

# Exam 2 - Practice Test

Saturday, June 11, 2016

Name: \_\_\_\_\_

There are seven problems on this exam. You must show all your work to get full credit.  
Please box your final answers.

**Problem 1:** Two balls are tossed into the air at the same moment and from the same initial position. One ball has a mass of 3 kg and is thrown vertically with an initial speed of 17 m/s. The other ball has a mass of 4 kg and is thrown with an initial velocity of 20 m/s at an angle of  $40^\circ$  above the horizontal. What is the velocity of the center-of-mass of the two-ball system after 1.5 s? Ignore air resistance.

### Solution

In this problem two balls are tossed up. Their velocity changes with time. We need the velocity of their center of mass after 1.5 seconds. There are two ways of calculating this.

One, calculate their individual velocities after 1.5 s and plug those into the center of mass formula. If we label the balls A and B, we have

$$v_{A,x} = 0$$

$$v_{A,y}^i = 17 \text{ m/s}$$

$$v_{A,y}^f = v_{A,y}^i - g\Delta t = 2 \text{ m/s}$$

$$v_{B,x} = 20 \cos 40^\circ = 15.32 \text{ m/s}$$

$$v_{B,y}^i = 20 \sin 40^\circ = 12.86 \text{ m/s}$$

$$v_{B,y}^f = v_{B,y}^i - g\Delta t = -2.14 \text{ m/s}$$

So the velocity of the centre of mass after 1.5 s will be

$$v_{\text{CoM},x} = \frac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \mathbf{8.75 \text{ m/s}}$$

$$v_{\text{CoM},y}^f = \frac{m_A v_{A,y}^f + m_B v_{B,y}^f}{m_A + m_B} = \mathbf{-0.37 \text{ m/s}}$$

Alternatively, you can find the velocity of the centre of mass at the time of launch using the initial velocities, and the acceleration of the center of mass. Using these we can calculate the final velocity of the centre of mass. The  $x$ -calculation will be the same as before. In the  $y$ -direction

$$v_{\text{CoM},y}^i = \frac{m_A v_{A,y}^i + m_B v_{B,y}^i}{m_A + m_B} = 14.63 \text{ m/s}$$

This is the initial velocity of the center of mass in the  $y$ -direction. Next we need to find the (constant) acceleration of the center of mass. Notice that both A and B have the same acceleration of  $g$  downwards, and so, so does their center of mass. This can also be seen by plugging into the formula for acceleration of center of mass. As a result we have

$$v_{\text{CoM},y}^f = v_{\text{CoM},y}^i - g\Delta t = \mathbf{-0.37 \text{ m/s}}$$

**Problem 2:** A cat is dropped from rest from a height of 5 m above the ground. It falls freely through the air before coming to rest on the ground below (safely). What is the impulse caused on the cat by the ground to bring it to rest? EDIT: The mass of the cat is 5 kg.

### Solution

The mass of the cat, which wasn't provided in the original version, is required to solve this problem.

Using conservation of mechanical energy or kinematics we can find the velocity of the cat just before landing.

$$v_{f,y}^2 = v_{i,y}^2 - 2g\Delta y = 0 - 2 \times 10 \times (-5) = 100 \text{ m}^2/\text{s}^2$$

which gives the velocity of the cat to be -10 m/s (we need to take the negative root since the cat is moving downwards). For this part, the mass isn't required.

With this velocity we can find the change in momentum that happens during the landing/collision using the impulse-momentum theorem. Momentum just before collision is  $mv_{f,y} = -50 \text{ kg m/s}$ . Momentum after collision is 0. So the change in momentum

$$\Delta p_y = p_{f,y} - p_{f,i} = 0 - (-50) = +50 \text{ kg m/s}$$

Now, impulse-momentum theorem tell us that the impulse caused by the ground is **+50 kg m/s** or **+50 N s**.

**Problem 3:** Maureen is standing in a boat and wishes to get to shore. Unfortunately she has lost her paddles! While searching the boat she finds a large can of tomato soup, mass  $m_{\text{soup}} = 15$  kg, and she decides to throw it overboard in frustration. To her surprise, the boat starts to move after she has thrown the can! If the can was thrown with an initial velocity of 35 m/s at  $20^\circ$  above the horizontal, how fast is the boat traveling just after the can leaves Maureen's hand? Assume that Maureen and the boat have a combined mass of 300 kg and that the water does not resist the boat's horizontal travel.

### Solution

In this problem, we use conservation of momentum, but only in the (horizontal)  $x$ -direction. Before the 'collision' (throw), the total momentum in the  $x$ -direction is 0. After collision it is

$$\begin{aligned} p_{\text{tot},x}^f &= p_{\text{soup},x}^f + p_{\text{boat},x}^f \\ \Rightarrow 0 &= m_{\text{soup}} \times 35 \cos 20^\circ + m_{\text{boat}} \times v_{\text{boat},x}^f \end{aligned}$$

Here, I've chosen my positive  $x$ -direction in the direction of the horizontal component of the can's velocity. Plugging in the numbers, we get

$$v_{\text{boat},x}^f = -1.64 \text{ m/s}$$

I've implicitly assumed in the solution that the velocity of the can given in the problem is with respect to the ground, and not as seen by Maureen.

**Problem 4:** A block of mass 1 kg is launched up an inclined plane of angle  $30^\circ$  at a speed of 10 m/s. If it slides a total of 5 m along the incline before coming to rest, calculate the work done by kinetic friction in slowing down the block.

### Solution

The only non-conservative force acting on the block is due to kinetic friction. We know that the change in mechanical energy of a system equals the work done by non-conservative forces. So, by calculating the change in mechanical energy, we can find the work done by kinetic friction.

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2} \times 1 \times 10^2 = 50 \text{ J}$$

$$KE_f = 0$$

$$\Delta KE = KE_f - KE_i = -50 \text{ J}$$

$$\Delta PE = PE_f - PE_i = mg(h_f - h_i)$$

$$= 1 \times 10 \times (5 \sin 30^\circ) = 25 \text{ J}$$

$$\Delta ME = \Delta KE + \Delta PE = -50 + 25 = \mathbf{-25 \text{ J}}$$

This is the total work done by friction.

**Problem 5:** An angry bird of mass 0.5 kg is launched from the ground at an angle of  $60^\circ$  above the horizontal from a massless catapult (spring) by stretching it by 50 cm and then releasing it. If the bird reaches a maximum height of 50 m above the ground, calculate the effective spring constant of the catapult.

### Solution

In this problem, initially when the bird is loaded in the catapult, it has 0 kinetic energy and some elastic potential energy. When it is released, all of the elastic PE gets converted to KE. After release, as the bird goes up, this KE gradually gets converted to gravitational PE. So, you might think the solution will involve setting 'elastic PE when the bird is loaded' equal to the 'gravitational PE at the maximum height'. But the catch is that not all of the KE will get converted to gravitational PE, since at the max height the object still has a non-zero  $x$ -component of motion.

A way of solving the problem is to find the speed of launch using kinematics and then using conservation of energy just before and after the launch to find the spring constant.

#### Projectile motion

$$\begin{aligned} v_{f,y}^2 - v_{i,y}^2 &= -2g\Delta y \\ \Rightarrow v_{i,y}^2 &= 2g\Delta y = 2 \times 10 \times 50 = 1000 \text{ m}^2/\text{s}^2 \end{aligned}$$

which gives  $v_{i,y} = 31.62$  m/s. Now using  $v_{i,y} = v_i \sin \theta$ , we can calculate  $v_i$  to be 36.61 m/s.

#### Catapult release

This initial velocity for projectile motion is the 'final' velocity of the bird immediately after releasing the catapult. Conserving energy before and after release, we get

$$\frac{1}{2}k(\Delta L)^2 = \frac{1}{2}mv^2$$

Plugging in the numbers, we get  $k = 2666.67$  N/m.

Note that the projectile motion part can be solved using conservation of energy too, if you want. The math will go as

$$\begin{aligned} \text{KE}_{\text{bottom}} &= \text{KE}_{\text{top}} + mgh \\ \Rightarrow \frac{1}{2}mv_i^2 &= \frac{1}{2}mv_f^2 + mgh \\ \Rightarrow \frac{1}{2}m(v_{i,x}^2 + v_{i,y}^2) &= \frac{1}{2}m(v_{f,x}^2 + v_{f,y}^2) + mgh \end{aligned}$$

Here I've used Pythagorean theorem to write  $v^2$  in terms of its components. Using  $v_{i,x} = v_{f,x}$  and  $v_{f,y} = 0$ , we get

$$\frac{1}{2}mv_{i,y}^2 = mgh$$

Multiplying throughout by 2 and dividing by  $m$ , we get

$$v_{i,y}^2 = 2gh$$

which is what we have from kinematics as well. Also, because we used the  $x$  and  $y$  component of velocity in the expression for KE, don't mistake KE for a vector, or  $\frac{1}{2}mv_x^2$  for the  $x$ -component of KE.

**Problem 6:** At what height *above the Earth's surface* is the acceleration due to Earth's gravitational field exactly 75 % the acceleration felt at the surface? Use the following value for the Earth's radius:  $R_E = 6.37 \times 10^6$  m.

### Solution

The expression for acceleration due to Earth's gravitational field, aka, gravitational field strength is

$$g(R) = \frac{GM_E}{R^2}$$

where  $G$  is the gravitational constant,  $M_E$  is the mass of Earth and  $R$  is the distance from earth's center at which  $g$  is measured.

At the surface of Earth,  $R$  is the radius of Earth  $R_E$ . At height  $h$  above the Earth's surface,  $R = R_E + h$ .

**This is important, do not set  $R$  to  $h$ .**

The information given in the problem translates to

$$\begin{aligned} \frac{g(R_E + h)}{g(R_E)} &= \frac{75}{100} \\ \Rightarrow \frac{GM_E}{(R_E + h)^2} \times \frac{R_E^2}{GM_E} &= \frac{75}{100} \\ \Rightarrow \frac{R_E^2}{(R_E + h)^2} &= \frac{75}{100} \\ \Rightarrow \frac{R_E}{R_E + h} &= \sqrt{0.75} \end{aligned}$$

Solving this we get

$$h = \frac{1 - \sqrt{0.75}}{\sqrt{0.75}} R_E = \mathbf{985000 \text{ m}}$$

So, at a height of **985 km** above the surface, the gravitational field strength will be 75 % of the value at the surface.

Note that we didn't need the value of  $G$  and  $M_E$  to solve this problem. A lot of physics of planets can be done without these values, since they can be canceled out in calculations. **This (cancelling variables) is a major theme of physics 1 problems from this topic (gravity).**

**Problem 7:** A ceiling fan starts rotating from rest. Its angular velocity increases at a constant rate from 0 rad/s to some final value in 10 s. If, during the speeding-up process, the fan makes 10 full rotations, what is the final angular speed of the fan?

**Solution**

Kinematics of circular motion is very similar to the kinematics of linear motion, with equations in one having analogous equations in the other.

$$\omega_i = 0$$

$$\Delta t = 10 \text{ s}$$

$$\Delta\theta = 10 \times 2\pi \text{ rad}$$

$$\omega_f = ?$$

Using  $\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$ , we get  $\omega_f = 4\pi \text{ rad/s} = \mathbf{12.75 \text{ rad/s}}$ . The equation we've used is analogous to  $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ .