Uniformly Accelerated Motion Super Problem

A ball is thrown upward at 25 m/s from the ground.

1. What is the initial velocity of the ball?
2. What is the acceleration of the ball?
3. What is the ball's velocity after 2 seconds?
4. What is the ball's velocity after 4 seconds?
5. What is the maximum height of the ball?
6. How long until the ball hits the ground?
7. When is the magnitude of the velocity 5 m/s?
8. What distance has the ball travelled after 5 seconds?
9. What is the average velocity and average speed of the ball after 5 seconds?
10. Another ball is thrown one second later. What speed does it need to hit the ground simultaneously with the first ball?

Solution:

1. $v_{iy} = +25$ m/s

2. $a_y = -9.8$ m/s^2

3. The final velocity is

$$v_{fy} - v_{iy} = a_y \Delta t$$
$$v_{fy} = v_{iy} + a_y \Delta t = 25\,\text{m/s} + (-9.8\,\text{m/s}^2)(2\,\text{s}) = 5.4\,\text{m/s}$$

4. The final velocity is

$$v_{fy} - v_{iy} = a_y \Delta t$$
$$v_{fy} = v_{iy} + a_y \Delta t = 25\,\text{m/s} + (-9.8\,\text{m/s}^2)(4\,\text{s}) = -14.2\,\text{m/s}$$

5. At the highest point, what is the velocity?

$$v_{fy}{}^2 - v_{iy}{}^2 = 2a_y \Delta y$$
$$0 - v_{iy}{}^2 = 2a_y \Delta y$$
$$\Delta y = \frac{-v_{iy}{}^2}{2a_y} = \frac{-(25\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = 31.9\,\text{m}$$

6. What is the ball's displacement when it returns to the ground?

$$\Delta y = v_{iy} \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$
\begin{aligned}
0 &= v_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2 \\
&= \Delta t (v_{iy} + \tfrac{1}{2} a_y \Delta t)
\end{aligned}
$$

$$\Delta t = \frac{-2v_{iy}}{a_y} = \frac{-2(25\,\text{m/s})}{-9.8\,\text{m/s}^2} = 5.10\,\text{s}$$

How long does it take for the ball to reach its maximum height?

7. The velocity has magnitude 5 m/s when its value is +5 m/s and −5 m/s. For +5 m/s,

$$v_{fy} - v_{iy} = a_y \Delta t$$

$$\Delta t = \frac{v_{fy} - v_{iy}}{a_y} = \frac{5\,\text{m/s} - 25\,\text{m/s}}{-9.8\,\text{m/s}^2} = 2.04\,\text{s}$$

For −5 m/s

$$v_{fy} - v_{iy} = a_y \Delta t$$

$$\Delta t = \frac{v_{fy} - v_{iy}}{a_y} = \frac{-5\,\text{m/s} - 25\,\text{m/s}}{-9.8\,\text{m/s}^2} = 3.06\,\text{s}$$

Notice these times are equidistant from the time it take to reach the highest point, 2.55 s. The velocities are symmetric about the highest point.

8. The ball is descending at 5 s. The position at 5 seconds is

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2 = (25\,\text{m/s})(5\,\text{s}) + \tfrac{1}{2}(-9.8\,\text{m/s}^2)(5\,\text{s})^2 = 2.5\,\text{m}$$

The ball rises to a maximum height of 31.9 m and falls to a height of 2.5 m, a distance of (31.9 m – 2.5 m) = 29.4 m below the highest point. The total distance travelled is

$$d = 31.9\,\text{m} + 29.4\,\text{m} = 61.3\,\text{m}$$

9. The average velocity is the displacement divided by the elapsed time

$$v_{av,y} = \frac{\Delta y}{\Delta t} = \frac{2.5\,\text{m}}{5\,\text{s}} = 0.5\,\text{m/s}$$

The average speed is the distance divided by the time

$$\text{speed} = \frac{d}{\Delta t} = \frac{61.3\,\text{m}}{5\,\text{s}} = 12.3\,\text{m/s}$$

10. The second ball must be in the air 1 second shorter than the first ball.

$$\Delta t_2 = \Delta t_1 - 1\text{s} = 5.10\text{s} - 1.00\text{s} = 4.10\text{s}$$

Again, when it hits the ground, $\Delta y = 0$

$$\begin{aligned}
\Delta y &= v_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2 \\
0 &= v_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2 \\
&= \Delta t(v_{iy} + \tfrac{1}{2} a_y \Delta t) \\
v_{iy} &= -\tfrac{1}{2} a_y \Delta t = -\tfrac{1}{2}(-9.8\,\text{m/s}^2)(4.10\text{s}) = 20.1\,\text{m/s}
\end{aligned}$$

NOW, CAN YOU DO THIS PROBLEM WITH A DIFFERENT STARTING POINT? For example, what if you were given the maximum height. What is the initial speed? What if you knew its velocity at 2 seconds. What is the speed at 4 seconds?