REVIEW AND SYNTHESIS: CHAPTERS 9–12

Review Exercises

1. **Strategy** The magnitude of the buoyant force on an object in water is equal to the weight of the water displaced by the object.

   Solution

   (a) Lead is much denser than aluminum, so for the same mass, its volume is much less. Therefore, $\boxed{\text{aluminum}}$ has the larger buoyant force acting on it; $\boxed{\text{since it is less dense it occupies more volume}}$.

   (b) Steel is denser than wood. Even though the wood is floating, it displaces more water than does the steel. Therefore, $\boxed{\text{wood}}$ has the larger buoyant force acting on it; $\boxed{\text{since it displaces more water than the steel}}$.

   (c) Lead: $\rho_\text{w} g V_\text{Pb} = \rho_\text{w} g \dfrac{m_\text{Pb}}{\rho_\text{Pb}} = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\dfrac{1.0\ \text{kg}}{11{,}300\ \text{kg/m}^3} = \boxed{0.87\ \text{N}}$

   Aluminum: $\rho_\text{w} g V_\text{Al} = \rho_\text{w} g \dfrac{m_\text{Al}}{\rho_\text{Al}} = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\dfrac{1.0\ \text{kg}}{2702\ \text{kg/m}^3} = \boxed{3.6\ \text{N}}$

   Steel: $\rho_\text{w} g V_\text{Steel} = \rho_\text{w} g \dfrac{m_\text{Steel}}{\rho_\text{Steel}} = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\dfrac{1.0\ \text{kg}}{7860\ \text{kg/m}^3} = \boxed{1.2\ \text{N}}$

   Wood: $mg = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{9.8\ \text{N}}$ (Since the wood is floating, the buoyant force is equal to its weight.)

2. **(a) Strategy** The relationship between the fraction of a floating object's volume that is submerged to the ratio of the object's density to the fluid in which it floats is $V_\text{f}/V_\text{o} = \rho_\text{o}/\rho_\text{f}$.

   Solution Find the percentage of the plastic that is submerged in the water.

   $$\frac{V_\text{submerged}}{V_\text{plastic}} = \frac{\rho_\text{plastic}}{\rho_\text{water}} = \frac{890\ \text{kg/m}^3}{1.00\times10^3\ \text{kg/m}^3} = 0.89,\ \text{or}\ \boxed{89\%}$$

   (b) Strategy and Solution

   In Figure C, the plastic will actually rise. Think about it this way. In Figure B, the part of the plastic above the water has a buoyant force from the air lifting it up. Since air is not very dense, this force is not very strong. Diagram C is correct. The oil is very dense so the buoyant force from the oil is strong and the plastic rises.

   (c) Strategy The buoyant forces of the water and oil on the plastic are equal to the total buoyant force on the plastic.

   Solution Let V_w and V_o be the volume of the plastic within the water and oil, respectively. Then, the total volume is $V_\text{p} = V_\text{o} + V_\text{w}$. Find the percentage of the plastic submerged in the water.

   $$\rho_\text{w} g V_\text{w} + \rho_\text{o} g V_\text{o} = \rho_\text{p} g V_\text{p}$$
   $$\rho_\text{w} V_\text{w} + \rho_\text{o}(V_\text{p} - V_\text{w}) = \rho_\text{p} V_\text{p}$$
   $$\rho_\text{w} V_\text{w} - \rho_\text{o} V_\text{w} = \rho_\text{p} V_\text{p} - \rho_\text{o} V_\text{p}$$
   $$\frac{V_\text{w}}{V_\text{p}} = \frac{\rho_\text{p} - \rho_\text{o}}{\rho_\text{w} - \rho_\text{o}} = \frac{890 - 830}{1000 - 830} = 0.35,\ \text{or}\ \boxed{35\%}$$

3. **Strategy** Use the continuity equation for incompressible fluids and Bernoulli's equation.

Solution Let the entrance point be 1 and the faucet be 2. Find v_1, the speed of the water in the main pipe.

$$A_1v_1 = A_2v_2, \text{ so } v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1.$$

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

$$v_1^2 = \frac{2(P_2 - P_1)}{\rho} + 2g(y_2 - y_1) + \left(\frac{r_1^2}{r_2^2}v_1\right)^2$$

$$\left(\frac{r_1^2}{r_2^2}v_1\right)^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} - 2g(y_2 - y_1)$$

$$v_1 = \sqrt{\left[\frac{2(P_1 - P_2)}{\rho} - 2g(y_2 - y_1)\right]\left[\left(\frac{r_1}{r_2}\right)^4 - 1\right]^{-1}}$$

$$v_1 = \sqrt{\left[\frac{2(52.0\times10^3 \text{ Pa})}{1.00\times10^3 \text{ kg/m}^3} - 2(9.80 \text{ m/s}^2)(4.20 \text{ m} + 0.90 \text{ m})\right]\left[\left(\frac{5.00}{1.20}\right)^4 - 1\right]^{-1}} = \boxed{0.116 \text{ m/s}}$$

4. **(a) Strategy** As Arnold falls, gravity does $mgh = (82 \text{ kg})(9.80 \text{ m/s}^2)(10 \text{ m}) = 8.0 \text{ kJ}$ of work on him, so each of his legs must absorb about 4.0 kJ of energy when he lands; that is, each leg must do work $W = F_{\text{av}}\Delta y$ to bring him to rest.

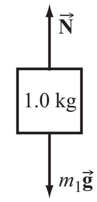
Solution Compute the compressive stress on Arnold's legs.

The average force on each femur is $F_{\text{av}} = \frac{W}{\Delta y} = \frac{4000 \text{ J}}{0.005 \text{ m}} = 8.0\times10^5 \text{ N}$. So, the compressive stress on each femur is $\frac{F_{\text{av}}}{A} = \frac{8.0\times10^5 \text{ N}}{5\times10^{-4} \text{ m}^2} = 2\times10^9 \text{ N/m}^2$. Since the compressive stress on each femur exceeds the maximum ultimate strength for compression, $\boxed{\text{Arnold's femur will break}}$.

(b) Strategy and Solution This time, Arnold has 30 cm instead of 5 mm to come to a stop, and it is the snow that does the work. So, the average force is $\frac{4000 \text{ J}}{0.30 \text{ m}} = 1.3\times10^4 \text{ N}$ and the compressive stress is $\frac{1.3\times10^4 \text{ N}}{5\times10^{-4} \text{ m}^2} = 3\times10^7 \text{ N/m}^2$. This time, the compressive stress is less than the maximum ultimate strength for compression, therefore, $\boxed{\text{Arnold's femur will not break}}$.

5. Strategy Use Eqs. (10-20a), (10-21), and (10-22), and Newton's second law.

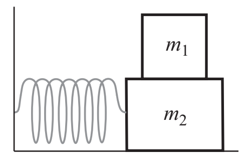
Solution The normal force on the 1.0-kg block m_1 is $N = m_1 g$. So, the force of friction on m_1 is $f = \mu N = \mu m_1 g$.



Find the maximum acceleration that the top block can experience before it starts to slip.

$\Sigma F = f = m_1 a$, so $a = \dfrac{f}{m_1} = \dfrac{\mu m_1 g}{m_1} = \mu g.$

For SHM, the maximum acceleration is $a_m = \omega^2 A = \dfrac{k}{m} A$, which in this case is equal to $\dfrac{kA}{m_1 + m_2}$. Equate the accelerations and solve for A.



$\dfrac{kA}{m_1 + m_2} = \mu g$, so $A = \dfrac{\mu(m_1 + m_2)g}{k}$.

The maximum speed is $v_m = \omega A$. Compute the maximum speed that this set of blocks can have without the top block slipping.

$$v_m = \omega A = \sqrt{\frac{k}{m_1 + m_2}}\left[\frac{\mu(m_1 + m_2)g}{k}\right] = \mu g\sqrt{\frac{m_1 + m_2}{k}} = 0.45(9.80 \text{ m/s}^2)\sqrt{\frac{1.0 \text{ kg} + 5.0 \text{ kg}}{150 \text{ N/m}}} = \boxed{0.88 \text{ m/s}}$$

6. Strategy Use Eq. (12-13) and conservation of energy.

Solution Find the maximum speed of the child, which occurs at the lowest point of her swing. As the child swings toward the whistle (1), $v_o < 0$. As she swings away (2), $v_o > 0$.

$$1.040 = \frac{f_1}{f_2} = \left(1 + \frac{|v_o|}{v}\right) f_s \left[\left(1 - \frac{|v_o|}{v}\right) f_s\right]^{-1} = \frac{v + |v_o|}{v - |v_o|}, \text{ so}$$

$$1.040(v - |v_o|) = v + |v_o|, \text{ or } |v_o| = \frac{0.040v}{2.040} = \frac{0.040(343 \text{ m/s})}{2.040} = 6.7 \text{ m/s}.$$

At the lowest point of the swing, the kinetic energy is at its maximum and the potential energy is at its minimum. Find how high the child is swinging; that is, her maximum height.

$$mgh_{max} = \frac{1}{2} m v_{max}^2, \text{ so } h_{max} = \frac{v_{max}^2}{2g} = \frac{(6.7 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{2.3 \text{ m}}.$$

7. (a) Strategy Use Eq. (11-7).

Solution Compute the wave speed for each traveling wave.

$$v_I = \frac{\omega_I}{k_I} = \frac{6.00 \text{ s}^{-1}}{4.00 \text{ cm}^{-1}} = 1.50 \text{ cm/s} \text{ and } v_{II} = \frac{\omega_{II}}{k_{II}} = \frac{3.00 \text{ s}^{-1}}{3.00 \text{ cm}^{-1}} = 1.00 \text{ cm/s}.$$

$\boxed{\text{Eq. I}}$ has the fastest wave speed of $\boxed{1.50 \text{ cm/s}}$.

(b) Strategy Use Eq. (11-7).

Solution Compute the wavelengths for each traveling wave.

$$\lambda_I = \frac{2\pi}{k_I} = \frac{2\pi}{4.00 \text{ cm}^{-1}} = 1.57 \text{ cm and } \lambda_{II} = \frac{2\pi}{k_{II}} = \frac{2\pi}{3.00 \text{ cm}^{-1}} = 2.09 \text{ cm}.$$

$\boxed{\text{Eq. II}}$ has the longest wavelength of $\boxed{2.09 \text{ cm}}$.

(c) Strategy Use Eq. (10-21).

Solution Compute the maximum speed of a point in the medium for each traveling wave.
$v_{\text{Im}} = \omega_{\text{I}} A_{\text{I}} = (6.00\text{ s}^{-1})(1.50\text{ cm}) = 9.00\text{ cm/s}$ and $v_{\text{IIm}} = \omega_{\text{II}} A_{\text{II}} = (3.00\text{ s}^{-1})(4.50\text{ cm}) = 13.5\text{ cm/s}$.
$\boxed{\text{Eq. II}}$ has the fastest maximum speed of a point in the medium of $\boxed{13.5\text{ cm/s}}$.

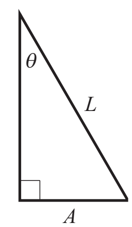
(d) Strategy and Solution $kx - \omega t$ indicates a wave is moving in the positive x-direction and $kx + \omega t$ indicates a wave is moving in the negative x-direction. So, $\boxed{\text{Eq. II}}$ is moving in the positive x-direction.

8. (a) Strategy Use Eq. (10-26a) and $\omega = 2\pi f$.

Solution Compute the frequency of the Foucault pendulum.

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{14.0\text{ m}}} = \boxed{0.133\text{ Hz}}$$

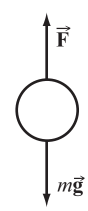
(b) Strategy Find the amplitude of the oscillation and use it and the frequency to find the maximum speed. Use Eq. (10-21).



Solution The amplitude of the oscillation is $A = L\sin\theta$. The maximum speed of the pendulum is

$$v_{\text{m}} = \omega A = \sqrt{\frac{g}{L}}L\sin\theta = \sqrt{gL}\sin\theta = \sqrt{(9.80\text{ m/s}^2)(14.0\text{ m})}\sin 6.10° = \boxed{1.24\text{ m/s}}.$$

(c) Strategy The maximum speed and tension occur at the equilibrium position. Use Newton's second law.



Solution Find the maximum tension.

$$\Sigma F_y = F - mg = ma_{\text{r}} = m\frac{v^2}{r} = m\frac{v_{\text{m}}^2}{L}, \text{ so}$$

$$F = m\left(g + \frac{v_{\text{m}}^2}{L}\right) = (15.0\text{ kg})\left[9.80\text{ m/s}^2 + \frac{(1.24\text{ m/s})^2}{14.0\text{ m}}\right] = \boxed{149\text{ N}}.$$

(d) Strategy Use Eqs. (11-2) and (11-13).

Solution Find the fundamental frequency of the wire.

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{FL}{m}} = \sqrt{\frac{F}{4Lm}} = \sqrt{\frac{149\text{ N}}{4(14.0\text{ m})(0.0100\text{ kg})}} = \boxed{16.3\text{ Hz}}$$

9. (a) Strategy Use Eqs. (11-2) and (11-13).

Solution Find the tension in the guitar string.

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{FL}{m}} = \sqrt{\frac{F}{4Lm}}, \text{ so } F = 4Lmf_1^2 = 4(0.655\text{ m})(0.00331\text{ kg})(82\text{ Hz})^2 = \boxed{58\text{ N}}.$$

(b) Strategy Use Eqs. (11-4) and (11-13).

Solution Find the length of the lowest frequency string when it is fingered at the fifth fret.

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}, \text{ so } L = \frac{1}{2f_1}\sqrt{\frac{F}{\mu}} = \frac{1}{2(110\text{ Hz})}\sqrt{\frac{58\text{ N}}{0.00331\text{ kg}/(0.655\text{ m})}} = \boxed{49\text{ cm}}.$$

10. Strategy Use Eq. (11-4) and $\Delta x = v\Delta t$.

Solution Find the time it takes for the wave pulse to travel from one child to the other.

$$\Delta t = \frac{\Delta x}{v} = \Delta x\sqrt{\frac{\mu}{F}} = (12\text{ m})\sqrt{\frac{0.0013\text{ kg/m}}{8.0\text{ N}}} = \boxed{0.15\text{ s}}$$

11. Strategy For two adjacent steps, the extra distance traveled by the wave reflected from the upper step is twice the tread depth. For the waves to cancel, they must be one-half wavelength out of phase or an odd multiple of one-half wavelength. Therefore, the minimum tread depth is half of one-half wavelength or one-quarter wavelength.

Solution Find the minimum tread depth d_{min}.

$$\lambda = \frac{v}{f},\text{ so } d_{\text{min}} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{343\text{ m/s}}{4(400.0\text{ Hz})} = \boxed{21.4\text{ cm}}.$$

12. (a) Strategy First, treat the brick wall as the observer. Next, treat the wall as the source. Use Eq. (12-12).

Solution The motion of Akiko is in the direction of sound propagation, so $v_{\text{Akiko}} > 0$.

$$f_{\text{wall}} = f_{\text{o}} = \frac{1}{1-\frac{v_{\text{s}}}{v}}f_{\text{s}} = \frac{1}{1-\frac{v_{\text{Akiko}}}{v}}f_{\text{Akiko}}$$

Now, since the wall and Haruki are stationary, there is no further Doppler effect. So,

$$f_{\text{Haruki}} = f_{\text{wall}} = \frac{1}{1-\frac{v_{\text{Akiko}}}{v}}f_{\text{Akiko}} = \frac{1}{1-\frac{7.00\text{ m/s}}{343\text{ m/s}}}(512.0\text{ Hz}) = \boxed{522.7\text{ Hz}}.$$

(b) Strategy The situation is similar to that of part (a), but instead of a stationary Haruki, we have Junichi as an observer moving in the direction of propagation of the sound reflected from the wall $(v_{\text{Junichi}} > 0)$. Use Eq. (12-13).

Solution

$$f_{\text{Junichi}} = f_{\text{o}} = \left(1-\frac{v_{\text{o}}}{v}\right)f_{\text{s}} = \left(1-\frac{v_{\text{Junichi}}}{v}\right)f_{\text{wall}} = \left(1-\frac{2.00\text{ m/s}}{343\text{ m/s}}\right)(522.67\text{ Hz}) = \boxed{519.6\text{ Hz}}$$

13. Strategy The frequency of the sound is increased by a factor equal to the number of holes in the disk. Use Eq. (11-6).

Solution The frequency of the sound is

$$f = 25(60.0\text{ Hz}) = \boxed{1500\text{ Hz}}.$$

Compute the wavelength that corresponds to this frequency.

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{1.50\times 10^3\text{ Hz}} = \boxed{22.9\text{ cm}}$$

14. Strategy Use Eqs. (11-4) and (11-13).

Solution Find the frequency in terms of the tension.

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Form a proportion to find the new fundamental frequency.

$$\frac{f_2}{f_1} = \frac{1}{2L}\sqrt{\frac{3F}{\mu}}\left(2L\sqrt{\frac{\mu}{F}}\right) = \sqrt{3}, \text{ so } f_2 = \sqrt{3}f_1 = \sqrt{3}(847\text{ Hz}) = \boxed{1470\text{ Hz}}.$$

15. Strategy and Solution There are 1000 cm^3 in 1 L. The heart pumps blood at a rate of $\frac{80\text{ cm}^3}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} = 4800\text{ cm}^3/\text{min}$. Since $5\text{ L} = 5000\text{ cm}^3 \approx 4800\text{ cm}^3$, it takes $\boxed{\text{about 1 minute}}$ for the medicine to travel throughout the body.

16. Strategy Use Eq. (11-6).

Solution

(a) The initial wavelength is $\lambda = \frac{v}{f} = \frac{341\text{ m/s}}{1231\text{ Hz}} = \boxed{27.7\text{ cm}}$.

(b) The frequency of the sound in the wall is the same as it was in the air. Therefore, the wavelength of the sound in the wall is $\lambda = \frac{v}{f} = \frac{620\text{ m/s}}{1231\text{ Hz}} = \boxed{50\text{ cm}}$.

(c) The frequency and the speed of sound are the same as before they entered the wall, so the wavelength is $\boxed{27.7\text{ cm}}$.

17. Strategy The source is moving in the direction of propagation of the sound, so $v_s > 0$. The observer is moving in the direction opposite the propagation of the sound, so $v_o < 0$. Use Eq. (12-14).

Solution Find the frequency heard by the passenger in the oncoming boat.

$$f_o = \frac{v - v_o}{v - v_s} f_s = \frac{343\text{ m/s} - (-15.6\text{ m/s})}{343\text{ m/s} - 20.1\text{ m/s}}(312\text{ Hz}) = \boxed{346\text{ Hz}}$$

18. (a) Strategy The wavelength of the diaphragm is 0.20 m. The frequency of the gas in the tube is the same as that of the diaphragm. Use Eq. (11-6).

Solution Find the speed of sound in the gas.

$$v = f\lambda = (1457\text{ Hz})(0.20\text{ m}) = \boxed{290\text{ m/s}}$$

(b) Strategy and Solution

The piles of sawdust represent displacement nodes: regions where the air remains at rest. They also represent pressure antinodes.

19. (a) Strategy Use the continuity equation for incompressible fluids and Bernoulli's equation.

Solution Let the lower end be 1 and the upper end be 2. Find the speed of the water as it exits the pipe.

$$A_1v_1 = A_2v_2,\ \text{so}\ v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1 = \left(\frac{10.0}{6.00}\right)^2(15.0\ \text{cm/s}) = \boxed{41.7\ \text{cm/s}}.$$

Find the pressure at the lower end.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 = P_2 + \rho g(y_2 - y_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$P_1 = 101.3\ \text{kPa} + (1.00\times10^3\ \text{kg/m}^3)\left\{(9.80\ \text{m/s}^2)(1.70\ \text{m}) + \frac{1}{2}\left[\left(\tfrac{10.0}{6.00}\right)^4 - 1\right](0.150\ \text{m/s})^2\right\}$$

$$P_1 = \boxed{118\ \text{kPa}}$$

(b) Strategy Use the equations of motion for a changing velocity.

Solution Find the time it takes for the water to fall to the ground.

$$\Delta y = v_\text{i}\sin\theta\Delta t - \frac{1}{2}g(\Delta t)^2$$

$$0 = g(\Delta t)^2 - 2v_\text{i}\sin\theta\Delta t + 2\Delta y$$

$$\Delta t = \frac{2v_\text{i}\sin\theta \pm \sqrt{4v_\text{i}^2\sin^2\theta - 8g\Delta y}}{2g}$$

$$\Delta t = \frac{2(0.4167\ \text{m/s})\sin 60.0° \pm \sqrt{4(0.4167\ \text{m/s})^2\sin^2 60.0° - 8(9.80\ \text{m/s}^2)(-0.300\ \text{m})}}{2(9.80\ \text{m/s}^2)}$$

$\Delta t = 0.287\ \text{s}$ (-0.213 s is rejected, since time is positive.)

The horizontal distance from the pipe outlet where the water lands is

$$\Delta x = v_\text{i}\cos\theta\Delta t = (41.7\ \text{cm/s})\cos 60.0°(0.287\ \text{s}) = \boxed{5.98\ \text{cm}}.$$

20. Strategy The standing wave is the first harmonic, f_2. The wave is in position B one-quarter period after it is in position A. It is in position C one-half period after it is in position A. Use Eqs. (11-4) and (11-13).

Solution

The speed of the wave on the string is $v = \sqrt{\frac{F}{\mu}}$, so $f_2 = \frac{v}{L} = \frac{1}{L}\sqrt{\frac{F}{\mu}}$.

The period is $T = \frac{1}{f_2} = L\sqrt{\frac{\mu}{F}} = (0.720\ \text{m})\sqrt{\frac{0.200\times10^{-3}\ \text{kg/m}}{2.00\ \text{N}}} = 7.20\ \text{ms}.$

Since the string is at A when $t = 0$, it will be at least one period before a photo of the string can be taken at position A; then it will be another period before the second photo can be taken. Therefore, the two earliest times after $t = 0$ that the string can be photographed in position A are $\boxed{\text{7.20 ms and 14.4 ms}}$. Unlike positions A and C, which only appear once per period, position B occurs twice per period. The string is at position B one-quarter period $(T/4)$ after it is at position A, and it will be at position B again an additional one-half period later $(T/4 + T/2 = 3T/4)$; therefore, the two earliest times after $t = 0$ that the string can be photographed in position B are $\boxed{\text{1.80 ms and 5.40 ms}}$. The string is at position C one-half period $(T/2)$ after it is at position A, and it will be at position C again an additional period later $(T/2 + T = 3T/2)$; therefore, the two earliest times after $t = 0$ that the string can be photographed in position C are $\boxed{\text{3.60 ms and 10.8 ms}}$.

21. Strategy Use the equations describing waves on a string fixed at both ends.

Solution

(a) The wavelength of the fundamental mode is $\lambda = 2L = 2(0.640\text{ m}) = \boxed{1.28\text{ m}}$.

(b) The wave speed on the string is $v = \lambda f = (1.28\text{ m})(110.0\text{ Hz}) = \boxed{141\text{ m/s}}$.

(c) Find the linear mass density of the string.

$v = \sqrt{\dfrac{F}{\mu}}$, so $\mu = \dfrac{F}{v^2} = \dfrac{133\text{ N}}{(140.8\text{ m/s})^2} = \boxed{6.71\text{ g/m}}$.

(d) Find the maximum speed of a point on the string.

$v = \omega A = 2\pi fA = 2\pi(110.0\text{ Hz})(0.00230\text{ m}) = \boxed{0.253\text{ m/s}}$

(e) The frequency in air is that same as that in the bridge and body of the guitar, which is the same as the frequency of the string: $\boxed{110.0\text{ Hz}}$.

(f) Find the speed of sound in air.

$v = 331\text{ m/s} + (0.60\text{ m/s})(20) = 343\text{ m/s}$

Find the wavelength of the sound wave in air.

$\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{110.0\text{ Hz}} = \boxed{3.12\text{ m}}$

22. Strategy Use conservation of energy, conservation of momentum, and Newton's second law.

Solution

(a) Find the speed of the oranges just before they land on the pan.

$\dfrac{1}{2}mv^2 = mgh$, so $v = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})} = 2.425\text{ m/s}$.

Find the speed of the oranges and the pan immediately after the oranges land on the pan.

$mv_{\text{oi}} = (m+M)v$, so $v = \dfrac{m}{m+M}v_{\text{oi}} = \dfrac{2.20\text{ kg}}{2.20\text{ kg} + 0.250\text{ kg}}(2.425\text{ m/s}) = \boxed{2.18\text{ m/s}}$.

(b) Find the new equilibrium point.

$\Sigma F = k\Delta y - mg$, so $\Delta y = \dfrac{mg}{k} = \dfrac{(2.20\text{ kg})(9.80\text{ m/s}^2)}{450\text{ N/m}} = \boxed{4.8\text{ cm}}$.

(c) Find the amplitude of the oscillations.

$m_{\text{tot}}g\Delta y + \dfrac{1}{2}k(\Delta y)^2 + \dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2$, so

$$A = \sqrt{\frac{2m_{\text{tot}}g\Delta y + k(\Delta y)^2 + m_{\text{tot}}v^2}{k}}$$

$$= \sqrt{\frac{2(2.45\text{ kg})(9.80\text{ m/s}^2)(0.0479\text{ m}) + (450\text{ N/m})(0.0479\text{ m})^2 + (2.45\text{ kg})(2.18\text{ m/s})^2}{450\text{ N/m}}} = \boxed{0.18\text{ m}}$$

(d) Find the frequency of the oscillations.

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{450\text{ N/m}}{2.45\text{ kg}}} = \boxed{2.2\text{ Hz}}$.

23. (a) Strategy Find the mass of the balloon and helium. Then use Newton's second law.

Solution Find the mass.

$$m = m_{\text{He}} + m_{\text{b}} = \rho_{\text{He}}V + m_{\text{b}} = (0.179\ \text{kg/m}^3)\frac{4}{3}\pi(0.120\ \text{m})^3 + 2.80\times10^{-3}\ \text{kg} = 0.00410\ \text{kg}$$

Find the tension in the ribbon.

$\Sigma F = F_{\text{B}} - mg - T = 0,$ so

$$T = F_{\text{B}} - mg = \rho_{\text{air}}Vg - mg = \left[(1.29\ \text{kg/m}^3)\frac{4}{3}\pi(0.120\ \text{m})^3 - 0.004096\ \text{kg}\right](9.80\ \text{m/s}^2) = \boxed{5.13\times10^{-2}\ \text{N}}.$$

(b) Strategy Use the small angle approximation for $\sin\theta$. Consider Hooke's law.

Solution Find the period of oscillation.

$F = F_{\text{T}}\sin\theta \approx F_{\text{T}}\theta \approx F_{\text{T}}\dfrac{\Delta x}{L}$, which implies that $k \approx \dfrac{F_{\text{T}}}{L}$. Thus, we can find the period using

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mL}{F_{\text{T}}}} = 2\pi\sqrt{\frac{(4.10\times10^{-3}\ \text{kg})(2.30\ \text{m})}{5.13\times10^{-2}\ \text{N}}} = \boxed{2.69\ \text{s}}.$$

24. Strategy Use Bernoulli's equation and the pressure difference of a static liquid.

Solution We have the two relations:

$P_2 - P_1 = \rho_{\text{liquid}}gh$ and $P_1 + \dfrac{1}{2}\rho_{\text{air}}v^2 = P_2.$

Eliminate the pressure difference and find the minimum speed of the air.

$$\frac{1}{2}\rho_{\text{air}}v^2 = \rho_{\text{liquid}}gh,\ \text{so}\ v = \sqrt{\frac{2\rho_{\text{liquid}}gh}{\rho_{\text{air}}}} = \sqrt{\frac{2(800\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.030\ \text{m})}{1.29\ \text{kg/m}^3}} = \boxed{19\ \text{m/s}}.$$

25. Strategy Use Newton's second law and Hooke's law.

Solution

(a) Find the tension in the string when hanging straight down.

$\Sigma F = T_1 - mg = 0,$ so $T_1 = mg.$

Find the tension in the string while it is swinging.

$\Sigma F_y = T_2\cos\theta - mg = 0,$ so $T_2 = \dfrac{mg}{\cos\theta}.$

Find the stretch of the string.

$Y = \dfrac{\Delta T/A}{\Delta L/L}$, so

$$\Delta L = \frac{L\Delta T}{AY} = \frac{Lmg}{AY}\left(\frac{1}{\cos\theta} - 1\right) = \frac{(2.200\ \text{m})(0.411\ \text{kg})(9.8\ \text{m/s}^2)}{\pi(0.00125\ \text{m})^2(4.00\times10^9\ \text{Pa})}\left(\frac{1}{\cos 65.0^\circ} - 1\right) = \boxed{6.17\times10^{-4}\ \text{m}}.$$

(b) Find the kinetic energy.

$\Sigma F_x = T_2\sin\theta = ma_x = \dfrac{mv^2}{r} = \dfrac{2K}{r}$, so

$$K = \frac{rT_2\sin\theta}{2} = \frac{L\sin\theta\frac{mg}{\cos\theta}\sin\theta}{2} = \frac{mgL\tan\theta\sin\theta}{2} = \frac{(0.411\ \text{kg})(9.80\ \text{m/s}^2)(2.200\ \text{m})\tan 65.0^\circ\sin 65.0^\circ}{2}$$
$$= \boxed{8.61\ \text{J}}$$

(c) Find the time it takes a transverse wave pulse to travel the length of the string. Neglect the small change due to the stretch when finding the linear mass density.

$v=\sqrt{\frac{T_2}{\mu}}=\sqrt{\frac{T_2}{\rho A}}$ and $L=vt$, so

$$t=\frac{L+\Delta L}{v}=(L+\Delta L)\sqrt{\frac{\rho A}{T_2}}=(L+\Delta L)\sqrt{\frac{\rho A\cos\theta}{mg}}=(2.2006\text{ m})\sqrt{\frac{(1150\text{ kg/m}^3)\pi(0.00125\text{ m})^2\cos 65.0^\circ}{(0.411\text{ kg})(9.80\text{ m/s}^2)}}$$
$$=\boxed{0.0536\text{ s}}$$

26. Strategy The tension is 0.93 of the tensile strength. Use Eqs. (10-2), (11-2), and (11-13).

Solution Set the stress equal to the strength.

$\frac{F}{A}=\text{strength}=\frac{\frac{T}{0.93}}{A}$, so $T=0.93A(\text{strength})$. $f_n=\frac{nv}{2L}$ and $v=\sqrt{\frac{TL}{m}}$. Find f_1.

$$f_1=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{TL}{m}}=\frac{1}{2}\sqrt{\frac{TL}{mL^2}}=\frac{1}{2}\sqrt{\frac{0.93A(\text{strength})L}{mL^2}}=\frac{1}{2}\sqrt{\frac{0.93(\text{strength})V}{mL^2}}=\frac{1}{2}\sqrt{\frac{0.93(\text{strength})}{\rho L^2}}$$
$$=\frac{1}{2}\sqrt{\frac{0.93(6.3\times10^8\text{ Pa})}{(8500\text{ kg/m}^3)(0.094\text{ m})^2}}=\boxed{1.4\text{ kHz}}$$

MCAT Review

1. Strategy The buoyant force on the brick is equal in magnitude to the weight of the volume of water it displaces.

Solution The brick is completely submerged, so its volume is equal to that of the displaced water. The weight of the displaced water is $30\text{ N}-20\text{ N}=10\text{ N}$. Find the volume of the brick.

$$\rho_\text{w}gV_\text{w}=\rho_\text{w}gV_\text{brick}=10\text{ N, so }V_\text{brick}=\frac{10\text{ N}}{\rho_\text{w}g}=\frac{10\text{ N}}{(1000\text{ kg/m}^3)(10\text{ m/s}^2)}=1\times10^{-3}\text{ m}^3$$

The correct answer is $\boxed{\text{A}}$.

2. Strategy The expansion of the cable obeys $F=k\Delta L$.

Solution Compute the expansion of the cable.

$$\Delta L=\frac{F}{k}=\frac{5000\text{ N}}{5.0\times10^6\text{ N/m}}=10^{-3}\text{ m}$$

The correct answer is $\boxed{\text{A}}$.

3. Strategy Solve for the intensity in the definition of sound level.

Solution Find the intensity of the fire siren.

$$\text{SL}=10\log_{10}\frac{I}{I_0},\text{ so }I=I_0 10^{\text{SL}/10}=(1.0\times10^{-12}\text{ W/m}^2)10^{100/10}=1.0\times10^{-2}\text{ W/m}^2.$$

The correct answer is $\boxed{\text{D}}$.

4. Strategy and Solution The two centimeters of liquid with a specific gravity of 0.5 are equivalent to one centimeter of water; that is, the 6-cm column of liquid is equivalent to a 5-cm column of water. Therefore, the new gauge pressure at the base of the column is five-fourths the original. The correct answer is $\boxed{\text{C}}$.

5. Strategy The wavelength is inversely proportional to the index n. Let $\lambda_n = 8$ m and $\lambda_{n+2} = 4.8$ m.

Solution Find n.

$$L = \frac{n\lambda_n}{4} = \frac{(n+2)\lambda_{n+2}}{4}, \text{ so } \frac{\lambda_n}{\lambda_{n+2}} = \frac{n+2}{n} = 1 + \frac{2}{n}, \text{ or } n = 2\left(\frac{\lambda_n}{\lambda_{n+2}} - 1\right)^{-1}.$$

Compute L.

$$L = \frac{n\lambda_n}{4} = 2\left(\frac{\lambda_n}{\lambda_{n+2}} - 1\right)^{-1}\frac{\lambda_n}{4} = \frac{1}{2}\left(\frac{8\text{ m}}{4.8\text{ m}} - 1\right)^{-1}(8\text{ m}) = 6\text{ m}$$

The correct answer is $\boxed{\text{C}}$.

6. Strategy The wave may interfere within the range of possibility of totally constructive or totally destructive interference.

Solution For totally constructive interference, the amplitude of the combined waves is $5 + 3 = 8$ units. For totally destructive interference, the amplitude of the combined waves is $5 - 3 = 2$ units. The correct answer is $\boxed{\text{B}}$.

7. Strategy and Solution As the bob repeatedly swings to and fro, it speeds up and slows down, as well as changes direction. Therefore, its linear acceleration must change in both magnitude and direction.
The correct answer is $\boxed{\text{D}}$.

8. Strategy Since $K \propto v^2$ and $v \propto r^{-4}$, $K \propto r^{-8}$.

Solution Compute the ratio of kinetic energies.

$$\frac{K_2}{K_1} = \left(\frac{2}{1}\right)^{-8} = \frac{1}{256} = \frac{1}{4^4} \text{ or } K_2 : K_1 = 1 : 4^4.$$

The correct answer is $\boxed{\text{B}}$.

9. Strategy The buoyant force on a ball is equal to the weight of the volume of water displaced by that ball.

Solution Since B_1 is not fully submerged and B_2 and B_3 are, the buoyant force on B_1 is less than the buoyant forces on the other two. Since B_2 and B_3 are fully submerged, the buoyant forces on each are the same.
The correct answer is $\boxed{\text{B}}$.

10. Strategy and Solution Ball 1 is floating, so its density is less than that of the water. Since Ball 2 is submerged within the water, its density is the same as that of the water. Ball 3 is sitting on the bottom of the tank, so its density is greater than that of the water. The correct answer is $\boxed{\text{A}}$.

11. Strategy The supporting force of the bottom of the tank is equal to the weight of Ball 3 less the buoyant force of the water.

Solution Compute the supporting force on Ball 3.

$$\begin{aligned}\rho_{\text{Ball 3}}gV_{\text{Ball 3}} - \rho_{\text{water}}gV_{\text{Ball 3}} &= gV_{\text{Ball 3}}(\rho_{\text{Ball 3}} - \rho_{\text{water}})\\ &= (9.80\text{ m/s}^2)(1.0\times10^{-6}\text{ m}^3)(7.8\times10^3\text{ kg/m}^3 - 1.0\times10^3\text{ kg/m}^3) = 6.7\times10^{-2}\text{ N}\end{aligned}$$

The correct answer is $\boxed{\text{B}}$.

12. Strategy The relationship between the fraction of a floating object's volume that is submerged to the ratio of the object's density to the fluid in which it floats is $V_\text{f}/V_\text{o} = \rho_\text{o}/\rho_\text{f}$. So, the fraction of the object that is not submerged is $V_\text{ns} = 1 - V_\text{f}/V_\text{o} = 1 - \rho_\text{o}/\rho_\text{f}$.

Solution Find the fraction of the volume of Ball 1 that is above the surface of the water.

$$1 - \frac{\rho_\text{Ball 1}}{\rho_\text{water}} = 1 - \frac{8.0\times10^2\ \text{kg/m}^3}{1.0\times10^3\ \text{kg/m}^3} = \frac{1}{5}$$

The correct answer is $\boxed{\text{D}}$.

13. Strategy The pressure difference at a depth *d* in water is given by $\rho_\text{water}gd$.

Solution Compute the approximate difference in pressure between the two balls.

$$\rho_\text{water}gd = (1.0\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.20\ \text{m}) = 2.0\times10^3\ \text{N/m}^2$$

The correct answer is $\boxed{\text{C}}$.

14. Strategy For the force exerted on Ball 3 by the bottom of the tank to be zero, Ball 3 must have the same density as the water.

Solution Find the volume of the hollow portion of Ball 3, V_H.

$$\rho_\text{Ball 3} = \frac{m_\text{Ball 3}}{V_\text{Ball 3}} = \frac{\rho_\text{Fe}V_\text{Fe}}{V_\text{Ball 3}} = \rho_\text{water} \text{ and } V_\text{H} = V_\text{Ball 3} - V_\text{Fe}, \text{ so}$$

$$V_\text{H} = V_\text{Ball 3} - \frac{\rho_\text{water}}{\rho_\text{Fe}}V_\text{Ball 3} = V_\text{Ball 3}\left(1 - \frac{\rho_\text{water}}{\rho_\text{Fe}}\right) = (1.0\times10^{-6}\ \text{m}^3)\left(1 - \frac{1.0\times10^3\ \text{kg/m}^3}{7.8\times10^3\ \text{kg/m}^3}\right) = 0.87\times10^{-6}\ \text{m}^3.$$

The correct answer is $\boxed{\text{C}}$.