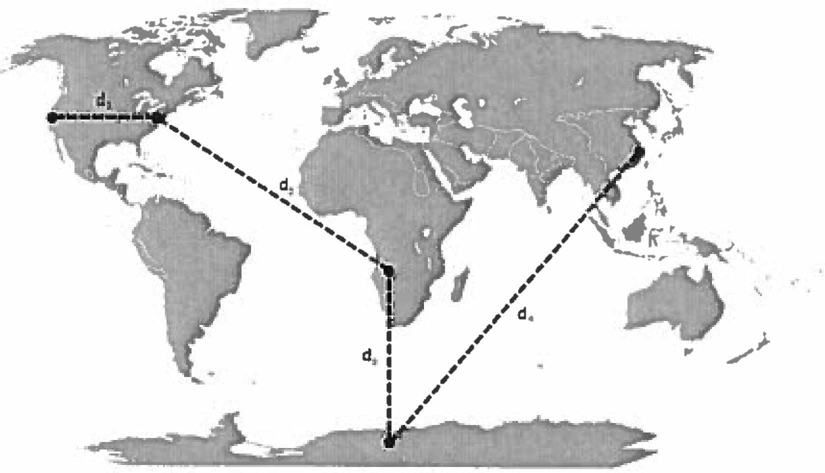
Exam 1

Thursday, May 26, 2016
5:00-6:15 p.m.

Name: KEY

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: Indiana Jones yearns to find the next world-changing artifact. He sets out on a globe-trotting adventure, which we view as dashed lines being drawn on a flat, two-dimensional map:



He first travels $\vec{d_1} = 3,200$ km due East across the United States to New York, where he boards a ship and sails to Africa. The ship travels along the roughly straight path $\vec{d_2} = 10,000$ km at 30° South of East. From Africa, Dr. Jones treks $\vec{d_3} = 7,000$ km due South to the frigid Antarctic tundras. Finally, he sets sail to Beijing along a route $\vec{d_4} = 18,000$ km at 60° North of East.

(a) What is the total distance traveled by Dr. Jones?

(b) What are the magnitude and direction of his total displacement?

(c) The film lasts 2 hours. Find the average speed and average velocity for his trip (as measured by the film goer, not the adventurer), assuming that his trip begins when the movie starts and finishes just as the movie ends.

$\vec{d_1} = 3200$ km E $\quad \vec{d_2} = 10000$ km 30° S of E $\quad \vec{d_3} = 7000$ km S $\quad \vec{d_4} = 18000$ km 60° N of E

a) Total distance is sum of magnitudes

$$\text{distance} = |\vec{d_1}| + |\vec{d_2}| + |\vec{d_3}| + |\vec{d_4}|$$
$$= (3200 + 10000 + 7000 + 18000)\text{ km} = \boxed{38,200 \text{ km}}$$

b) To get displacement, add as vectors:

$$disp_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = 3200 + 10000\cos(-30°) + 0 + 18000\cos(60)$$
$$= 20,860.3 \text{ km}$$

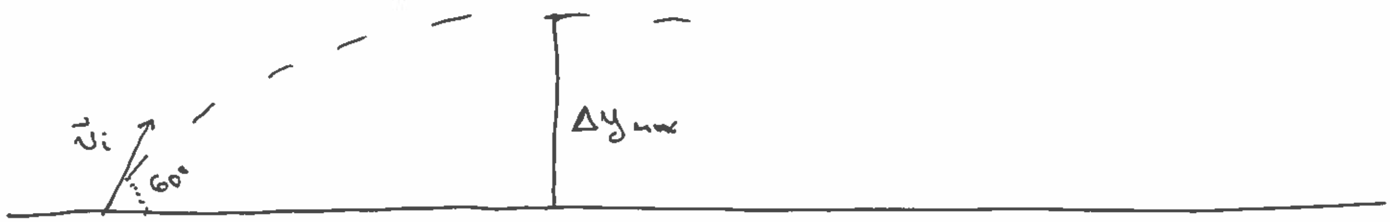
$$disp_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = 0 + 10000\sin(-30) - 7000 + 18000\sin(60)$$
$$= 3588.5 \text{ km}$$

$$\theta = \arctan\left(\frac{disp_y}{disp_x}\right)$$

$$\text{displacement} = \sqrt{disp_x^2 + disp_y^2} = \boxed{21,166.7 \text{ km at } 9.8° \text{ N of E}}$$

c) $$\text{average speed} = \frac{\text{total distance}}{2 \text{ hrs}} = \boxed{19,100 \frac{\text{km}}{\text{hr}}} \qquad \text{avg velocity} = \frac{\text{displacement}}{2 \text{ hrs}} = \boxed{10,583.4 \frac{\text{km}}{\text{hr}} \text{ at } 9.8° \text{ N of E}}$$

Problem 2: A baseball is thrown (from ground level) at an angle of 60° above the horizontal. If it reaches a maximum height of 20 m above the ground, what is its speed when it is thrown?



$\vec{v}_i = ?$ at 60° above horizontal $\qquad \Delta y_{max} = 20\,m$

At max height, we know $v_y = 0$:

Known

$\Delta y_{max} = 20\,m$

$v_y = 0$

$a_y = -g$

Find v_{iy} $\Rightarrow$ $v_y^2 - v_{iy}^2 = 2a_y \Delta y$

$-v_{iy}^2 = -2g\Delta y_{max}$

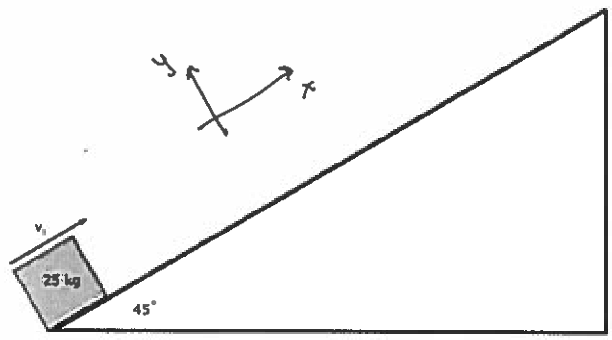
$v_{iy} = \sqrt{2g\Delta y_{max}} = 20\,m/s$

We know that v_{iy} is related to v_i by the sine of the angle:

$v_{iy} = v_i \sin\theta \longrightarrow v_i = \frac{v_{iy}}{\sin\theta} = \frac{20\,m/s}{\sin 60°}$

$\boxed{v_i = 23.1\ m/s}$

Problem 3: A 25 kg block is launched up a frictionless incline with initial speed $v_i = 12$ m/s. The incline is 45 m long and angled at $\theta = 45°$ above the horizontal.



(a) What is the value of the block's normal force on the ramp?

(b) How far along the ramp does the block travel before coming to rest (for an instant, before sliding back down)?

a) As the block remains in contact with the incline, $a_y = 0$. Thus $F_{net,y} = 0$.

From FBD →

$$F_N - W_y = 0 \quad \text{so}$$

$$F_N = W_y = mg\cos 45° \longrightarrow \boxed{F_N = 176.8\ \text{N}}$$

b) Nothing balances the weight in x-dir, so we have a net accel. in x:

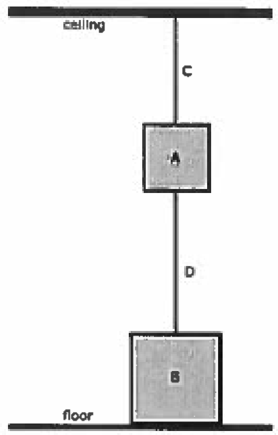
$$-W_x = m a_x \longrightarrow a_x = -g\sin\theta.$$

To find how far we travel until we stop, find Δx when $v_x = 0$:

$$v_x^2 - v_{ix}^2 = 2a_x\Delta x \longrightarrow -v_{ix}^2 = -2g\sin\theta\,\Delta x$$

$$\text{or} \quad \Delta x = \frac{v_{ix}^2}{2g\sin\theta} = \boxed{10.2\ \text{m}}$$

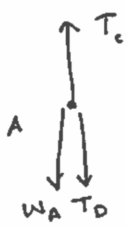
Problem 4: Block A of mass $m_A = 1$ kg is tied to the ceiling by the massless rope C. A second block, block B with mass $m_B = 2$ kg, is tied to block A by the massless rope D. Block B rests on the floor. The tension in rope C is $T_C = 25$ N.



Everything at rest $\longrightarrow a = 0.$

(a) Calculate the magnitude of the tension in rope D.

(b) Calculate the normal force between block B and the floor.



$T_c - T_D - W_A = 0$ ①

$T_D + F_N - W_B = 0$ ②

a) We are given $T_c = 25$N and can calculate $W_A = m_A g = 10$ N, so from ①:

$$T_c - T_D - W_A = 0 \longrightarrow T_D = T_c - W_A = \boxed{15 \text{ N}}$$

b) Now we know T_D and $W_B = m_B g = 20$ N, so from ②:

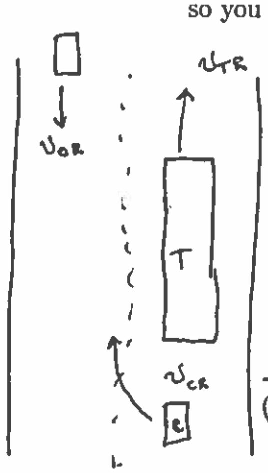
$$T_D + F_N - W_B = 0 \longrightarrow F_N = W_B - T_D = \boxed{5 \text{ N}}$$

Problem 5: You are driving on a two-lane highway when you come up behind a 12-meter-long semi-truck driving at 25 m/s relative to the road. You decide to pass the semi-truck, and so you pull into the opposing lane of traffic and speed up until your speedometer reads 35 m/s. For this problem, ignore the time it takes for you to accelerate and change lanes.

(a) If the speed limit on the road is 30 m/s, how fast are you traveling with respect to the oncoming traffic?

(b) How long does it take you to pass the semi-truck?

(c) Looking up from your dashboard, you notice a car heading toward you in the oncoming lane as you start to pass the semi-truck. If the oncoming car is 200 m away when you are at the back of the semi-truck, do you pass it in time or collide with the oncoming car? (Assume your car has no length, so you can get back in the correct lane immediately when you reach the front of the semi-truck.)



$v_{TR} = +25$ m/s

$v_{CR} = +35$ m/s

$v_{OR} = -30$ m/s

(a) We want v_{CO}, the velocity of your car relative to the oncoming traffic:

$$v_{CO} = v_{CR} + v_{RO}$$
$$= v_{CR} - v_{OR}$$
$$= 35\text{ m/s} - (-30\text{ m/s}) = \boxed{65\text{ m/s}}$$

(b) We are moving at

$$v_{CT} = v_{CR} + v_{RT}$$
$$= v_{CR} - v_{TR} = 10\text{ m/s}$$

relative to truck, and the truck is 12 m long:

$$\Delta t_T = \frac{\Delta x}{v} = \frac{12\text{ m}}{10\text{ m/s}} = \boxed{1.2\text{ s}}$$

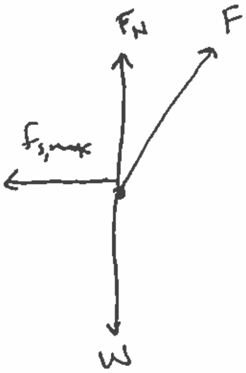
c) It takes us 1.2 s to pass the truck. How long does it take the oncoming traffic to reach us? The oncoming traffic is moving at a speed of 65 m/s (from (a)), and they are 200 m away:

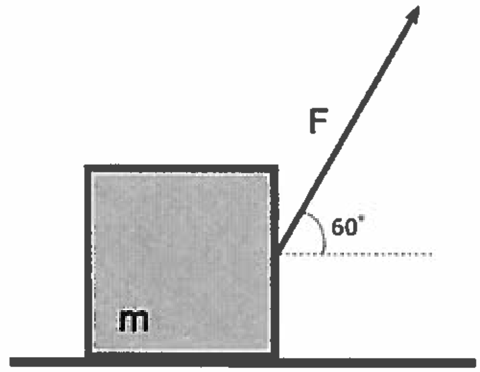
$$\Delta t_{onc} = \frac{200\text{ m}}{65\text{ m/s}} = 3.1\text{ s}$$

which is longer than it takes to pass the truck.

$\boxed{\text{Pass in time}}$

Problem 6: A block of mass $m = 50$ kg rests on a horizontal surface as shown below. The coefficients of static and kinetic friction between the surfaces in contact are $\mu_s = 0.20$ and $\mu_k = 0.18$, respectively. You wish to slide the object along the surface by exerting a force $\vec{F}$ on the block at an angle of 60° above the horizontal. What is the *minimum* force required to set the block in motion?





$$F_{net,y} = F_N + F\sin 60° - W = 0 \quad (1)$$

$$F_{net,x} = \underbrace{F\cos 60° - f_{s,max} = 0}_{\text{this is true just as it starts to slip.}} \quad (2)$$

Use (1) to solve for F_N $\longrightarrow$ plug into $f_{s,max} = \mu_s F_N$. Then use this in (2) to find F:

$$(1):\ F_N + F\sin 60° - W = 0 \longrightarrow F_N = mg - F\sin 60°$$

thus $f_{s,max} = \mu_s (mg - F\sin 60°)$.

$$(2) \rightarrow F\cos 60° - \mu_s (mg - F\sin 60°) = 0$$

or $F\cos 60° - \mu_s F \sin 60° = \mu_s mg$

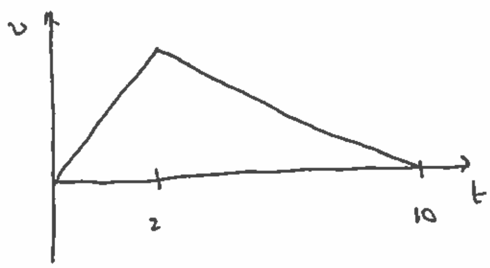
$$\longrightarrow F = \frac{\mu_s mg}{\cos 60° - \mu_s \sin 60°} = \boxed{306\text{ N}}$$

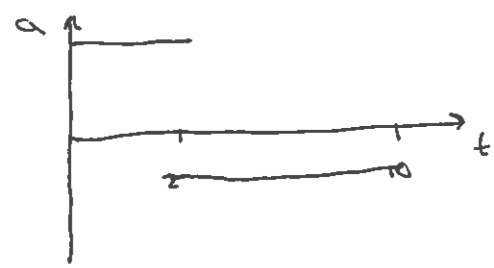
Problem 7: A drag-racing car sits at rest on the starting line. When the light turns green, the car accelerates forward down the track at a rate of $a = 40$ m/s^2. After 2 s of driving, the engine is cut and the brakes are applied (along with little parachutes) in order to slow the car down. While braking, the car experiences a net acceleration of magnitude $a = 10$ m/s^2 directed opposite its motion. The car comes to rest a total time of 10 s after the light turned green.

(a) Sketch the velocity vs. time and acceleration vs. time graphs for the car's motion in these ten seconds.

(b) How far did the car travel from the starting line to where it stops?

(a) $a = +40$ for $0 < t \le 2s$ $\quad a = -10$ for $2 \le t \le 8s$





b) We have two intervals of const. $\vec{a}$:

$0 \le t \le 2s$

$v_{i_1} = 0$, $a_1 = +40$ m/s^2, $\Delta t_1 = 2s$ $\longrightarrow$ $\Delta X_1 = \frac{1}{2} a_1 \Delta t_1^2 = 80$ m

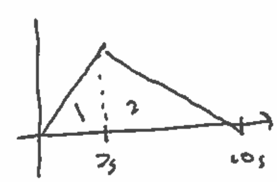
$2 \le t \le 8s$

$v_{i_2} = a_1 \Delta t_1 = 80$ m/s, $a_2 = -10$ m/s^2, $\Delta t_2 = 8s$

$$\Delta X_2 = v_{i_2} \Delta t_2 + \frac{1}{2} a_2 (\Delta t_2)^2$$
$$= 80 \text{ m/s} (8s) - \frac{1}{2} (10 \text{ m/s}^2)(8s)^2$$
$$= 320 \text{ m}$$

so $\boxed{\Delta X = \Delta X_1 + \Delta X_2 = 400 \text{ m}}$

Could also use area under v vs. t graph



$A_1 = \frac{1}{2} (2s)(a_1 \cdot 2s)$
$= 80$ m

$A_2 = \frac{1}{2} (8s)(80 \text{ m/s})$
$= 320$ m

$\Delta X = A_1 + A_2 = 400$ m