string is

$$v = \sqrt{\frac{F}{\mu}}$$

$$v^2 = \frac{F}{\mu}$$

$$\mu = \frac{F}{v^2}$$

$$\frac{m}{L} = \frac{F}{v^2}$$

$$m = \frac{F}{v^2} L = \frac{75\,\text{N}}{(140\,\text{m/s})^2}(5\,\text{m}) = 0.019\,\text{kg}$$

14. The velocity of the wave is related to the wavenumber and angular frequency

$$v = \frac{\omega}{k}$$

$$\omega = vk = (60\,\text{m/s})(3.0\,\text{rad/m}) = 180\,\text{rad/s}$$

The general equation of a wave traveling in the –*x* direction is

$$y = A\cos(\omega t + kx)$$

Substituting the values above,

$$y = A\cos((180\,\text{rad/s})t + (3.0\,\text{rad/m})x)$$

15. Since the end of the rope is a free end, the reflected pulse will have the same orientation as the incident pulse. Both pulses will point upward when they meet and

$$A = A_1 + A_2 = (3\,\text{cm}) + (4\,\text{cm}) = 7\,\text{cm}$$

16. The speed of the wave can be found

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{mg}{\mu}} = \sqrt{\frac{(2.10\,\text{kg})(9.8\,\text{m/s}^2)}{3.55\times10^{-5}\,\text{kg/m}}} = 761\,\text{m/s}$$

For the lowest frequency, the wavelength is twice the length of the string, $\lambda = 2L = 4$ m. The frequency is

$$f = \frac{v}{\lambda} = \frac{761\,\text{m/s}}{4\,\text{m}} = 190\,\text{Hz}$$

17. The decibel scale is defined as

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

Solving for I,

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$
$$\beta/10 = \log\left(\frac{I}{I_0}\right)$$
$$\frac{I}{I_0} = 10^{\beta/10}$$
$$I = I_0 10^{\beta/10} = (1.0\times10^{-12}\ \text{W/m}^2)10^{70/10} = 1.0\times10^{-5}\ \text{W/m}^2$$

18. The frequencies for a pipe open at both ends is given by

$$f_n = n\frac{v}{2L}$$

where n = 1, 2, 3, etc.

$$f_1 = 1\frac{v}{2L} = \frac{340\text{m/s}}{2(1\text{m})} = 170\text{Hz}$$

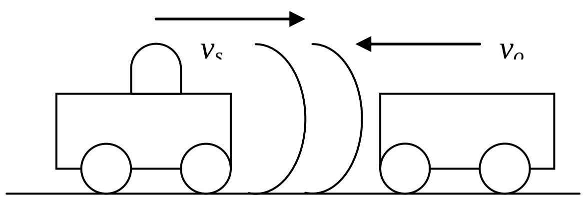
The next frequency corresponds to n = 2

$$f_1 = 2\frac{v}{2L} = \frac{340\text{m/s}}{1\text{m}} = 340\text{Hz}$$

The frequencies are 170 Hz and 340 Hz.

19. Two waves with similar frequencies create **beats**.

20. The vehicles are traveling in different directions



The sound travels in the same direction as the sound. Therefore $v_s > 0$. The car moves against the sound waves and $v_o < 0$. Using the equation for the Doppler effect

$$f_0 = f_s \left(\frac{1 - \frac{v_o}{v}}{1 - \frac{v_s}{v}} \right) = (550\text{Hz}) \left(\frac{1 - \frac{(-25\text{m/s})}{340\text{m/s}}}{1 - \frac{35\text{m/s}}{340\text{m/s}}} \right) = 660\text{Hz}$$